Nested-Fermi-liquid theory

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The susceptibility and quasiparticle self-energy are found to exhibit anomalous behavior in nested-Fermi-liquid (NFL) systems that have nearly parallel sections of the Fermi surface. Electron-electron scattering yields damping much stronger than the conventional electron-gas result and predicts a linear temperature variation of the resistivity. The susceptibility $\chi''_{\rm NFL}(\mathbf{q},\omega)$ for nested fermions is calculated at $\mathbf{q}\simeq\mathbf{Q}$, where $\mathbf{Q}$ is a typical nesting wave vector. The NFL susceptibility is linear in frequency up to a crossover region near $\omega\simeq 4T$ where a saturation to a constant value occurs. The above features, as well as various theoretical constraints, are highly sensitive to the strength of the electron-electron coupling and to the degree of nesting. The relevance of the NFL results to superconducting oxides is briefly examined, with emphasis on the resistivity and the photoemission data, which supports the calculated damping $\Gamma(\omega>T)\simeq\alpha\omega$ with an intermediate on-site Coulomb coupling.

I. INTRODUCTION

A conventional Fermi liquid (FL) is characterized by electrons with a long lifetime near the Fermi energy, in part because of the limited phase space for electron-electron scattering. Thus the electron processes yield a resistivity $\rho=AT^2$, where the constant A is small and barely detectable in ordinary metals.

Puzzling deviations from FL behavior have been observed in high-temperature superconducting oxides, even though the photoemission data demonstrate the existence of a well-defined Fermi surface. For example, the cuprates exhibit large resistivities with a linear temperature variation down to 10 K in some cases, and anomalous electronic features are seen in photoemission spectra, as well as in other experiments.

Motivated by the above anomalies, and the two-dimensional electronic structure of the Cu-O superconductors, we analyze the response of a system of electrons or holes whose Fermi surface allows "nesting" in a substantial region of momentum space. The susceptibility $\chi(\mathbf{q},\omega)$ and the quasiparticle self-energy $\Sigma(\mathbf{k},\omega)$ are calculated including the strong electron-electron scattering terms corresponding to a typical nesting momentum $\mathbf{Q}$. The resulting analytic structure for χ and Σ are quite different from the conventional FL, and the novel energy dependence has a direct impact on the quasiparticle response to external probes.

We begin with a single-quasiparticle energy $E(\mathbf{k})$ and the Hamiltonian

$$H=\sum_{\mathbf{k}\sigma}E(\mathbf{k})c^\dagger_{k\sigma}c_{k\sigma}+U\sum_i n_{i\uparrow}n_{i\downarrow}\ , \qquad (1)$$

where $U>0$ denotes the on-site Coulomb repulsion, $c^\dagger_{k\sigma}(c_{k\sigma})$ are the creation (destruction) operators for an itinerant electron or hole within a band $E(\mathbf{k})$ of width W. Measuring energies $\varepsilon(\mathbf{k})=E(\mathbf{k})-\mu$ relative to the chemical potential μ, we consider the Green's function G given by

$$G^{-1}(\mathbf{k},i\omega)=i\omega-\varepsilon(\mathbf{k})-\Sigma(\mathbf{k},i\omega)\ , \qquad (2)$$

where the self-energy caused by electron-electron scattering is $\Sigma=\Sigma'+\Sigma''$. The imaginary part is[1]

$$\Sigma''(\mathbf{k},\omega)=g^2\frac{1}{(2\pi)^4}\int d\omega' F(\omega')\int d^3q\,\chi''(\mathbf{q},\omega')\times{\rm Im}G(\mathbf{k}+\mathbf{q},\omega-\omega')\ , \qquad (3)$$

where the coupling is $g=UV_c$ with V_c the cell volume, and

$$F(\omega')=\coth\left[\frac{\omega'}{2T}\right]-\tanh\left[\frac{\omega'-\omega}{2T}\right]\ . \qquad (4)$$

The imaginary part of the susceptibility is

$$\chi''(\mathbf{q},\omega)=\frac{1}{(2\pi)^4}\int d\omega' M(\omega')\int d^3k\,{\rm Im}G(\mathbf{k},\omega'+\omega/2)\times{\rm Im}G(\mathbf{k}-\mathbf{q},\omega'-\omega/2)\ , \qquad (5)$$

where

$$M(\omega')=\tanh\left[\frac{\omega'+\omega/2}{2T}\right]-\tanh\left[\frac{\omega'-\omega/2}{2T}\right]\ . \qquad (6)$$

Within the Born approximation the Green's function $G(\mathbf{k},i\omega)$ is replaced by the unperturbed value

$$G^{(0)}=\frac{1}{i\omega-\varepsilon(\mathbf{k})}\ . \qquad (7)$$

Then the corresponding self-energy diagram for the lowest-order damping is of the form shown in Fig. 1.

In the analysis of the self-energy in Eq. (3), we consider the case that the temperature T, the frequency ω, the quasiparticle energy $\varepsilon(\mathbf{k})$ and the damping $\Gamma=-\Sigma''$ are all small in comparison to the bandwidth W. Moreover,

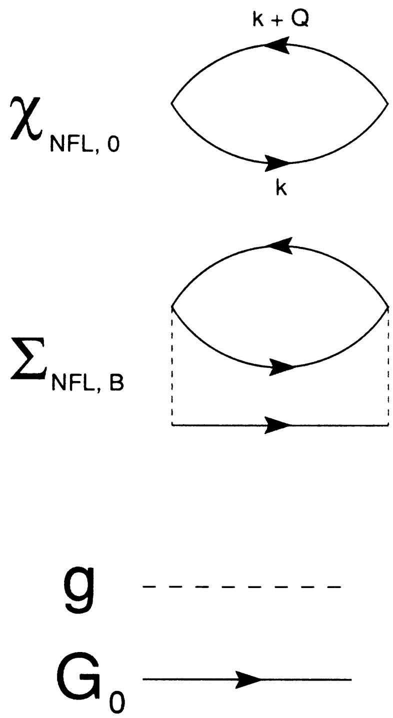

FIG. 1. Diagrams for the quasiparticle self-energy in the Born approximation. A solid line represents the unperturbed Green's function $G^{(0)}$, the dotted line is the on-site Coulomb coupling g, and the susceptibility in lowest order is shown by the electron-hole "bubble."

