Theory of the effects of destruction of localization by inelastic scattering in the resistivity of pure thin potassium wires

Mary Eileen Farrell*
Department of Physics and Atmospheric Science, Drexel University, Philadelphia, Pennsylvania 19104

Marilyn F. Bishop
Department of Physics, Virginia Commonwealth University, 1020 West Main Street, P.O. Box 2000, Richmond, Virginia 23284-2000

N. Kumar
Department of Physics, Indian Institute of Science, 560 012 Bangalore, Mysore, India

W. E. Lawrence
Department of Physics and Astronomy, Dartmouth College, Hanover, New Hampshire 03755
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Measurements of the electrical resistivity of thin potassium wires at temperatures near 1 K have revealed a minimum in the resistivity as a function of temperature. By proposing that the electrons in these wires have undergone localization, albeit with large localization length, and that inelastic-scattering events destroy the coherence of that state, we can explain both the magnitude and shape of the temperature-dependent resistivity data. Localization of electrons in these wires is to be expected because, due to the high purity of the potassium, the elastic mean free path is comparable to the diameters of the thinnest samples, making the Thouless length l_T (or inelastic diffusion length) much larger than the diameter, so that the wire is effectively one dimensional. The inelastic events effectively break the wire into a series of localized segments, whose resistances can be added to obtain the total resistance of the wire. The ensemble-averaged resistance for all possible segmented wires, weighted with a Poisson distribution of inelastic-scattering lengths along the wire, yields a length dependence for the resistance that is proportional to $[L^3/l_{in}(T)]$, provided that $l_{in}(T) \geq L$, where L is the sample length and $l_{in}(T)$ is some effective temperature-dependent one-dimensional inelastic-scattering length. A more sophisticated approach using a Poisson distribution in inelastic-scattering times, which takes into account the diffusive motion of the electrons along the wire through the Thouless length, yields a length- and temperature-dependent resistivity proportional to $(L/l_T)^4$ under appropriate conditions. Inelastic-scattering lifetimes are inferred from the temperature-dependent bulk resistivities (i.e., those of thicker, effectively three-dimensional samples), assuming that a minimum amount of energy must be exchanged for a collision to be effective in destroying the phase coherence of the localized state. If the dominant inelastic mechanism is electron-electron scattering, then our result, given the appropriate choice of the channel number parameter, is consistent with the data. If electron-phason scattering were of comparable importance, then our results would remain consistent. However, the inelastic-scattering lifetime inferred from bulk resistivity data is too short. This is because the electron-phason mechanism dominates in the inelastic-scattering rate, although the two mechanisms may be of comparable importance for the bulk resistivity. Possible reasons why the electron-phason mechanism might be less effective in thin wires than in bulk are discussed.

I. INTRODUCTION

A surprising minimum in the electrical resistivity as a function of temperature characterizes the behavior of pure thin potassium wires having diameters close to 0.1 mm, as observed by Yu *et al.*[1] Up to about 1.3 K in the thinnest samples observed, the resistivity decreases with increasing temperature. This "resistivity dip" is dramatic since the classical theory of the resistivity of a metal predicts that inelastic-scattering contributions cause an increase of the resistivity with increasing temperature. In this paper we present a theory to describe the observed minimum by employing a model of localized electrons in a long thin wire, albeit with large localization length. The inelastic-scattering events that these electrons experience effectively break the wire into a series of segments, each of which has an elastic resistance that increases exponentially with the length of segment.[2] To obtain the total resistance, we perform an ensemble average of the elastic-scattering resistances over all possible segmentations of the wire, assuming that segments add Ohmically.

The experiments of Yu *et al.*[1] included studies of the temperature-dependent part of the resistivity $\rho(T)$ from 1.8 down to 0.08 K for free-hanging high-purity K wires

of length 5 cm with diameters d in the range $0.09 \leq d \leq 1.5$ mm. The data were plotted as $\rho_{4.2\,\mathrm{K}}\Delta[\ln(\rho)]/\Delta T$. Since the temperature-dependent resistivity in these samples is about one part in 10^4 of the total resistivity, $\rho_{4.2\,\mathrm{K}}$ is nearly equal to ρ, so that the quantity that is plotted is essentially $d\rho/dT$. This quantity is observed to change sign from positive to negative with decreasing temperature, indicating a resistivity minimum. In order to see this minimum explicitly, we integrate the data from Yu *et al.* and plot the resistivity $\rho(T)=\rho-\rho_0$ in Fig. 1, where ρ_0 is the residual resistivity. Although all samples were prepared under the same conditions, it is clear that as the sample diameter decreases, a minimum develops and eventually becomes quite pronounced. Since these localization effects should be greatest in the thinnest samples (diameters 0.09 and 0.10 mm), for the simple reason that the localization length is directly proportional to the cross-sectional area for thin wires,[3] we concentrate on those data sets.

