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Proximity effect in zero field with the Landau-Ginzburg equation. II

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Based on the results of the preceding paper, we find that for a film sandwich composed of a superconductor with higher transition temperature T_S and a superconductor with lower transition temperature T_N (S-S lamina), the transition temperature is determined by the following equation: $B_0 d_N \xi_S / d_S(1-T_{cSS}/T_S) - d_S = B_1 d_N/\sqrt{T_{cSS}/T_N - 1}$, which can be converted into an equation of third order in T_{cSS}. Numerical calculations indicate that this model fits experimental results very well, and a new general treatment of the proximity effect can be developed.

I. INTRODUCTION

The transition temperature of S-S laminae, together with that of S-N laminae, has been studied since the 1960's. A substantial experimental and theoretical literature has accumulated (Ref. 1–11). The successful (to some extent) theories of de Gennes–Werthamer[6,7] Moormann,[4] and Abrikosov-Gor'kov[9] etc. are based on Gor'kov's Green's-function treatment of superconductivity. Recent reviews of the theoretical situation have been given by Lechevet[8] and Gilabert.[10]

In this paper we present a new solution to this problem that is directly obtained from the first Landau-Ginzburg (LG) equation.

In the preceding paper we found a simple, explicit expression for the transition temperature of a S-N lamina by solving the complete LG equation. These results are invalid for the S-S lamina, although they can be used as a basis for treating this problem. For a S-S lamina, further development using the LG equation is necessary. On the side of the high-T_c superconductor, the treatment is similar to that used in the preceding paper.

On the side of the low-T_c superconductor the treatment must be slightly modified. It is possible to develop a theory to unify the solutions for S-N laminae and S-S laminae.

II. THEORETICAL MODEL

For convenience we discuss the case of the pure limit in the following. It is obvious that the results are valid for the dirty limit. Following convention, we will call the superconductor with lower T_c the "normal" component of the lamina, and label associated quantities with the subscript N, and call the superconductor with higher T_c the "superconducting" component, and label associated quantities with S.

Consider the geometry of Fig. 1. The quantities d_S and d_N are the thicknesses of the superconductors on both sides, respectively. Similar to the model discussed in the preceding paper, it is a one-dimensional problem. On the superconducting side, the LG equation and its solutions are the same as Eq. (3), Eq. (6a), and Eq. (6b) in the preceding paper. At the boundary ($y=0$), we have

$$\Psi_S/\Psi'_S|_{y=0} = C_5/Y^2c + Y/3 ,$$

where $Y|_{y=0} = -d_S$; and $c=\alpha_S/d_S$; C_5 is a constant. Thus

$$\Psi_S/\Psi'_S|_{y=0} = C_6\xi_S^2/d_S(1-T_{CSS}/T_S) - d_S/3 . \quad (1)$$

On the "normal" superconductor side, a solution can be supplied by neglecting the cubic term of the LG equation. Since $T_N < T_{cSS} < T_S$, then

$$\alpha_N = 1.83(1-T_{cSS}/T_N)/\xi_N^2 < 0 .$$

A solution with physical meaning is obtained as

$$\Psi_N = Qe^{\sqrt{|\alpha_N|}y}, \quad -d_N \le y \le 0 .$$

Then the first derivative is $\Psi'_N = \sqrt{|\alpha_N|}\Psi_N$. The ratio of Ψ_N to Ψ'_N at the boundary ($y=0$) is

$$\Psi_N/\Psi'_N|_{y=0} = 1/\sqrt{|\alpha_N|} = \xi_N/\sqrt{T_{cSS}/T_N - 1} . \quad (2)$$

To include the influence of the thickness d_N on this side, we replace ξ_N by d_N and introduce the attenuation constant k. Thus Eq. (2) becomes

$$\Psi_N/\Psi'_N|_{y=0} = d_N/k\sqrt{T_{cSS}/T_N - 1} . \quad (3)$$

Connecting Eq. (1) and Eq. (3) through the continuity of Ψ and Ψ' at the boundary ($y=0$), we get

$$C_6^*\xi_S^2/d_S(1-T_{cSS}/T_S) - d_S/3 = d_N/k\sqrt{T_{cSS}/T_N - 1} . \quad (4)$$

Using the same approach as in the preceding paper, we

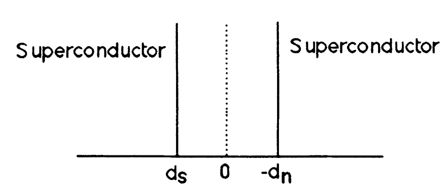

FIG. 1. A S-N layer.

can write $C_6\xi_S^2$ as $C_0 d_N \xi_S$. Thus

$$C_0 d_N \xi_S / d_S (1 - T_{cSS}/T_S) - d_S/3 = d_N / k\sqrt{T_{cSS}/T_N - 1}\,. \quad (5)$$