we assume that the quasiparticle density of states $N(\varepsilon)$ can be approximated by a constant $N(0)$ in the vicinity of the Fermi energy, such that

$$\tilde{N}(\omega)=-\frac{1}{8\pi^4}\int d^3k\,\mathrm{Im}G(\mathbf{k},\omega)$$
$$=-\frac{1}{\pi}\int d\varepsilon N(\varepsilon)\frac{\Sigma''}{(\omega-\Sigma'-\varepsilon)^2+(\Sigma'')^2}\simeq N(0)\ , \quad (8)$$

and $N(0)\simeq 1/WV_c$. These approximations allow an analytic solution of the higher-order self-consistent equations which become important as the dimensionless coupling $\bar{g}=gN(0)=U/W$ approaches unity. A constant $N(0)$ approximation is familiar in the original derivation of the Fermi liquid lifetime.[2]

A one-dimensional electron gas is exceptional because momentum conservation in the scattering process determines the energy of the scattering electron in terms of the allowed states of the target electron. Luttinger realized that this dimensionality constraint yields a quasiparticle damping linear in frequency ω. By comparison, his classic proof of the quadratic variation of the damping in three dimensional systems relies on treating the energies of the scattering electrons as independent variables.[2]

The case of an ordinary electron model is summarized in Sec. II, and the nested region analysis is presented in Sec. III with emphasis on the self-energy and susceptibility. Renormalization of the effective mass is considered in Sec. IV, and the theoretical constraints are in Sec. V. Conclusion of our analysis in Sec. VI emphasize the necessary ingredients for dominant electron-electron scattering of the NFL form.

II. CONVENTIONAL FERMI LIQUID

A conventional electron gas in three dimensions yields the following Fermi-liquid (FL) behavior.[2,3] The susceptibility $\chi''_{\mathrm{FL}}(\mathbf{q},\omega')$, in general, does not correlate with $\mathrm{Im}G(\mathbf{k}+\mathbf{q},\omega-\omega')$ with respect to the variable $\mathbf{q}$, and therefore the product $\chi''_{\mathrm{FL}}\ \mathrm{Im}G$ in Eq. (3) can be factorized using a susceptibility averaged over momentum $\mathbf{q}$. The appropriate averaging procedure in Eq. (5) then yields

$$\langle\chi''(\mathbf{q},\omega)\rangle_{\mathbf{q}}\simeq\frac{\pi N(0)\omega}{W}\ , \quad (9)$$

and a resulting quasiparticle damping

$$\Gamma_{\mathrm{FL}}(\omega)=\frac{\pi\bar{g}^2}{2W}(\pi^2T^2+\omega^2)\ . \quad (10)$$

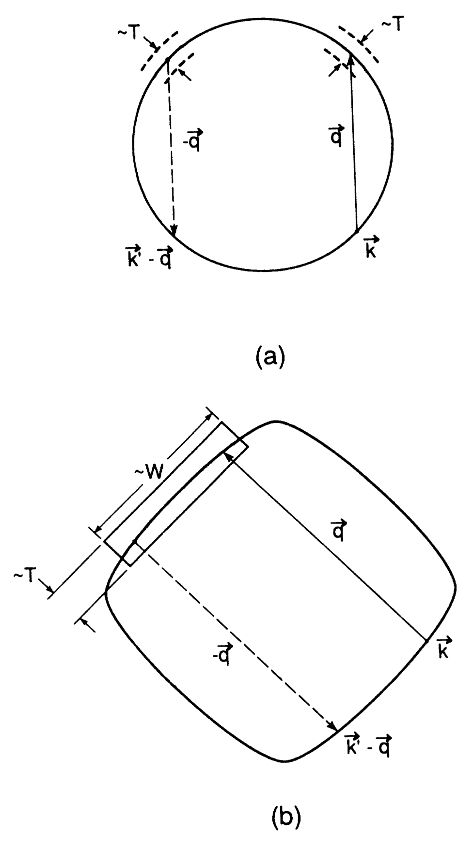

FIG. 2. Phase-space restrictions for the electron-electron cross section at finite temperature T in (a) conventional electron gas, and (b) a Fermi liquid with nesting wave vector $\mathbf{Q}$. The incoming quasiparticle momenta are $\mathbf{k}$ and $\mathbf{k}'$, with a scattering momentum transfer $\mathbf{q}$. Shaded regions are allowed by the Pauli exclusion principle. The nesting broadens the range of momenta available to the scattering electron to a width proportional to W in comparison to the free particle width proportional to T. In the NFL case $\mathbf{q}\simeq\mathbf{Q}$. The solid curves represent the Fermi surface.

This conventional Fermi-liquid damping is quite small in ordinary metals because the bandwidth is much larger than the typical temperatures of interest. Physically the variation $\Gamma_{FL} \sim T^2$ follows from the Pauli exclusion requirement that limits the scattering electron energies within an interval $\sim k_B T$ of the Fermi energy. As shown in Fig. 2(a), the limited phase space for the scattered electron and the other electron that does the scattering results in a cross section that is proportional to $(T/\mu)^2 \sim (T/W)^2$.

Exact calculations[4,5] for electrons with an effective-mass model in two dimensions yield

$$\Gamma_{FL} \propto \left[\frac{T}{\mu}\right]^2 \ln\left[\frac{\mu}{T}\right] , \tag{11}$$

which exhibits a small derivation from the T^2 dependence. Evidence for the FL behavior has been found in resistivity measurements of superconducting $Nd_{1.85}Ce_{0.15}CuO_4$ films.[6]

However, most of the high-temperature cuprate superconductors exhibit a resistivity which is linear in temperature. The observed linearity extends to as low as 10 K in some oxides.[7] Such a behavior can originate from Fermi surface nesting, as we demonstrate below.

III. NESTED FERMI LIQUID

In the case of quasiparticles on a Fermi surface that exhibits nesting, i.e., $\varepsilon(\mathbf{k})+\varepsilon(\mathbf{k}-\mathbf{Q})\simeq 0$, there is an extended region of phase space for the scattering events as illustrated in Fig. 2(b). This feature enhances the electron-electron cross section and also alters the constraints on the momentum integration which ultimately determine the frequency and temperature variation of electron (or hole) lifetime.