Yu *et al.*[1] originally proposed that these data could be explained by the interference between normal electron-electron scattering (NEES) and surface scattering, the Gurzhi effect.[4] If one assumes potassium to have a nearly spherical closed Fermi surface, NEES would not contribute to the resistivity because the total electron momentum is conserved at each collision. However, if the mean free path due to NEES is much less than the diameter of the sample, i.e., $l_{ee}^N \ll d$, but the mean free path due to electron-impurity scattering is larger than the diameter, then NEES can reduce the number of electron-surface collisions, and thus reduce the resistance. While Yu *et al.* admitted that, in fact, $l_{ee}^N \gg d$, they suggested that the reduction in resistance might apply in this opposite regime as well. Previous Monte Carlo calculations by Black[5] were not of sufficiently high resolution to determine the sign of the NEES contribution to the resistivity at low temperatures. However, recent high-resolution calculations by Movshovitz and Wiser[6] have shown that in this regime the resistivity increases with increasing temperature, so that this mechanism cannot account for the observed effect.

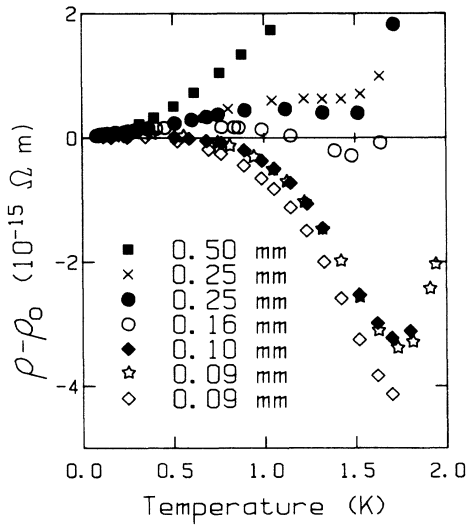

FIG. 1. Resistivity vs temperature for samples with decreasing diameters for data of Ref. 1. Original data were integrated to yield $(\rho-\rho_0)$, total resistivity minus zero-temperature resistivity. Symbols used for data sets are those of Ref. 1 and diameters are indicated in the figure. Corresponding to the order of the symbols listed in the figure from top to bottom, ρ_0 values are (in units of pΩ m) 20, 19.82, 20.07, 20.07, 134.4, 127.2, and 113.1; $\rho_{4.2\,\mathrm{K}}$ values are (in units of pΩ m): 17.9, 22.5, 23.6, 31.3, 138, 130, and 116.

Although thermal expansion of the wire can result in a slightly increasing diameter as a function of temperature and thus a decreasing resistance, this mechanism is about 4 orders of magnitude too small to explain the observed effects. The resistance $R(T)=\rho L/A$, where ρ is the resistivity, L is the length, and A is the cross-sectional area, will increase with temperature due to an increase in both resistivity and length, but will decrease due to an increase in diameter, as

$$R(T+\Delta T)\approx R(T)(1+\Delta\rho/\rho+\Delta L/L-2\,\Delta d/d)\,, \tag{1.1}$$

where d is the diameter of the wire. Since all linear dimensions increase with temperature according to $(1/l)\Delta l/\Delta T\approx\alpha(T)$, where $\alpha(T)$ is the thermal expansion coefficient, it is clear that thermal expansion has the tendency to decrease the resistance of the wire. The value measured by Schouten and Swenson[7] for the thermal coefficient for potassium at $T=1$ K is $\alpha(T)=7.6\times10^{-9}$ K^{-1}, or about 4 orders of magnitude smaller than observed changes with temperature $(1/\rho)\Delta\rho/\Delta T\approx1.3\times10^{-4}$ K^{-1}.

There are several experimental observations of localization effects in effectively one-dimensional systems.[8-10] Those data conform in both magnetic field and temperature dependences to previous theoretical predictions.[11,12] Although the samples studied in those experiments are much narrower and much more disordered than those being considered here, the present samples are still effectively one dimensional for localization because the inelastic-scattering (phase-breaking) length l_T is greater than the diameter of the sample. The present data differ significantly from those of Refs. 8–10 in that the temperature dependence here varies as an inverse power of l_T, rather than as l_T itself. We will show that this behavior is to be expected if l_T is sufficiently long compared with the length of the sample, whereas the other data conform to the opposite regime.

