Superconductivity in the anyon model

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We analyze a two-dimensional system of electrons or holes, interacting via, in addition to the electromagnetic fields, the Chern-Simons gauge fields (a_ν) equipped with the action $(\mu/2)(\epsilon^{\lambda\nu\rho}a_\lambda\partial_\nu a_\rho$. For $|\mu|=e^2N/2\pi$ ($N=2,4,6,\ldots$), the ground state is unique and a spin singlet, having completely filled Landau levels with respect to the Chern-Simons magnetic fields. For these values of μ, equations are derived that replace the London equations in BCS theory. It is shown that a complete Meissner effect operates at zero temperature. Integer quantum Hall effects in the Chern-Simons fields play a crucial role. The magnetic penetration depth coincides with that in BCS theory as long as $N \ll 1000$.

The motion of the conducting electrons in the newly discovered high-T_c superconductors seems to be confined to the copper oxide planes, and so is essentially two dimensional. It is customary in the literature to describe experimental data in terms of the Ginzburg-Landau theory, assuming the existence of a scalar order parameter. Years ago Hohenberg[1] proved that in a strictly two-dimensional space a Cooper pair condensate $\langle \psi(\mathbf{x})\psi(\mathbf{x})\rangle$ must vanish at finite temperature ($T\neq 0$), at least in a translationally invariant continuum theory. A few ways out from this theorem have recently been discussed by several authors,[2] which allow a phase transition at $T\neq 0$ of the Kosterlitz-Thouless type. In this paper we confine ourselves to $T=0$ and show that a complete Meissner effect operates in this simple two-dimensional model. It remains an open question if the model analyzed in this paper describes the new high-T_c materials adequately.

The ground state of this model differs from that of the BCS theory. There is no Ginzburg-Landau order parameter. Instead, new gauge fields, the Chern-Simons fields, play the role of the order parameters. It is reserved for future publications to show how flux quantization and the Josephson phenomenon emerge in this theory.

This work was motivated by Laughlin's claim[3] that fractional statistics[4] in two-dimensional space plays a crucial role in high-T_c superconductivity. Similar models have recently been analyzed by several authors.[5–9] In particular, Fetter *et al.*[6] have also shown the existence of a zero-temperature Meissner effect by analyzing the linear response of the system to external electromagnetic fields. Such models of superconductivity are sometimes referred to as semion superconductivity or more generally, as anyon superconductivity in the literature.

In quantum-field theory one consistent way of incorporating fractional statistics is to introduce the Chern-Simons gauge fields at the Lagrangian level.[10] Therefore we are led to consider a system consisting of electrons (or holes) ψ_σ, electromagnetic fields A_μ (and $F_{\mu\nu}$), and Chern-Simons gauge fields a_μ (and $f_{\mu\nu}$), whose Lagrangian is given by

$$\mathcal{L}=-\tfrac{1}{4}F_{\mu\nu}^2-\frac{\mu}{2}\epsilon^{\lambda\nu\rho}a_\lambda\partial_\nu a_\rho+\psi^\dagger iD_0\psi -\frac{1}{2m}|D_k\psi|^2-\frac{e}{2m}B\psi^\dagger\sigma_3\psi-eA_0n_e\ , \quad (1)$$

where $D_\mu=\partial_\mu-ie(A_\mu+a_\mu)$ and $B=F_{12}(b=f_{12})$. n_e is the electron (or hole) density. The last term accounts for the background neutralizing charges. We have intentionally dropped a possible magnetic moment interaction for the Chern-Simons gauge fields so that the ground state is a spin singlet. Note that one can always rescale a_μ such that it has the same coupling e to electrons as A_ν does; the relevant quantity is μ/e^2. The appearance of the Chern-Simons fields a_μ in our model must be due to effective interactions from a more fundamental Hamiltonian describing the high-T_c superconductors. We shall show that the model described by (1) exhibits a complete Meissner effect for $|\mu|=e^2N/2\pi$ ($N=2,4,6,\ldots$) when $N\ll 1000$. For any integer value of N, (1) describes a system of anyons. $N=2$ corresponds to the semion model. However, the magnetic penetration depth turns out to be independent of N, coinciding with that of the BCS theory.