Formally, the influence of nesting may be seen in the correlation between the susceptibility and the Green's function that enter in the definition of the self-energy in Eq. (3). In fact, there is an extended momentum region near $\mathbf{q}\simeq\mathbf{Q}$ where a crest in $\chi''(\mathbf{q},\omega')$ coincides with large values of $\mathrm{Im}G(\mathbf{k}+\mathbf{q},\omega-\omega')$: An illustration of such a situation is shown in Fig. 3 for a "nested" Fermi surface reminiscent of a nearly half-filled tight-binding energy band in two dimensions. Under the nesting circumstances, we may derive the quasiparticle self-energy from Eq. (3) by replacing the susceptibility by $\chi''(\mathbf{Q},\omega')$ and then performing the standard integration over $\mathbf{q}$ for the Green's function to obtain

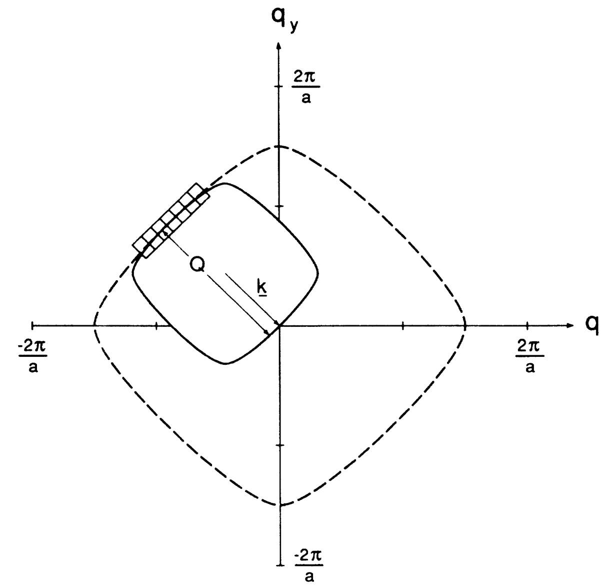

FIG. 3. The dominant regions of the momentum $\mathbf{q}$ integration which determine the quasiparticle self-energy in Eq. (3). For the NFL the function ImG is peaked at the Fermi surface shown by the solid curve, while the susceptibility χ'' is enhanced on the dashed curve as a result of nesting at wave vectors $\mathbf{q}\simeq\mathbf{Q}$. The shaded overlap region yields the NFL damping.

$$\Gamma_{NFL}(\omega)=\tfrac{1}{2}g^2N(0)\int d\omega' F(\omega')\chi''(\mathbf{Q},\omega') . \tag{12}$$

The magnitude of Γ_{NFL} will be considerably larger than Γ_{FL}, and the temperature variation of $\Gamma_{NFL}(T)$ turns out to be unorthodox as we show below.

A self-consistent set of equations relating to the self-energy and susceptibility is derived by using the Green's function including the self-energy, as indicated in Fig. 4. Thus the susceptibility at the nesting wave vector $\mathbf{Q}$ follows from the condition $\varepsilon(\mathbf{k})+\varepsilon(\mathbf{k}-\mathbf{Q})=0$ in Eq. (5), which yields

$$\chi''_{NFL}(\mathbf{Q},\omega)=\frac{1}{2\pi}\int d\omega' M(\omega')\int d\varepsilon\, N(\varepsilon)\frac{\Gamma_+}{[(\omega-\Sigma')_+-\varepsilon]^2+\Gamma_+^2}\frac{\Gamma_-}{[(\omega-\Sigma')_-+\varepsilon]^2+\Gamma_-^2} , \tag{13}$$

where the subscript $\pm$ denotes the argument $(\omega'\pm\omega/2)$. Performing the ε integration, we find

$$\chi''_{NFL}(\mathbf{Q},\omega)=\tfrac{1}{2}N(0)\int d\omega' M(\omega')\frac{\Gamma_+ + \Gamma_-}{[(\omega-\Sigma')_+ + (\omega-\Sigma')_-]^2+(\Gamma_+ + \Gamma_-)^2} . \tag{14}$$

Neglecting, for the moment, the mass renormalization, i.e., assuming $\Sigma'\simeq 0$, Eqs. (12) and (14) form a closed system for $\chi''_{NFL}(\mathbf{Q},\omega)$ and $\Gamma_{NFL}(\omega)$ which is amenable to a self-consistent solution.

A. Weak coupling

The physical origin of the strong anomalous damping can be traced to the enhanced susceptibility $\chi''_{NFL}(\mathbf{Q},\omega)$

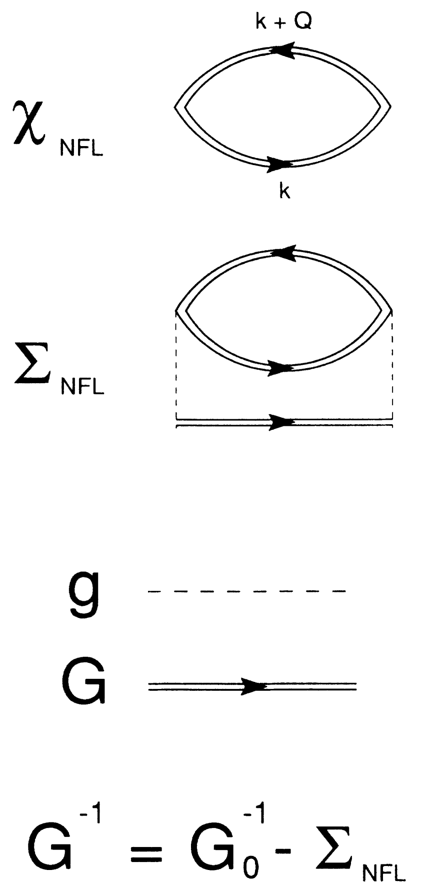

FIG. 4. Quasiparticle self-energy Σ_{NFL} and susceptibility χ_{NFL} in the self-consistent treatment that is needed for intermediate coupling $\bar{g}=U/W\sim 1$ in the event of substantial nesting of the Fermi surface. The double lines refer to the dressed Green's function G which includes the self-energy Σ_{NFL}.