In Sec. II we review the fundamental properties of localized electron states; in particular, we explain the distinction between our approach in the weakly localized regime and approaches to the strongly localized regime and present the framework for the derivation of the total resistance of a thin wire for which the electrons are accommodated in localized states, in which the role of inelastic collisions is to disrupt the coherence of the localized state, breaking the wire into a series of localized segments. In Sec. III A we obtain an expression for the total resistance of the wire by averaging with a Poisson distribution over all possible scattering lengths along the wire.

In Sec. III B we improve on the calculations in Sec. III A by averaging with a Poisson distribution in inverse-scattering times, which allows for the diffusive nature of the electrons. In Sec. IV we discuss the choice of appropriate inelastic-scattering mechanisms for the destruction of phase coherence in the temperature range of interest. In Sec. IV A we derive the quasiparticle rate for electron-phason scattering, in which we assume that a minimum energy would be required for the destruction of phase coherence, and in IV B we derive the corresponding rate for electron-electron scattering. Our theoretical results are compared with the experimental data in Sec. V, and in Sec. VI we state our conclusions.

II. LOCALIZATION OF ELECTRONIC STATES

In a three-dimensional disordered metal, the randomness must first exceed a certain finite amount before the electron states at the Fermi level can become localized. However, the situation in one dimension is well known to be quite different. As first shown by Mott and Twose,[13] and later by Landauer,[14] any amount of disorder, regardless how small, will cause all the electrons in a one-dimensional system to exist in localized states, leading to residual resistances increasing exponentially with the wire length.

A negative temperature coefficient of resistance for such a disordered wire is expected on general grounds. For the relatively simple case of a strongly disordered wire, with a small localization length, the transport at low temperature is by the phonon-assisted hopping of electrons between well localized states. This is the activated "exponential hopping" or "strongly localized" regime. On the other hand, for the case of a weakly disordered wire with a large localization length, as in the case in question, the inelastic-scattering processes randomly interrupt the phase-coherent propagation of the electron wave packet long before the finiteness of the localization length is *sensed* by the wave packet. This is the "weakly localized" regime. We will presently make precise the fundamental distinction between these two regimes. It is, however, clear that, in both these cases, increasing temperature should offset the effect of localization and hence lead to $(d\rho/dT)<0$.

The first approach to electrical conduction in one dimension, due to Mott and Twose,[13] involves solving the Schrödinger equation as a boundary-value problem to obtain expressions for the localized electron wave functions. An alternative method, introduced by Landauer,[14] consists in solving the quantum-mechanical scattering problem of reflection and transmission of an incident electron through an obstacle. He finds that the relationship between the reflection and transmission coefficients $\mathcal{R}$ and $\mathcal{T}$ and the resistance r of the obstacle is

$$r=2(\pi\hbar/e^2)\mathcal{R}/\mathcal{T}\ . \tag{2.1}$$

Henceforth in this paper all resistances will be given in units of $\pi\hbar/e^2=1.29\times10^4\ \Omega$. In order to obtain the resistance of a one-dimensional system consisting of a series of obstacles, one can then average the quantity $\mathcal{R}/\mathcal{T}$ over the distance between successive ones, with the interspaces treated as independent random variables. This treatment incorporates the element of disorder into the model by allowing the scatterers to be of the same type, but located at random positions, so that their phasing is automatically random at this stage. (This type of spatial disorder should be distinguished from shape disorder, in which the obstacles are evenly spaced, but their potentials differ in shape.) The resulting average ratio $\langle\mathcal{R}/\mathcal{T}\rangle$, for a spatially disordered array of scatters, is proportional to $(\mathcal{R}/\mathcal{T})^n$, where n is the number of obstacles. Then, if the distribution of obstacles (or scatterers) is uniform along the length of the wire, the resistance will increase exponentially with length.

Abrikosov and Ryzhkin[15] also examined the zero-temperature resistance of one-dimensional systems containing random impurities. They introduced randomness through a continuous random potential with a Gaussian distribution. Their result for the elastic resistance can be shown to be equivalent to Landauer's result, if one realizes that they include only terms of inverse transmission coefficient, while Landauer expresses resistance in terms of the ratio $\mathcal{R}/\mathcal{T}$. Thus, the apparent discrepancy between the two results arises simply from a difference in notation; it is not due to any failure of the scaling method, as proposed by Abrikosov.[16]