A few comments relating the present work to that of FHL (Ref. 6) are in order. The FHL Hamiltonian can be obtained from Eq. (1) by expressing a_μ in terms of the fermionic field operators and by turning off the electromagnetic gauge field A_μ. The approximation scheme used in these two computations are different. FHL retain the effects of the Chern-Simons interaction at the random-phase approximation level. We treat both the gauge fields (a_μ, and A_μ) equally but only upto the self-consistent mean-field level.

Equations of motion are given in the mean-field approximation by

$$
\begin{aligned}
&\tfrac{1}{2}\mu\epsilon^{\lambda\nu\rho}f_{\nu\rho}=\langle j^\lambda\rangle,\quad \partial_\nu F^{\lambda\nu}=\langle J^\lambda\rangle-en_e\delta^{\lambda 0}\,,\\
&j^0=J^0=e\psi^\dagger\psi\,,\\
&j^k=-\frac{ie}{2m}(\psi^\dagger D_k\psi-D_k\psi^\dagger\psi)\\
&\quad=J^k-\frac{e}{2m}\epsilon^{kj}\partial_j(\psi^\dagger\sigma_3\psi)\,.
\end{aligned}
\tag{2}
$$

Here $\langle j^\mu\rangle$ is the expectation value of j^μ: $\langle\Psi|j^\mu|\Psi\rangle$. The electron field satisfies

$$
i\partial_0\psi=H\psi=\left[-\frac{1}{2m}(D_k^2+eB\sigma_3)-e(A_0+a_0)\right]\psi\,. \tag{3}
$$

We solve Eqs. (2) and (3) self-consistently by explicit construction of $|\Psi\rangle$.

In the absence of external electromagnetic fields the ground state is expected to be uniform. Then one of Eq. (2) implies that $\mu b^{(0)}=en_e$, i.e., electrons move in a uniform Chern-Simons magnetic field. In the Landau gauge ($a_1^{(0)}=-b^{(0)}x_2$), the Hamiltonian is given by

$$
H_0=-\frac{1}{2m}\left[\left[\partial_1+i\frac{x_2}{l_2}\right]^2+\partial_2^2\right]\,, \tag{4}
$$

where the magnetic length l is given by $l^{-2}=|eb^{(0)}|=e^2n_e/|\mu|$. We impose a periodic boundary condition in the x_1 direction: $\psi(t,x_1+L,x_2)=\psi(t,x_1,x_2)$. The limit $L\to\infty$ is taken at the end. Solutions to the Schrödinger equation are

$$
\begin{aligned}
&H_0\psi_{nk\sigma}^{(0)}(\mathbf{x})=\varepsilon_n\psi_{nk\sigma}^{(0)}(\mathbf{x}),\quad \varepsilon_n=(n+\tfrac{1}{2})\frac{1}{ml^2}\,,\\
&\psi_{nk\sigma}^{(0)}(\mathbf{x})=\frac{1}{\sqrt{lL}}e^{-ikx_1}u_n\left[\frac{x_2}{l}-kl\right]v_\sigma\,,
\end{aligned}
\tag{5}
$$

where $n=0,1,2,\ldots$, $k=2\pi p/L$, $(p\in Z)$,

$$
u_n(z)=(2^n\sqrt{\pi}n!)^{-1/2}e^{z^2/2}(d^n/dz^n)e^{-z^2}\,,
$$

and $\sigma_3v_\sigma=\sigma v_\sigma$.