even in the weak-coupling limit $\bar{g}\rightarrow 0$. In this limit Γ_+ and Γ_- tend to vanish and the lowest-order response function becomes

$$\chi''_{\mathrm{NFL},0}(\mathbf{Q},\omega)\simeq\frac{\pi}{2}N(0)\int d\omega' M(\omega')\delta(2\omega') , \qquad (15)$$

which becomes

$$\chi''_{\mathrm{NFL},0}(\mathbf{Q},\omega)\simeq\frac{\pi N(0)}{2}\tanh\left[\frac{\omega}{4T}\right] . \qquad (16)$$

This noninteracting quasiparticle susceptibility is much larger than the conventional FL average, and it exhibits a crossover at $\omega\simeq 4T$ as shown in Fig. 5. Hence it is reasonable to expect the temperature and frequency variation of $\chi''_{\mathrm{NFL},0}(\mathbf{Q},\omega)$ to influence the self-energy of Eq. (12) in a substantial way.

The Born approximation (as in Fig. 1) should suffice for small $\bar{g}\ll 1$, and it yields the self energy from Eqs. (12) and (16)

$$\Gamma_{\mathrm{NFL},B}\simeq\frac{\pi\bar{g}^2}{4}\int d\omega' F(\omega')\tanh\left[\frac{\omega'}{4T}\right] . \qquad (17)$$

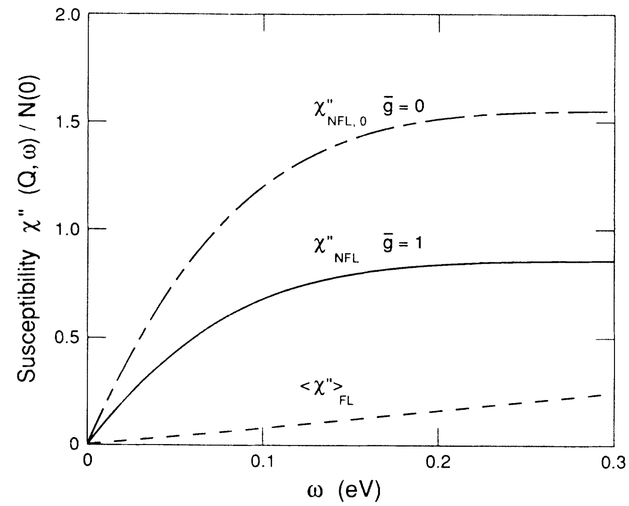

FIG. 5. Susceptibility as a function of frequency at $T=290$ K. The dotted curve is the conventional Fermi-liquid (FL) average for $W=4$ eV. Nested regions of the Fermi surface yield the NFL contributions in the Born approximation (dashed curve) and in the higher-order self-consistent analysis indicated by the solid curve with $\bar{g}=1$.

The asymptotic forms of the quasiparticle damping follow from Eq. (17), and already show anomalous variations, i.e.,

$$\Gamma_{\mathrm{NFL},B}(|\omega|\ll T)\simeq\frac{\pi^3}{16}\bar{g}^2 T \qquad (18)$$

and

$$\Gamma_{\mathrm{NFL},B}(|\omega|\gg T)\simeq\frac{\bar{g}^2\pi}{2}|\omega| . \qquad (19)$$

The static case provides an explanation for the linear T variation of the resistivity, and the evidence for the linear frequency variation of the damping has just been discovered in $Bi_2Sr_2CaCu_2O_8$ by photoemission spectroscopy.[8] However, the magnitudes of the damping suggested by the transport and optical data require at least intermediate values of the on site Coulomb coupling $\bar{g}$, and therefore a self-consistent analysis of the self-energy and susceptibility is considered in the next section.

Alternate models of the density of states would be interesting to consider in view of the present nesting analysis. Another example of a two-dimensional tight-binding band, with $N(\omega)\sim\ln\omega$, was shown[9] earlier to produce a linear T variation of the electron-electron scattering rate as a consequence of nesting. Further calculations[10] on the latter model reveal a sensitivity of the scattering to electron or hole doping which shifts the Fermi energy. Nevertheless, even the partially nested Fermi surface in the two-dimensional (2D) band is found to yield a linear T dependence of the self-energy. Also, the nearly half-filled tight-binding band yields a real part of $\chi'(\mathbf{Q},\omega)$ which is frequency dependent: Hence the Born approximation is inadequate[10] close to half-filling for two reasons. First of all there is an instability[11,12] toward the formation of an insulating SDW state, and the higher-order ladder series of diagrams that generate the phase transition have also been shown[10] to suppress the electron (or hole) mobility. In addition, as the coupling increases, the "dressing" of quasiparticle propagators by large self-energy corrections becomes important, and that relevant correction is considered next.

At some point of diminishing nesting, the net scattering rate should revert to an ordinary FL behavior. A quantitative estimate of the nesting required for the anomalous NFL behavior is presented in Sec. V.

B. Intermediate coupling

Our initial weak-coupling basis tends to break down if the combined effect of the electron-electron coupling and the extent of Fermi surface nesting exceed certain stability requirements examined directly in Sec. V. By intermediate coupling for a highly nested topology, we refer to the crude estimate $\bar{g} \leq 1$.

Since the quasiparticle damping $\Gamma_{\rm NFL}$ is substantial even in the Born approximation results of Eqs. (18) and (19), we need to include the self-energy corrections in the Green's function $G(\mathbf{k},i\omega)$ that determines the susceptibility $\chi''_{\rm NFL}(\mathbf{Q},\omega)$ and simultaneously use the "dressed" susceptibility to calculate the self-energy. A graphical representation of these contributions is shown in Fig. 4.

We have been able to deduce a solution for the damping and susceptibility in Eqs. (12) and (14) by considering the asymptotic behavior of the self-consistent equations. We find that

$$\Gamma_{\rm NFL}(\omega)=\alpha\max(\beta T,|\omega|)\ , \tag{20}$$

is a valid solution, with the constants α and β determined by the coupling $\bar{g}$, where β is of the order of unity.