It was suggested by Yuval[17] that any wire will behave in a one-dimensional manner for a sufficiently low temperature and sufficiently large ratio of length to cross-sectional area. Anderson employed scaling methods to develop a multichannel formalism,[18] appropriate for a wire of length L with small nonzero diameter. He showed that the elastic-scattering resistance is (in units of $\pi\hbar/e^2$):

$$r(L)=\langle R(L)\rangle=(r_s/\nu)\{\exp[\nu r_c(L)/r_s]-1\}\ , \tag{2.2}$$

where ν denotes the number of "distinct" channels (to be discussed shortly); $r_c(L)=\alpha L/l_{\rm el}$ is the classically calculated additive resistance, which is the actual measured resistance when it is sufficiently small, $l_{\rm el}$ is an elastic mean free path for backscattering of the electrons, and r_s is the scale resistance. For the thinnest samples studied, $r_c(L)\approx7\times10^{-8}$, where the value of α is chosen to give the correct classical resistance. The scale resistance is in the range $1.764\le r_s\le2$, and is equal to 2 in the small-resistance limit,[18] which applies in the present case. For the resistivity data of potassium below 1.3 K, the resistances of the wires are so small that the classical resistance is dominant, as can be seen by expanding the exponential, with the effects of localization a small fraction of the total resistance. However, Thouless[3] suggested that localization effects in quasi-one-dimensional wires should be expected to be significant only if the impurity resistances were greater than about 10 kΩ. The wires under consideration here have resistances that are less than $10^{-3}\ \Omega$, and the localization effects are less than 10^{-4} of the total resistance. In our case, it is the extremely high purity of the samples that allows one to observe localization effects in wires of macroscopic diameter, since $l_{\rm el}$ is comparable to the sample diameter d, and the inelastic

diffusion length $\sim(l_{el}l_{in})^{1/2}$ is larger than d. Our previous paper, Ref. 19, marks the first time that one-dimensional localization effects have been proposed to explain transport phenomena in such pure samples. The analogous case of two-dimensional localization effects was first observed by Van den dries *et al.*[20] in high-purity, low-resistivity thin Cu films (50–500 Å) between 1 and 20 K, where the elastic mean free path was of the order of the film thickness.

Setting $\nu=1$ in Eq. (2.2) gives the expression due to Anderson[18] for the elastic-scattering resistance of a one-dimensional system with a large number n of intimate channels, that is, channels that effectively communicate with another through interchannel scattering (crosstalk). Each channel may be associated with a transverse quantum number, and channels with no Fermi-level density in them will simply have no transmission $\mathcal{T}$. For a physical wire of finite diameter, one would expect the total number N of channels to be partitioned into ν sets, each of which contains n intimate channels, so that $N=\nu n$, with only weak coupling between the sets, assumed to be widely separated in transverse quantum numbers. Thus, we may consider the wire as consisting of ν distinct sets of Anderson-type multichannel conductors connected in parallel, where $\nu r_c(L)$ in Eq. (2.2) is the classical resistance of each of the ν conductors. In this way the higher-than-one-dimensionality of the system is expressed by allowing electrons to be scattered between transverse channels, thereby providing alternate parallel paths for the current in the wire. These paths serve to reduce the effect of highly resistive channels, while the series connection serves to diminish the influence of highly conductive sets.

The essential effect of inelastic-scattering processes is that, because they involve an exchange of energy between initial and final states, they interrupt the phase coherence of electron wave evolution under elastic scattering. Thus, if an electron encounters n inelastic-scattering events as it traverses the sample, its wave-function coherence will be destroyed at each of those n locations, and the sample will effectively be broken into $n+1$ segments, such that $0\le x_1\le x_2\le\cdots\le x_n\le L$, where the distances between adjacent inelastic-scattering events are varied independently of one another. Within each segment the electronic wave functions will remain coherent, and the corresponding elastic-scattering resistance will increase exponentially with the segment length, according to Eq. (2.2). If the segments are considered to be connected like series resistors, then their resistances will simply add. This means that the total resistance of the wire will be less if inelastic collisions occur within the length of the wire than if they did not, as depicted schematically in Fig. 2.

Apart from the total length L of the wire, two other length scales determine the efficacy of localization. The first is the localization length λ, which for a wire of finite cross-sectional area A is given by[3] $\lambda=(4Ak_F^2/3\pi^2)l_{el}\ge l_{el}$, where k_F is the Fermi wave vector. For the potassium wires of interest, $\lambda\sim 10^{11}l_{el}-10^{12}l_{el}$, while for a strictly one-dimensional wire, $\lambda=l_{el}$. (In the present treatment the effect of finite cross section is being taken into account through the multichannel parameter ν.)