The number density of states per Landau level is

$$
2(2\pi l^2)^{-1}=(e^2/\pi|\mu|)n_e\,.
$$

The extra factor 2 comes from electron spin. For

$$
|\mu|=e^2N/2\pi\quad(N=2,4,6,\ldots)
$$

the lowest $N/2$ levels are seen to be completely filled. The ground state is unique and a spin singlet. Note that if there were a magnetic moment interaction, $n+\frac{1}{2}$ in ε_n in (5) would be replaced by $n+\frac{1}{2}(1-\sigma)$ so that low-lying Landau levels would be completely filled for an odd integer N instead. The ground state in this case describes a ferromagnet.

It has been shown by one of the authors[11] that in the Chern-Simons gauge theory formulated on a two torus, gauge invariance can be maintained only if N is an integer. This is indicative of an underlying connection between gauge invariance on a torus and a unique feature of states with completely filled Landau levels.[12] This point, however, is reserved for future investigation.

Let us restrict ourselves to gauge field configurations uniform in the x_1 direction: $A_\nu=A_\nu(x_2)$, $a_\nu=a_\nu(x_2)$, and $A_2=a_2=0$. Note $E_1=0$ in this ansatz. We decompose the Hamiltonian into $H=H_0+V_1+V_2$, where

$$
\begin{aligned}
V_1&=-\frac{e}{m}(a_1^{(1)}+A_1)\left[\frac{x_2}{l^2}-i\partial_1\right]\\
&\quad-e(a_0+A_0)-\frac{e}{2m}B\sigma_3\,,\\
V_2&=\frac{e^2}{2m}(a_1^{(1)}+A_1)^2\,.
\end{aligned}
\tag{6}
$$

Here $a_1=a_1^{(0)}+a_1^{(1)}$. So long as V_1+V_2 is sufficiently small, the ground state $|\Psi\rangle$ is given by

$$
\begin{aligned}
&H\psi_{nk\sigma}(\mathbf{x})=\varepsilon_{nk\sigma}\psi_{nk\sigma}(\mathbf{x})\,,\\
&\psi_{nk\sigma}=\psi_{nk\sigma}^{(0)}+\psi_{nk\sigma}^{(1)}+\cdots\,,\\
&\varepsilon_{nk\sigma}=\varepsilon_n+\varepsilon_{nk\sigma}^{(1)}+\cdots\,,\\
&\psi(t,\mathbf{x})=\sum_{n,k,\sigma}a_{nk\sigma}\psi_{nk\sigma}(\mathbf{x})e^{-i\varepsilon_{nk\sigma}t}\,,\\
&|\Psi\rangle=\prod_{n=0}^{N/2-1}\prod_{k,\sigma}a_{nk\sigma}^\dagger|0\rangle\,.
\end{aligned}
\tag{7}
$$

$\psi_{nk\sigma}^{(1)}(\mathbf{x})$, and therefore $\langle j^\nu\rangle$, are determined as functions of δa_ν and δA_ν. With these $\langle j^\nu\rangle$ the gauge fields must solve (2) self-consistently.

It is important to recognize that for $N=2,4,6,\ldots$ the free Hamiltonian $\mathbf{H}_0$ yields the ground state which exactly solves Eqs. (2) and (3). Indeed, for $V_1+V_2=0$,

$$
\langle j^0\rangle=\sum_{n=0}^{N/2-1}\sum_{k,\sigma}e\psi_{nk\sigma}^{(0)*}\psi_{nk\sigma}^{(0)}=en_e\,.
$$

Similarly $\langle j^1\rangle=\langle j^2\rangle=\langle\psi^\dagger\sigma_3\psi\rangle=0$. Hence Eqs. (2) and (3) are solved consistently by $A_\nu=a_0=a_2=0, a_1=a_1^{(0)}$.

Weak external magnetic fields change the motion of the electrons, consequently generating nonvanishing δa_ν and δA_ν, which in turn affect the motion of the electrons. Solving these back reactions self-consistently, we show that magnetic fields are expelled from the interior of the material. Under small changes in the gauge fields the lowest $N/2$ Landau levels are kept completely filled so that the dependence of the ground state on the gauge fields can be determined in perturbation theory. Our strategy is to first derive the equations which replace the London equations in the BCS theory, relating the currents $\langle J^\nu\rangle$ to the electromagnetic fields.