Considering first the quasistatic limit $\omega \ll T$, the function $M(\omega')$ in Eq. (14) reduces to the derivative of the Fermi function: $M(\omega')\simeq -2\omega f'(\omega')$. Then the sharp cutoff for $\omega' \geq T$ allows us to use $\Gamma_+ + \Gamma_- = 2\Gamma_{\rm NFL}(\omega=0)=2\alpha\beta T$, which leads to

$$\chi''_{\rm NFL}(\mathbf{Q},\omega\ll T)=\frac{N(0)\omega}{4\pi T}\psi'\left[\frac{1}{2}+\frac{\alpha\beta}{2\pi}\right]\ , \tag{21}$$

where ψ' is the derivative of the diagamma function. Equation (21) reduces to $\pi N(0)\omega/8T$ for $\alpha \ll 1$, and to $N(0)\omega/2\alpha\beta T$ for $\alpha \gg 1$.

At high frequencies $\omega \gg T$, the $M(\omega')$ function in Eq. (14) simplifies to give

$$\chi''_{\rm NFL}(\mathbf{Q},\omega\gg T)=N(0)\int_{-\omega/2}^{\omega/2}d\omega'\frac{\Gamma_+ + \Gamma_-}{(2\omega')^2+(\Gamma_+ + \Gamma_-)^2}\ . \tag{22}$$

In this case

$$\Gamma_+ + \Gamma_- \simeq \alpha[|\omega'+\omega/2|+|\omega'-\omega/2|]=\alpha|\omega|\ ,$$

and we obtain

$$\chi''_{\rm NFL}(\mathbf{Q},\omega\gg T)=N(0)\tan^{-1}(1/\alpha){\rm sgn}(\omega)\ . \tag{23}$$

In essence, Eqs. (21) and (23) suggest that $\chi''(\mathbf{Q},\omega)$ is linear in ω for frequencies smaller than the temperature and then saturates to a constant value for higher frequencies. Clearly the following interpolation exhibits the proper asymptotic behaviors,

$$\chi''_{\rm NFL}(\mathbf{Q},\omega)=N(0)\tan^{-1}(1/\alpha)\tanh\left[\frac{\omega}{\gamma T}\right]\ , \tag{24}$$

where γ ranges from four in the weak-coupling ($\alpha \ll 1$) limit to 2β in the strong coupling ($\alpha \gg 1$) regime.

Now we use these asymptotic expressions for the susceptibility to determine a self-consistent solution for the damping $\Gamma(\omega)$ in Eq. (12). This method needs to validate the trial solution of Eq. (20) and fix the values of α and β. For $\omega \ll T$,

$$\Gamma_{\rm NFL}(\omega\ll T)=g^2N(0)\int\frac{\chi''(\mathbf{Q},\omega')}{\sinh(\omega'/T)}d\omega' \tag{25}$$

is sensitive only to the low frequency (linear ω) behavior of the susceptibility, and yields

$$\Gamma_{\rm NFL}(\omega\ll T)=\frac{\pi^2\bar{g}^2}{2\gamma}\tan^{-1}(1/\alpha)T\ . \tag{26}$$

By contrast, the high frequency limit $\omega \gg T$ reduces to

$$\Gamma_{\rm NFL}(\omega\gg T)=g^2N(0)\int_0^{\omega}\chi''(\mathbf{Q},\omega')d\omega'\ . \tag{27}$$

Here the high frequency (constant) behavior of the susceptibility dominates to give

$$\Gamma_{\rm NFL}(\omega\gg T)=\bar{g}^2\tan^{-1}(1/\alpha)|\omega|\ . \tag{28}$$

Clearly the linear temperature and frequency dependence of $\Gamma_{\rm NFL}$ in Eqs. (26) and (28) is consistent with the starting form in Eq. (20). Compatibility of the coefficients requires

$$\alpha=\bar{g}^2\tan^{-1}(1/\alpha)\ , \tag{29}$$

and $\beta=\pi^2/8$ for $\alpha \ll 1$ whereas $\beta=\pi/2$ for $\alpha \gg 1$, which indicates that β is of order unity in any event. Correspondingly, the constant γ in Eq. (24) satisfies the self-consistency condition with a value 4 (if $\alpha \ll 1$) and π (in the strong-coupling $\alpha \gg 1$ limit). Only α depends strongly on the coupling $\bar{g}$, in the weak-coupling case $\alpha \simeq \pi\bar{g}^2/2 \ll 1$, and in the other mathematical extreme $\alpha=\bar{g}\gg 1$. Nevertheless, the intermediate-coupling regime $\bar{g}\sim 1$ yields $\alpha \sim 1$ from Eq. (29).

The susceptibility of Eq. (24) is shown in Fig. 5, where its unusual frequency variation contrasts with the conventional FL result. The self-consistent susceptibility is closer to the Born approximation result of Eq. (16) although the slightly shifted crossover in the frequency variation and the coupling-dependent magnitude of $\chi''_{\rm NFL}(\mathbf{Q},\omega)$ in Eq. (24) reflect the influence of the self-energy corrections.

The quasiparticle damping is shown in Fig. 6 for a relatively strong value of the coupling $\bar{g}=1.0$ which corresponds to $\alpha=0.86$. For a typical metal bandwidth of $W=4$ eV, the NFL damping is much larger than the standard FL result. The magnitude of the damping extracted from the photoemission data is $\alpha \simeq 0.6$,[8] which is at the upper end of the intermediate-coupling range in

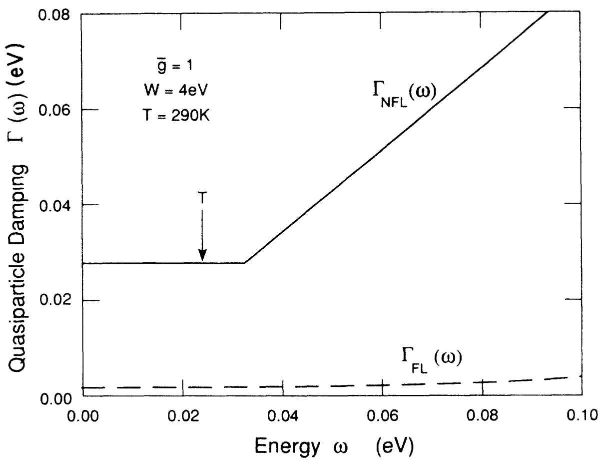

FIG. 6. Quasiparticle damping $\Gamma(\omega)$ caused by electron-electron scattering for coupling $\bar{g}=1$ at room temperature using a bandwidth $W=4$ eV. The conventional FL result shows small damping. Nesting yields the high damping Γ_{NFL} curve with a linear temperature dependence in the static limit and a linear frequency variation for $\omega \gtrsim T$.

our model of a fully nested Fermi surface.