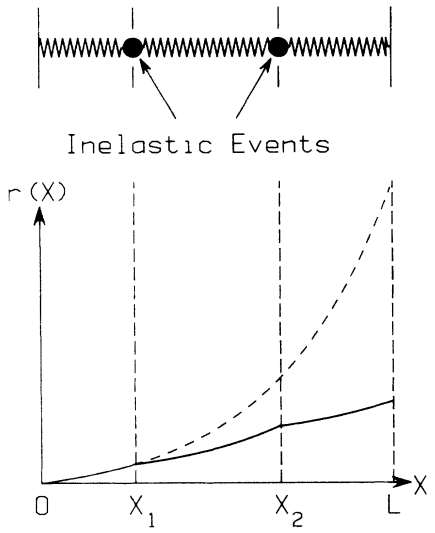

FIG. 2. Schematic illustration of the elastic resistance as a function of length in a one-dimensional wire for the specific case of two inelastic events. The resistance $r(x)$ rises exponentially with length between successive inelastic collisions. Segment lengths are varied independently of one another, since we assume that phase coherence is totally destroyed at the position of each inelastic event. Note that the total resistance of the wire is reduced if inelastic collisions occur within the length of the wire.

The second important dimension is the effective distance over which an electron wave packet loses phase coherence due to inelastic scattering. This distance is the Thouless length,[3] which is given by

$$l_T(T)=[D\tau_{in}(T)]^{1/2}\ , \tag{2.3}$$

where $\tau_{in}(T)$ is the inelastic-scattering time and D is the diffusion constant given by the Einstein relation

$$D=(l_{el}v_F)/3=(\tau_{el}v_F^2)/3\ . \tag{2.4}$$

We are now in a position to discern more precisely the two regimes mentioned earlier. The strongly localized regime corresponds to $l_T(T)\gg\lambda$, when the localization of wave functions is fully sensed by the electron. In the weakly localized regime, $l_T(T)\ll\lambda$, the electron loses its phase coherence due to inelastic scattering long before it can sense the finiteness of λ, so that the effects of localization will only be a small perturbation on the "classical" transport behavior. The data considered here (Ref. 1) and also the data of Refs. 8–10 are in the latter regime; different temperature dependences are possible within this regime, depending upon the phase-breaking mechanisms and upon the magnitude of $l_T(T)$ compared with the sample length. In the limit $l_T(T)/\lambda\to 0$, deviations from classical behavior become undetectable, and the

main effect of inelastic collisions is to cause the resistance to increase with increasing temperature. Presumably, this occurs in our case for temperatures above about 1.3 K. In Sec. III, we evaluate the resistance of a wire by performing an average over all segmentations by employing the two different distributions in segment length discussed earlier.

III. CALCULATION OF THE TEMPERATURE DEPENDENCE OF THE RESISTANCE

We begin by calculating the temperature-dependent resistance using a Poisson distribution over inelastic-scattering lengths $l_{\rm in}(T)$. This is done in Sec. III A, and in Sec. III B an improved result is obtained using a Poisson distribution over inelastic-scattering *times*. The former case is discussed because the resistance calculation can be done analytically. This case was also discussed in Ref. 19, where only the final result was quoted.

A. Poisson distribution over lengths

The distribution of length segmentations of the wire is obtained as follows: The probability for obtaining n scattering events in a length interval Δx is given by

$$P(n,\Delta x)=\langle n\rangle^n e^{-\langle n\rangle}/n! , \tag{3.1}$$

where $\langle n\rangle=\Delta x/l_{\rm in}(T)$ is the average number of scattering events occurring within Δx. Now suppose, for example, that only one inelastic event occurs over the total length of the wire, so that the wire is effectively divided into two segments, $(0,x_1)$ and (x_1,L), with the inelastic event taking place within the interval dx_1. The probability of this event is the product $P(0,x_1)P(1,dx_1)P(0,(L-x_1))=\exp(-x_1/l_{\rm in})(dx_1/l_{\rm in})\exp[(x_1-L)/l_{\rm in}]$, obtained by setting $n=0$ or 1 and $\exp(-dx_1/l_{\rm in})=1$. This probability is written as $p(x_1)dx_1$, with $p(x_1)$ the probability density for a single scattering event. The corresponding probability density for n events is

$$\begin{aligned}p(x_1,\ldots,x_n)&=(1/l_{\rm in})^n P(0,x_1-0)P(0,x_2-x_1)\cdots P(0,x_n-x_{n-1})P(0,(L-x_n)),\\&=(1/l_{\rm in})^n\exp(-x_1/l_{\rm in})\exp[-(x_2-x_1)/l_{\rm in}]\cdots\exp[-(x_n-x_{n-1})/l_{\rm in}]\exp[-(L-x_n)/l_{\rm in}] .\end{aligned} \tag{3.2}$$

Since we assume that the coherence is totally destroyed at the position of each inelastic-scattering event, we break the wire into separate coherent segments, whose resistances simply add. The total resistance of the wire is then the sum of the resistances in each segment multiplied by the probability that the wire has that resistance, summed over all possible configurations of inelastic events. The ensemble-averaged resistance, in units of $\pi\hbar/e^2$, then becomes

$$\langle R(L)\rangle=r(L)\exp(-L/l_{\rm in})+\sum_{n=1}^{\infty}\int_0^L dx_n\cdots\int_0^{x_3}dx_2\int_0^{x_2}dx_1[r(x_1)+r(x_2-x_1)+\cdots+r(L-x_n)]p(x_1,\ldots,x_n) , \tag{3.3}$$

where $r(x)$ is the elastic-scattering resistance given by Eq. (2.2). Physically, we see that the total resistance arises from a direct interplay between elastic and inelastic scattering.