The computations are straightforward. Note that $\langle n'k'\sigma'|V_1|nk\sigma\rangle\propto\delta_{kk'}\delta_{\sigma\sigma'}$ and

$$\langle n'k\sigma|V_1|nk\sigma\rangle=\langle nk\sigma|V_1|n'k\sigma\rangle=-\int_{-\infty}^{\infty}\frac{dy}{l}\frac{e}{m}\left[C(y,k)+\frac{\sigma}{2}B(y)\right]u_n\left[\frac{y}{l}-kl\right]u_{n'}\left[\frac{y}{l}-kl\right],$$

$$C(y,k)=\frac{1}{l}\left[\frac{y}{l}-kl\right][A_1(y)+a_1^{(1)}(y)]+m[A_0(y)+a_0(y)], \tag{8}$$

$$\psi_{nk\sigma}^{(1)}=\sum_{n'\neq n}\psi_{n'k\sigma}^{(0)}\frac{1}{\varepsilon_n-\varepsilon_{n'}}\langle n'k\sigma|V_1|nk\sigma\rangle .$$

Equations (7) and (8) determine $\langle j^\nu(x)\rangle$ as a function of gauge fields. We introduce

$$K[h(x_2,y,k);x_2]\equiv\frac{2}{\pi}\sum_{n=0}^{N/2-1}\sum_{n'\neq n}\frac{1}{n'-n}\int_{-\infty}^{\infty}\frac{dy}{l}\int_{-\infty}^{\infty}d(kl)\,h(x_2,y,k)u_n(\xi)u_{n'}(\xi)u_n(\eta)u_{n'}(\eta), \tag{9}$$

where $\xi=(y/l)-kl$ and $\eta=(x_2/l)-kl$. Then

$$\begin{aligned}
\langle j^0(x_2)\rangle&=e^2K[C(y,k);x_2],\\
\langle j^1(x_2)\rangle&=-\frac{e^2n_e}{m}(A_1+a_1^{(1)})+\frac{e^2}{ml}K[\eta C(y,k);x_2],\\
\langle\psi^\dagger\sigma_3\psi(x_2)\rangle&=\frac{e}{2}K[B(y);x_2],
\end{aligned} \tag{10}$$

and $\langle j^2(x_2)\rangle=0$.

The integrands for the $K[h(x_2,y,k)]$'s of (10) significantly contribute only in the vicinity of $y=x_2$ because of the Gaussian falloff of $u_n(\xi)$ and $u_n(\eta)$. Therefore, $h(x_2,y,k)$ can be expanded in a Taylor series around x_2, provided it is a smooth function of y, i.e., the penetration depth (λ) is much larger than the magnetic length (l). The relevant expansion parameter turns out to be $N(l/\lambda)^2\sim N^2 10^{-6}$.

Employing the orthogonality relations of the $u_n(z)$'s and the recursion relation $zu_n(z)=(\sqrt{n+1}u_{n+1}+\sqrt{n}\,u_{n-1})/\sqrt{2}$, one can easily show that

$$h(x_2,y,k)=\begin{bmatrix}1\\-\xi,\eta\\\xi\eta\end{bmatrix}f(y)\to K[h]=\frac{N}{2\pi}\begin{bmatrix}-l^2\partial_2^2 f\\ l\partial_2 f+\frac{3}{8}l^3N\partial_2^3 f\\ f+\frac{1}{2}l^2N\partial_2^2 f\end{bmatrix}. \tag{11}$$