Although quantitative estimates of the resistivity are hampered by uncertainties, a crude estimate gives

$$\rho \simeq 8\pi\Gamma/\omega_{pl}^2 \simeq 8\pi\alpha\beta T/\omega_{pl}^2 ,$$

where ω_{pl} is the plasma frequency. At room temperature, intermediate coupling of $\alpha=0.6$ and $\omega_{pl}\simeq 1$ eV gives $\rho \simeq 300\ \mu\Omega$ cm, which is in the neighborhood of measured values for single-crystal high-T_c superconductors.[13]

The long-wavelength susceptibility is also affected by strong quasiparticle damping of the NFL type and yields an unusual response to optical probes such as infrared reflectivity. We consider the impact of nesting on $\chi''(q\simeq 0,\omega)$ in a separate publication.[14]

The scattering in nested regions of the Fermi surface competes with the smaller conventional FL scattering of the remaining regions. From Fig. 6 it is reasonable to expect that the NFL and FL contributions will become comparable if the nested region is reduced to roughly 10% of the total Fermi surface. The extent of nesting also has a strong influence on the stability criterion which we examine in Sec. V.

IV. EFFECTIVE MASS AND QUASIPARTICLE STRENGTH

Previously, a phenomenological hypothesis has been proposed, which anticipated some of the spectral features derived here. Starting with a representation of the anomalous Raman scattering with an empirical function

$$\chi''(0,|\omega|<T) \propto N(0)\omega/T$$

and

$$\chi''(0,|\omega|>T) \propto N(0)\mathrm{sgn}\omega ,$$

and then presuming that $\chi''(\mathbf{q},\omega)$ is independent of $\mathbf{q}$, Varma *et al.*[15] calculated a self-energy Σ emanating from χ'' that is similar to our result in the case of weak coupling. Furthermore, they deduced the quasiparticle weight Z_k which is found to vanish logarithmically at the Fermi surface at zero temperature, thus naming the situation a "marginal" Fermi liquid. Also, they calculated the conductivity in connection with infrared data and developed a correlation between the susceptibility and NMR, specific heat, and thermal conductivity behaviors as well as the observed linear temperature variation of the resistivity.

Our NFL calculations reveal a substantial mass renormalization which follows from the real part of the self-energy $\Sigma'(\omega)$. Kramers-Kronig transform of Eq. (20) gives

$$\Sigma'_{NFL}(\omega) \simeq -\frac{2\alpha\omega}{\pi}\ln\frac{\omega_c}{\max(\beta T,|\omega|)} , \tag{30}$$

where the cutoff energy $\omega_c \gg \max(\beta T,|\omega|)$ is needed for convergence. The renormalization energy $\omega-\Sigma'(\omega)$ should be smaller than the bandwidth W even at the cutoff $\omega\simeq\omega_c$, which requires $\omega_c-\Sigma'(\omega_c)\simeq\omega_c+\alpha\omega_c\simeq W$. Hence the cutoff is at most

$$\omega_c \simeq \frac{W}{1+\alpha} . \tag{31}$$

This upper limit for $\max(\beta T,|\omega|)$ is more stringent than the damping condition $\Gamma \ll W$, which requires $\max(\beta T,|\omega|) \ll W/\alpha$.

The mass enhancement in a nested Fermi liquid exhibits a frequency dependence given by

$$m^*_{NFL} = m_0\left[1+\frac{2\alpha}{\pi}\ln\left[\frac{\omega_c}{\max(\beta T,|\omega|)}\right]\right] , \tag{32}$$

and this function is shown in Fig. 7. For electron-electron coupling at the high end of the intermediate

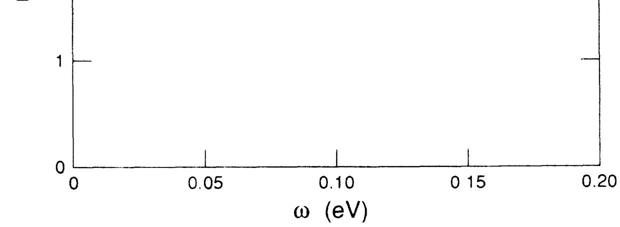

FIG. 7. Effective mass in the NFL analysis as a function of frequency for an electron-electron coupling $\alpha=0.6$ ($\bar{g}=0.76$) and a bandwidth $W=4$ eV. The temperature variation is indicated by the solid curve for 290 K and the dashed curve for 50 K.

scale, i.e., $\alpha \simeq 0.6$, $m^* \simeq 2-3m_0$. The frequency variation of m^* is weak, and therefore is unlikely to change the functional dependence of the damping and susceptibility. However, inclusion of the real part of the self-energy in the self-consistent analysis will produce corrections to the numerical coefficients α, β, and γ.

The self-energy allows the quasiparticle strength $Z(\omega)$ to be expressed as

$$Z_{\rm NFL}(\omega)=\frac{1}{1-\partial\Sigma'_{\rm NFL}/\partial\omega}=\frac{m_0}{m^*_{\rm NFL}(\omega,T)} . \quad (33)$$

The logarithmic temperature variation of Z and m^* are reminiscent of studies on ferromagnetic spin fluctuations.[16,17] There are also analogies in the $\ln T$ contributions to the self-energy of one dimensional systems, where higher-order Parquet diagrams have been shown to be important.[18]

It is also interesting to note that the restricted phase space of a one-dimensional electronic system yields a resistivity that is linear in temperature,[19] with a natural correspondence to the Luttinger[2] derivation of a linear ω variation of the damping.