In order to evaluate the summation in Eq. (3.3), it is useful to take the Laplace transform $\mathcal{L}$, of both sides of this equation, with the definition

$$r_{\mathcal{L}}(s)\equiv\mathcal{L}[r(L)]=\int_0^\infty dL\, e^{-sL}r(L) . \tag{3.4}$$

To avoid complicating the discussion, we illustrate the steps involved in determining the Laplace transform of the various integrals by considering the particular case in which only two inelastic events take place along the length of the wire ($n=2$), so that the quantity of interest becomes

$$\begin{aligned}L_2=(l_{\rm in})^{-2}\mathcal{L}\Big[\int_0^L dx_2\int_0^{x_2}dx_1[r(x_1)+r(x_2-x_1)+r(L-x_2)]\\\times\exp(-x_1/l_{\rm in})\exp[-(x_2-x_1)/l_{\rm in}]\exp[-(L-x_2)/l_{\rm in}]\Big] .\end{aligned} \tag{3.5}$$

By successively applying the convolution theorem, we find that

$$L_2=3r_{\mathcal{L}}(s+1/l_{\rm in})/[(l_{\rm in})^2(s+1/l_{\rm in})^2] . \tag{3.6}$$

By following the procedure outlined above, one can then show that the general expression for $\mathcal{L}[\langle R(L)\rangle]$ will be given by

$$\mathcal{L}[\langle R(L)\rangle]=r_{\mathcal{L}}(s+1/l_{\rm in})\left[1+\sum_{n=1}^{\infty}l_{\rm in}^{-n}(n+1)(s+1/l_{\rm in})^{-n}\right] . \tag{3.7}$$

The series can be summed exactly with the result,

$$\mathcal{L}[\langle R(L)\rangle]=r_{\mathcal{L}}(s+1/l_{\rm in})[1+2/(sl_{\rm in})+1/(sl_{\rm in})^2] . \tag{3.8}$$

Then, by taking the inverse Laplace transform of each term in Eq. (3.8), we obtain an integral expression for the total resistance of the wire,

$$\langle R(L)\rangle=(l_{\rm in})^{-2}\int_0^L dx(L-x)r(x)\exp(-x/l_{\rm in})+2(l_{\rm in})^{-1}\int_0^L dx\, r(x)\exp(-x/l_{\rm in})+r(L)\exp(-L/l_{\rm in})\ . \tag{3.9}$$

Upon substituting Eq. (2.2) for $r(x)$ and performing all the required integrations, we arrive at an exact expression for the resistance of a thin wire:

$$\langle R(L)\rangle=(r_s/\nu)(-\beta^2(l_{\rm in})\{1-\exp[\beta(L)-L/l_{\rm in}]\}/[1-\beta(l_{\rm in})]^2+\beta(L)/[1-\beta(l_{\rm in})])\ , \tag{3.10}$$

where $\beta(x)=\nu r_c(x)/r_s=\nu\alpha x/r_s l_{\rm el}$. Equation (3.10) correctly reduces to the Landauer result when $l_{\rm in}\to\infty$.

In order to obtain the temperature-dependent part, $R(L,l_{\rm in})$, of the total resistance $\langle R(L)\rangle$, we subtract from Eq. (3.10) the zero-temperature resistance $r(L)$ of Eq. (2.2). Since for the data of interest, $\beta(L),\beta(l_{\rm in})\ll 1$, we can expand Eq. (3.10) in these small quantities to write forms that show clearly the dependence on $l_{\rm in}$:

$$R(L,l_{\rm in})=-[r_s\beta^2(L)/2\nu]\{1-2(l_{\rm in}/L)+2(l_{\rm in}/L)^2[1-\exp(-L/l_{\rm in})]\} \tag{3.11a}$$

$$=-[\nu r_c^2(L)/6r_s](L/l_{\rm in})\left[1+3!\sum_{n=4}^{\infty}[(-1)^{n+1}(L/l_{\rm in})^{n-3}/n!]\right]\ , \tag{3.11b}$$