It immediately follows from (10) and (11) that

$$\begin{aligned}
\langle j^0\rangle&=+\frac{Ne^2}{2\pi}(b+B)+\frac{N^2e^2m}{4\pi^2n_e}\partial_2(f_{02}+E_2)+\frac{3N^3e^2}{32\pi^2n_e}\partial_2^2(b+B)+\cdots,\\
\langle j^1\rangle&=-\frac{Ne^2}{2\pi}(f_{02}+E_2)-\frac{N^2e^2}{4\pi m}\partial_2(b+B)-\frac{3N^3e^2}{32\pi^2n_e}\partial_2^2(f_{02}+E_2)+\cdots,\\
\langle\psi^\dagger\sigma_3\psi\rangle&=-\frac{N^2e}{8\pi^2n_e}\partial_2^2B+\cdots,
\end{aligned} \tag{12}$$

and $\langle j^2\rangle=0$. Here dots imply higher-order derivatives and $b=b^{(0)}+b^{(1)}$. The difference between $\langle J^1\rangle$ and $\langle j^1\rangle$ is numerically negligible to this order. Equations (12) replace the London equations of the BCS theory.

Let us also note that a term proportional to A_1+a_1, which plays a central role in the BCS theory, is absent. The first term in $\langle j^1\rangle$ exhibits the integer quantum Hall effect.[13] ($\hbar=c=1$ in our units.) The first term in $\langle j^0\rangle$ describes the change in the number density of states per Landau level due to additional (Maxwell as well as Chern-Simons) magnetic fields. Linearly increasing electric fields $(f_{02}+E_2)\propto x_2$ have two effects: they change the effective magnetic length and shift the average position in the x_2 direction for a given x_1 momentum k, which collectively gives the second term in $\langle j^0\rangle$.

The result (12) may be summarized by an effective Lagrangian $\mathcal{L}^{\mathrm{macro}}[A_\nu,a_\nu]=\mathcal{L}_0+\mathcal{L}_1$, where

$$\begin{aligned}
\mathcal{L}_0&=-\tfrac{1}{4}F_{\mu\nu}^2-\frac{Ne^2}{4\pi}\epsilon^{\lambda\nu\rho}a_\lambda\partial_\nu a_\rho-eA_0n_e,\\
\mathcal{L}_1&=\frac{Ne^2}{4\pi}\epsilon^{\lambda\nu\rho}(a_\lambda+A_\lambda)\partial_\nu(a_\rho+A_\rho)+\frac{N^2e^2m}{8\pi^2n_e}(f_{0k}+F_{0k})^2-\frac{N^2e^2}{8\pi m}\left[b+B-\frac{2\pi n_e}{eN}\right]^2.
\end{aligned} \tag{13}$$

$\mathcal{L}_1$ is related to the currents by $\delta\mathcal{L}_1/\delta a_\nu = \langle j^\nu \rangle$, which for configurations under consideration reproduces (12) to the leading order. The constant $-(2\pi n_e/eN)$ in the last term accounts for the fact that $B=0, b=2\pi n_e/eN$ in the ground state.

The effective Lagrangian $\mathcal{L}^{\rm macro}$ corresponds to the Ginzburg-Landau Hamiltonian in the BCS theory, from which macroscopic properties may be deduced. The Chern-Simons fields a_ν replace the Ginzburg-Landau order parameter. It is remarkable that the Chern-Simons term for a_ν in $\mathcal{L}_1$ exactly cancels the one in $\mathcal{L}_0$. This cancellation, which has been suggested by Banks and Lykken for a different reason,[8] is expected to suffer no higher-order corrections as it is enforced by the integer quantum Hall effect.[14]

It follows from (2) that

$$\frac{Ne^2}{2\pi} b^{(1)} = -\partial_2 E_2, \quad \frac{Ne^2}{2\pi} f_{02} = -\partial_2 B \ , \tag{14}$$

so that $b^{(1)}$ and f_{02} can be eliminated from (12). Inserting (14) into (12) and keeping numerically dominant terms, one finds that Eq. (2) is reduced to

$$\frac{Ne^2}{2\pi}\left[B - \frac{m}{e^2 n_e}\partial_2^2 B\right] + \frac{N^2 e^2 m}{4\pi^2 n_e}\partial_2 E_2 = 0, \quad \frac{Ne^2}{2\pi} E_2 = 0 \ . \tag{15}$$