The prospect of a vanishing strength $Z=0$ at $T=0$ is a key element in the phenomenological "marginal Fermi-liquid" theory.[15] At finite temperatures their functional form for $Z(T)$ is essentially identical to $Z_{\rm NFL}$ in Eq. (33), apart from the constants α and β which are determined by the electron-electron coupling in our analysis. However, Fermi surface nesting will not allow the limit $Z_{\rm NFL}=0$ to be reached as the temperature is lowered. If the electron (or hole) orbit deviates from perfect nesting, there will be a temperature T^* below which the damping reverts to ordinary FL behavior with a conventional finite quasiparticle strength $Z_{\rm FL}$ even at zero temperature. Estimates[14] of the crossover temperature T^* for a simple tight-binding band indicate that the crossover region may be observable in the oxide superconductors, and it is interesting to note that the resistivity $\rho(T)$ of $Nd_{2-x}Ce_xCuO_4$ single crystals has been found[20,21] to follow a T^2 variation up to $T^* \sim 100$ K, and then $\rho(T>150$ K) shows a linear T variation.

In the rare event of perfect nesting, the hypothetical $Z=0$ situation will be avoided because a phase transition is expected at a finite temperature. As the temperature is lowered for a partially nested Fermi surface, Z will remain finite as a result of (a) the crossover at T^* to ordinary FL behavior, or (b) a phase transition at a temperature which may be greater or less than T^*. The phase instability is examined in Sec. V.

V. REGION OF APPLICABILITY

A strong constraint on the NFL theory is imposed by the neglect of vertex corrections in the self-energy expression of Eq. (3). This approximation requires g $\chi'(\mathbf{Q},\omega=0) \ll 1$. Otherwise, as the temperature is lowered a phase transition[22] is expected to a SDW, CDW, or another ordered state determined by the details of the nesting and the quasiparticle interactions. In the ladder approximation for the multiple scattering of an electron-hole pair, the phase instability region is obtained from the real part of the susceptibility $\chi'(\mathbf{Q},\omega=0)$, which follows from a Kramers-Kronig transform of Eq. (24):

$$\chi'_{\rm NFL}(\mathbf{Q},\omega=0)=\frac{2N(0)}{\pi}\tan^{-1}\left[\frac{1}{\alpha}\right]\ln\left[\frac{4\gamma_E\omega_c}{\pi\gamma T}\right] , \quad (34)$$

where $\gamma_E=1.78$ is the Euler constant. Then the NFL region is bounded by $T>T_N$ with

$$T_N=\frac{4\gamma_E\omega_c}{\pi\gamma}\exp\left[\frac{-\pi\bar{g}}{2\alpha}\right] . \quad (35)$$

The limits imposed by Eqs. (31), (32), and (35) define a phase boundary which is presented in Fig. 8. In the case of complete nesting the vertex corrections become important for intermediate coupling $\bar{g}\sim 1$ even at temperatures $T\sim 0.1W$, which would restrict any realistic analysis to weak coupling. However, in typical cases nesting is achieved only over a partial fraction ν of the Fermi surface, and such a case extends the validity of the NFL calculation. Clearly the reduced nesting implies that the system is less unstable with respect to the low-temperature phase transition, thereby increasing the NFL region to a wider temperature range. We have estimated the effect of this reduced nesting by scaling down the momentum integrals by ν, in comparison to the previous equations which presume complete nesting. Consequently $\chi(\mathbf{Q},\omega)$ is reduced by a factor ν, and $\bar{g}$ is replaced by $\nu\bar{g}$ in Eqs. (29) and (35) to make the dark shading region in Fig. 8 for $\nu=\frac{1}{4}$. Therefore the suppression of the low-temperature vertex instability by imperfect nesting

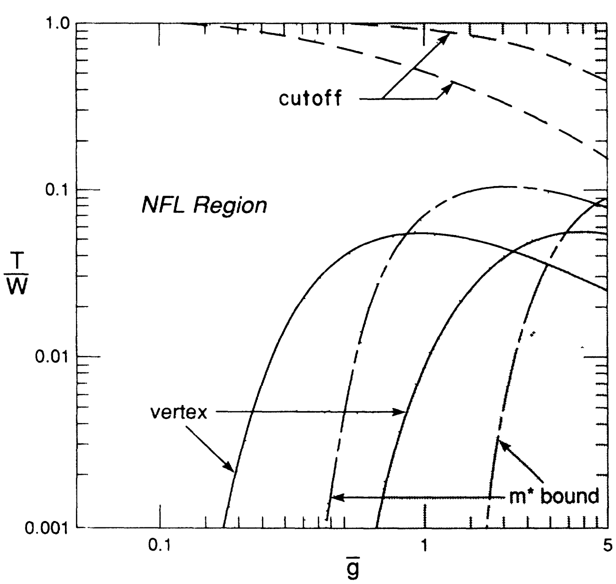

FIG. 8. Phase diagram for constraints on the NFL theory. The dashed curve depicts the energy cutoff requirement, the dot-dashed curve shows the bound for weak mass renormalization m^*, and the solid curve yields the phase transition instability region. Forbidden regions are shaded: Sandy shading illustrates the fully nested case of $\nu=1$, whereas the darkly shaded regions correspond to a more realistic $\nu=0.25$ fraction of the Fermi surface which exhibits nesting.

extends the NFL region by almost two decades in temperature, even for intermediate coupling. As the temperature is lowered further, the vertex corrections become important in the shaded region and a phase transition is expected with a clear need to consider higher-order corrections to the theory.

An example of variable nesting may be found in a tight-binding band as for example the two-dimensional Fermi surface shown in Fig. 2(b). Maximum nesting occurs exactly for a half-filled band which is unstable in any event. A shift in the Fermi energy caused by doping with electrons or holes will reduce the nesting fraction and reduce the region of phase instability. Thus the NFL theoretical development is more relevant to significantly doped electronic structures, although extreme doping would of course eliminate the nesting and restore the conventional Fermi-liquid behavior to the entire Fermi surface. From our estimates of the damping the NFL contribution is comparable to the conventional scattering when nesting occurs in less than 10% of the Fermi surface.

VI. CONCLUSIONS

A nested region of Fermi surface is shown to yield an electron-electron scattering rate that is linear in temperature at low frequencies and then becomes linear in frequency from $\omega \gtrsim T$. The NFL damping is typically an order of magnitude larger than the conventional Fermi-liquid result that a quasiparticle will experience in non-nested regions.