where in Eq. (3.11b) we have additionally assumed that $L\ll l_{\rm in}$ and have expanded the exponential in powers of $(L/l_{\rm in})$. For $(L/l_{\rm in}\ll 1$, since the classical resistance $r_c(L)$ is proportional to L, $R(L,l_{\rm in})$ is proportional to $(-L^3/l_{\rm in})$, as can be seen from Eq. (3.11b). Since $(1/l_{\rm in})$ increases with temperature, this represents a decrease in resistance with increasing temperature. We will assume here that $l_{\rm in}\gg L$, so that we can use the leading term in the expansion of Eq. (3.11b), which is proportional to $1/l_{\rm in}$. As $(1/l_{\rm in})$, and thus temperature, increases, Eq. (3.11a) reaches a minimum and turns upward, and in the limit $l_{\rm in}\ll L$, the leading terms in the localization contribution are equal to $R(L,l_{\rm in})\sim-[\nu r_c^2(L)/2r_s][1-2(l_{\rm in}/L)]$, which approaches a constant value that is less than the zero-temperature resistance.

In Ref. 19 we assumed for simplicity that $l_{\rm in}$ was the product of $\tau_{\rm in}$ and the Fermi velocity, which fails to account for the diffusive motion between inelastic events. This problem is addressed in Sec. III B.

B. Poisson distribution over times

Because the motion of an electron as it traverses a total path length $l_{\rm in}$ between inelastic collisions is diffusive, the net distance it travels along the length of the wire is actually the Thouless length l_T, given by

$$l_T^2(T)=(l_{\rm el}v_F\tau_{\rm in}/3)\ , \tag{3.12}$$

where $\tau_{\rm in}$ is the inelastic-scattering time. Thus we consider a Poisson distribution over inelastic times, with mean value $\tau_{\rm in}$ and segment the wire according to the relation

$$(x_n-x_{n-1})^2=(v_F l_{\rm el}/3)\tau_{n-1,n}\ , \tag{3.13}$$

where $\tau_{n-1,n}$ is the time elapsed between the $(n-1)$ and n collisions, and x_{n-1} and x_n are the positions of those collisions along the wire. (The positions at the beginning and end of the wire will be denoted by 0 and L, respectively.) The elastic-scattering resistance will retain the form of Eq. (2.2), with the x dependence written in terms of the τ variables. For convenience in the following derivation, we define the variable

$$\xi(\tau_{n-1,n})=r(x_n-x_{n-1})=(r_s/\nu)\{\exp[\nu r_c(x_n-x_{n-1})/r_s]\}-1]\ . \tag{3.14}$$

In the small resistance limit, the classical resistance must, of course, depend linearly on the length of the wire.

The probability distribution p_T in scattering time sequences is analogous to that of length segmentations along the wire constructed in Sec. III A, with

$$p_T(\tau_{01},\ldots,\tau_{0L})=(1/\tau_{\rm in})^n P(0,\tau_{01})P(0,\tau_{12})\cdots P(0,\tau_{n-1,n})P(0,\tau_{n,L})\ , \tag{3.15}$$

where $\tau_{\rm in}$ is the mean inelastic-scattering time. Substituting the explicit expressions for the Poisson distribution from Eq. (3.2), the probability density becomes

$$p_T(\tau_{01},\ldots,\tau_{0L})=(1/\tau_{\rm in})^n\exp(-\tau_{01}/\tau_{\rm in})\exp(-\tau_{12}/\tau_{\rm in})\cdots\exp(-\tau_{nL}/\tau_{\rm in})\ . \tag{3.16}$$

The resistances of individual segments are now written in the form of Eq. (3.14), so that the ensemble-averaged sum of resistances becomes

$$\langle R(L)\rangle=\sum_{n=0}^{\infty}\int_0^{\tau_{nL}}d\tau_{n-1,n}\cdots\int_0^{\tau_{23}}d\tau_{12}\int_0^{\tau_{12}}d\tau_{01}[\xi(\tau_{01})+\xi(\tau_{12})+\cdots+\xi(\tau_{nL})]p_T(\tau_{01},\ldots,\tau_{0L})\ . \tag{3.17}$$