Finally Eq. (5) can be written as

$$B_0 d_N \xi_S / d_S (1 - T_{cSS}/T_S) - d_S = B_1 d_N / k\sqrt{T_{cSS}/T_N - 1} \quad (6)$$

with $B_1 = 3/k$ and $B_0 = 3C_0$. It is easy to prove that Eq. (6) can be converted into a equation of third order in T_{cSS}:

$$A_3 T_{cSS}^3 + A_2 T_{cSS}^2 + A_1 T_{cSS} + A_0 = 0 \quad (7)$$

with

$$A_3 = d_S^4 / T_S^2 T_N\ ,$$

$$A_2 = (2B_0 d_N \xi_S d_S^2 - 2d_S^4)/T_S T_N - (d_S^4 + B_0 d_N^2 d_S^2)/T_S^2\ ,$$

$$A_1 = (B_0 d_N \xi_S - d_S^2)^2 / T_N + (2d_S^4 + 2B_1 d_N^2 d_S^2 - 2B_0 d_N \xi_S d_S^2)/T_S\ ,$$

$$A_0 = -(B_0 d_N \xi_S - d_S)^2 - B_1 d_N^2 d_S^2\ .$$

III. DISCUSSION

We apply Eq. (6) to experimental results which are abundant in the literature.[1-5] The experimental data have been reported on lamina of Pb-Hg, Pb-Sn, Pb-In, Pb-Tl, Pb-Zn, and Pb-Cd, etc. (Refs. 1–5). All these curves have similar shapes. Here we take two experimental examples from Hauser *et al.* into account. The first is a lamina of lead ($T_c = 7.2$ K) with a dilute Pb:Gd magnetic alloy. As Ref. 1 indicates, the Pb:Gd films were produced using a lead target with gadolinium inserts and the gadolinium concentration of 2.9 at. % was determined from the ratios of areas and sputtering rates. The transition temperature of an isolated Pb:(2.9 at. % Gd) film was 1.52 K. The thickness of the film is 5200 Å. In principle, two experimental points can determine the values of B_0 and B_1. Reading data directly from Fig. 8 in Ref. 1, we choose point $P_1(d_{\mathrm{Pb}} = 100$ Å, $T_{cSS} = 1.7$ K) and point $P_2(d_{\mathrm{Pb}} = 400$ Å, $T_{cSS} = 5.5$ K). Then the values of B_0 and B_1 are found as $B_0 = 0.011\,243\,9$, $B_1 = 0.035\,424\,1$. By numerical calculations we obtain a plot of the transition temperature T_{cSS} versus d_{Pb}, the thickness of the lead film, in Fig. 2. The agreement between the experimental data and theory is quite good.

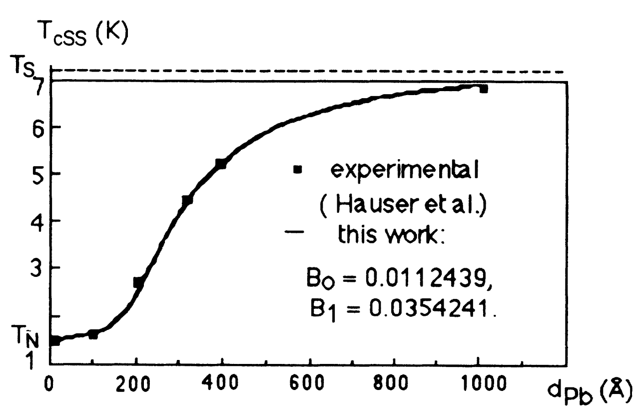

FIG. 2. Transition temperature for a Pb-Pb:2.9% Gd lamina with a constant Pb:2.9% Gd film of 5200 Å as a function of the lead-film thickness.

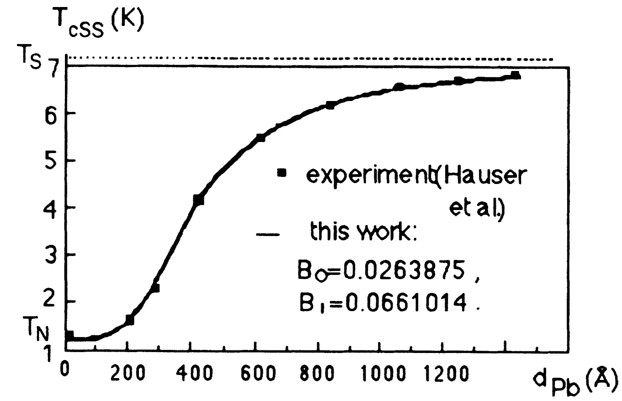

FIG. 3. Transition temperature of a Pb-Al lamina for a constant aluminum-film thickness of 4400 Å as a function of the lead-film thickness.

The second example is the Pb-Al lamina in Ref. 2. The thickness of the Al film is 4400 Å. The transition temperature varies with the thickness of the lead film. Reading data directly from Fig. 1 in Ref. 2, we choose point $P_1(d_{\mathrm{Pb}} = 407.69$ Å, $T_{cSS} = 4.3077$ K) and point $P_2(d_{\mathrm{Pb}} = 90.38$ Å, $T_{cSS} = 1.27$ K) to find the values of B_0 and B_1 as

$$B_0 = 0.026\,387\,5,\quad B_1 = 0.066\,101\,4\ .$$

The theoretical plot of T_{cSS} versus d_{Pb} is shown in Fig. 3, which is consistent with the experimental results. For other experimental curves of similar shape, we have no trouble to obtain proper values of B_0 and B_1 that fit the experimental results well.

IV. CONCLUSION

This model can be developed into a general treatment of the proximity effect. The appealing aspect of this theory is its theoretical consistence with the LG equation, its universal agreement with experimental measurements and its simplicity of application. It can be applied to various situations involving lamina. For a *S-S* lamina, Eq. (5) determines the transition temperature. For a *S-N* lamina, the term $\sqrt{T_{cSS}/T_N - 1}$ disappears, Eq. (5) becomes

$$B_0 d_N \xi_S / d_S (1 - t) - d_S/3 = d_N / k\ ,$$

which produces a simple expression for t:

$$t = 1 - B_0 d_n \xi_S / (d_S^2 + 3 d_S d_N / k)\ ,$$

as was pointed out in the preceding paper.

All these formulas are valid for both the pure and the dirty limit. However, in the dirty limit we can replace the ξ_S by l, the mean free path, as an alternative.

The values of B_0 and k can be determined experimentally. How to interpret them theoretically depends on the further study.

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