Physically the expanded phase space for allowed scattering in nested regions yields the anomalous damping even in the Born approximation. Estimates of the magnitude of the scattering provided by resistivity and photoemission data point to intermediate on-site Coulomb coupling U comparable to the bandwidth of the itinerant charge carriers, although uncertainties arise in the high-temperature superconductors whose Fermi surface topology has not yet been determined. A nested region comprising more than 10% of the total Fermiesurface orbit should allow the NFL scattering to dominate over other ordinary contributions.

Nesting introduces analytic structure in the susceptibility that is relatively model independent at low frequencies as seen in an elegant proof shown to us by I. E. Dzyaloshinski. On the basis of the fluctuation-dissipation theorem the mean-square fluctuation component $\langle \rho_q^\dagger \rho_q \rangle_\omega$ is related to the susceptibility $\chi''(\mathbf{q},\omega)$. If the fluctuations at wavevector Q are quasistatic up to a cutoff frequency ω_c, then the fluctuation dissipation relation yields a susceptibility whose frequency variation is qualitatively similar to our NFL result. This view also elucidates the correspondence of our NFL results for the self-energy with similar behavior in one-dimensional systems[2,19] which are also a likely source of quasistatic charge and spin fluctuations.

Deviations from perfectly parallel sections of the Fermi surface will yield a corresponding crossover temperature T^* below which ordinary Fermi-liquid behavior should set in, thus assuring a finite quasiparticle strength Z_{FL} at zero temperature. This scenario may explain the recent discovery[6,20,21] of conventional T^2 resistivity contributions at low temperatures in $Nd_{2-x}Ce_xCuO_4$, and a crossover to $\rho(T) \sim T$ above 150 K in single-crystal samples.[20,21] The nesting mechanism is sensitive to delicate changes in structure and chemical composition which may be related to the observed variations in the high-temperature behavior of the resistivity in films[6] as well as in crystals.

Theoretically, intermediate coupling requires the consideration of higher-order corrections to the self-energy and susceptibility which we have included by a self-consistent method. The basic features of the weak-coupling results prevail, although there are corrections to the numerical coefficients at stronger coupling.

Nesting of the Fermi surface is amenable to changes induced by shifting the Fermi energy with suitable chemical substitutions. Thus the case of a nearly half-filled tight-binding band of the form expected in copper oxide superconductors provides an ideal testing ground for the predictions of our theory.

Our simple electronic structure model with a constant density of states was chosen to yield analytic solutions for the damping and susceptibility over a wide frequency range. It would be worthwhile to extend the basic analysis to sophisticated cases such as the pseudogap structure invoked in spin-bag calculations,[23] or the electronic spectrum derived from $1/N$ expansions.[24]

The validity criteria for the theory as a function of temperature and electron coupling suggest that vertex corrections may be particularly important at lower temperatures near a phase transition instability. The nature of the phase transition requires further study, especially in view of the variety of SDW, CDW, and other phase transitions that often accompany situations with considerable Fermi surface nesting.

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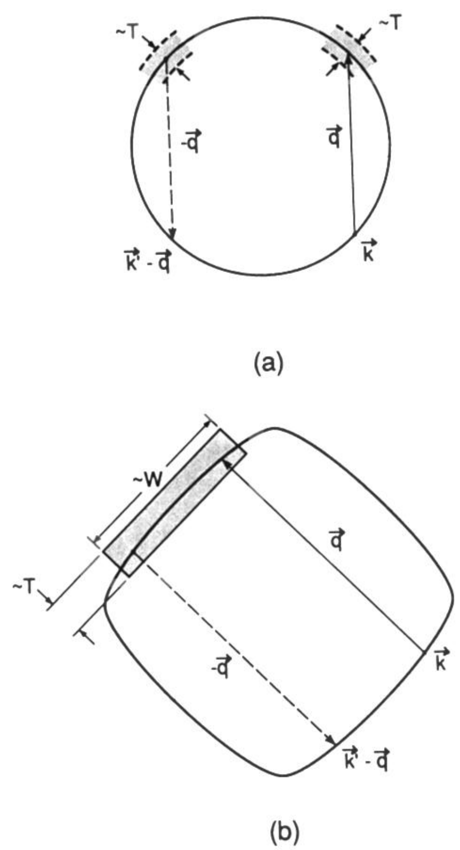
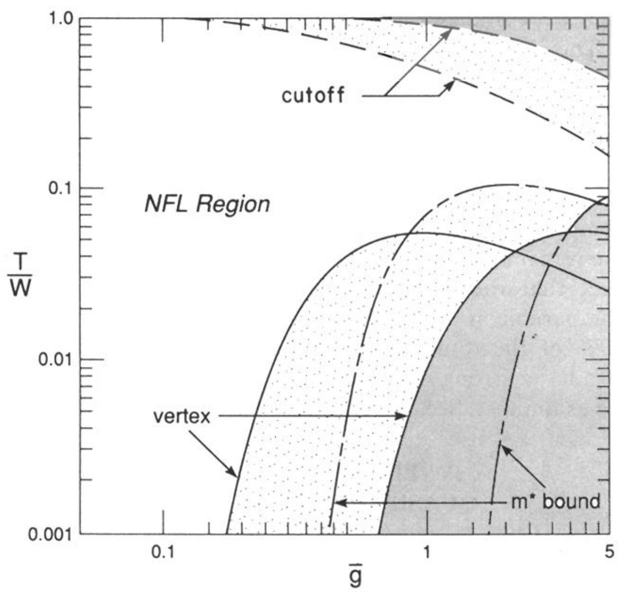


FIG. 2. Phase-space restrictions for the electron-electron cross section at finite temperature T in (a) conventional electron gas, and (b) a Fermi liquid with nesting wave vector $\mathbf{Q}$. The incoming quasiparticle momenta are $\mathbf{k}$ and $\mathbf{k}'$, with a scattering momentum transfer $\mathbf{q}$. Shaded regions are allowed by the Pauli exclusion principle. The nesting broadens the range of momenta available to the scattering electron to a width proportional to W in comparison to the free particle width proportional to T. In the NFL case $\mathbf{q}\simeq\mathbf{Q}$. The solid curves represent the Fermi surface.



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