The penetration depth characterizing the Meissner effect ($B \propto e^{-x_2/\lambda}$) is given by

$$\lambda = \sqrt{m/e^2 n_e} \ . \tag{16}$$

Although the mechanism is completely different, the expression for the penetration depth is exactly the same as in the BCS theory. This can be understood as a result of the Galilean invariance of the system.[15] In the boundary region the kinetic energy density associated with currents is

$$\tfrac{1}{2} m n_e v^2 = m J_1^2 / 2e^2 n_e \ ,$$

while the field energy density is $\frac{1}{2}B^2$. Hence, upon making use of $\partial_2 B = J_1$, the total surface energy is minimized when $(1-\lambda^2\partial^2)B = 0$ where λ is given by (16). We remark that the Meissner effect results for either sign of B with given μ. The nonvanishing current $\langle J^1 \rangle$ is a supercurrent, since $E_1 = 0$.

The same conclusion concerning the Meissner effect has been obtained by Fetter *et al.*[6] (FHL), but the mechanism discovered above differs in its detail from FHL's. There is no term proportional to $A_1 + a_1$ in the expression for $\langle j^1 \rangle$ in Eq. (12), which plays a central role in the BCS theory and in the FHL approach. The gauge invariance does not allow such a term in the absence of Ginzburg-Landau order parameter. Unlike the ground state of the BCS theory, there is no mixture of creation and annihilation operators in the ground state $|\Psi\rangle$ in Eq. (7).

Crucial elements here are the integer quantum Hall effect and the balance represented by (14) between the electromagnetic and the Chern-Simons fields.[16] The Maxwell equation involving J^1 is automatically satisfied, provided $E_2 = 0$. The charge neutrality condition (the Maxwell equation involving J^0) enforces the complete screening of the total magnetic fields.

So far everything has been defined in two-dimensional space. To relate it to the (real) three-dimensional one, we recall that high-T_c superconductors have a layered structure, and denote the interplanar spacing by δ. Then, $e^2/4\pi = \alpha/\delta$ where $\alpha = \frac{1}{137}$, and $F_{\mu\nu}^{d=2} = \delta^{1/2} F_{\mu\nu}^{d=3}$. As typical values we take $n_e = 10^{14}$ cm^{-2} and $\delta = 10^{-7}$ cm. We choose $m = 7.5 \times 10^{10}$ cm^{-1} thrice the value of the bare electron mass. With these values one finds $\lambda = 2.9 \times 10^{-5}$ cm. The two relevant expansion parameters in the problem are seen to be

$$\begin{aligned} &\frac{N^2 e^2}{4\pi m} = \frac{N}{2}\left[\frac{l}{\lambda}\right]^2 = 9.7 \times 10^{-7} N^2 \ , \\ &\frac{\pi}{e^2 m}\frac{1}{\lambda^2} = \frac{\pi n_e}{m^2} = 5.6 \times 10^{-8} \ . \end{aligned} \tag{17}$$

The expansion is valid for $N \ll 1000$. The correction to the electric field is evaluated as $E_2 = -10^{-7} NB$, which with (17) justifies our approximation scheme.

The Meissner effect results for magnetic fields perpendicular to the copper-oxide layers in the three-dimensional material, even if the Chern-Simons coefficient μ takes a different sign from one layer to the next.[17] If this is the case, the effects of P- or T-violation inherent in the Chern-Simons theory would be very difficult to see.[18]

In this paper we have shown that a coupled system of electrons (or holes), electromagnetic fields and Chern-Simons fields exhibits a complete Meissner effect at zero temperature. Our results are best summarized by (12) and (14), or by (13). It is likely that the magnetic flux quantum in this theory will turn out to be $2\pi/Ne$, reflecting to the fact that the number of states in $\frac{1}{2}N$ Landau levels changes in unit of N. If this is the case, $N=2$ (the semion model) describes the known high-T_c superconductors. We are planning to come back to this point, in addition to finite temperature effects and the Josephson effect, in separate papers.

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