This integral is more complicated than Eq. (3.3). However, since we are interested in the case $\tau_{0L}/\tau_{\rm in} \ll 1$, we may expand the exponential factors and evaluate the resulting integrals analytically. Let us consider then $n=0$ term in the sum

$$R_0 = \xi(\tau_{0L})\exp(-\tau_{0L}/\tau_{\rm in}) . \tag{3.18}$$

The elastic resistance $\xi(\tau_{0L})$ is given by

$$\xi(\tau_{0L}) = (r_s/\nu)[\exp(\gamma\tau_{0L}^{1/2}/r_s\tau_{\rm el}^{1/2}) - 1] , \tag{3.19a}$$

where

$$\gamma \equiv \nu\alpha/\sqrt{3} . \tag{3.19b}$$

Expanding the exponentials in Eqs. (3.18) and (3.19a) for small argument, we have for the $n=0$ term

$$\begin{aligned} R_0 \approx{}& \gamma(\tau_{0L}/\tau_{\rm el})^{1/2} + \gamma^2(\tau_{0L}/\tau_{\rm el})/2r_s \\ & - \gamma(\tau_{0L}/\tau_{\rm el})^{1/2}(\tau_{0L}/\tau_{\rm in}) \\ & - \gamma^2(\tau_{0L}/\tau_{\rm el})(\tau_{0L}/\tau_{\rm in})/2r_s . \end{aligned} \tag{3.20}$$

For the $n=1$ case we obtain

$$\begin{aligned} R_1 = \int_0^{\tau_{0L}} d\tau_{01}[\xi(\tau_{01}) + \xi(\tau_{1L})](1/\tau_{\rm in}) \\ \times \exp(-\tau_{01}/\tau_{\rm in})\exp(-\tau_{1L}/\tau_{\rm in}) . \end{aligned} \tag{3.21}$$

Since the length segments of Eq. (3.13) must add to produce the total length (L) of the wire, we have the constraint

$$L = (v_F l_{\rm el}/3)^{1/2}(\tau_{01}^{1/2} + \tau_{12}^{1/2} + \cdots + \tau_{nL}^{1/2}) . \tag{3.22}$$

For the $n=1$ case, this relation may be used to determine τ_{1L} in terms of τ_{01}:

$$\tau_{1L} = \tau_{0L} - 2\tau_{0L}^{1/2}\tau_{01}^{1/2} + \tau_{01} .$$

Carrying out the various integrations yields the result

$$R_1 \approx \gamma(\tau_{0L}/\tau_{\rm el})^{1/2}(\tau_{0L}/\tau_{\rm in}) + \gamma^2(\tau_{0L}/\tau_{\rm el})(\tau_{0L}/r_{\rm in})/3r_s . \tag{3.23}$$

We note that the first term in this expression cancels with the third term in the expression for R_0 in Eq. (3.20). Since the $n>1$ cases contribute only higher-order terms in $(\tau_{0L}/\tau_{\rm in})$, the dominant contribution to the resistance comes from the $n=0$ and $n=1$ terms. Subtracting the residual resistance from the total resistance gives

$$R(L,\tau_{\rm in}) = -[\nu r_c^2(L)/6r_s][L/(v_F\tau_{\rm in})]^2 . \tag{3.24}$$

Since the classical resistance varies linearly in L, $R(L,\tau_{\rm in}) \sim (L^4/\tau_{\rm in}^2)$. As in the model of Sec. III A, to leading order in $(\tau_{0L}/\tau_{\rm in})$, the temperature-dependent resistance is negative.

IV. INELASTIC-SCATTERING MECHANISMS

Our next objective is to determine the inelastic-scattering lifetime $\tau_{\rm in}$, which produces the temperature dependence of $R(L,\tau_{\rm in})$. Rather than attempt first-principles calculations, we instead estimate this quantity, to the extent possible, from experimental data for bulk samples. We consider two mechanisms: electron-electron collisions and electron-phason scattering, since a combination of these have been proposed[21,22] to explain the temperature dependence of the electrical resistivity of large diameter samples (in the classical regime)[23–26] in the temperature range below 1.3 K. The large variations in magnitude and temperature dependence of resistivity in these samples suggests that both mechanisms are present, and that both are sample dependent.[22] Phasons are excitations associated with phase fluctuations of an incommensurate charge-density wave (CDW),[27] assuming that the ground state has the CDW structure. The sample dependence in both electron-electron scattering and electron-phason scattering contributions to the resistivity have been attributed to[22] a sample dependence of CDW domain structure, which is believed to be sensitive to surfaces.[27]

Of course the temperature-dependent classical resistivity by itself does not provide the appropriate inelastic-scattering lifetime, because the effectiveness of a scattering event is different for classical resistivity (where momentum is degraded) than for localization (where the phase coherence of a localized wave function is degraded). The inelastic quasiparticle lifetime (which involves essentially energy transfer) is more appropriate to localization effects, although it is not always correct. Altshuler *et al.*[28] have emphasized that when $\Delta E\tau_{\rm in} \leq 2\pi$, where ΔE is the energy change during an individual scattering event, then many such events are necessary in order to disrupt the phase coherence, so that the effective inelastic-scattering lifetime is longer than $\tau_{\rm in}$. Both mechanisms of interest here, when considered to be three-dimensional in character and estimated from bulk data (in a manner to be discussed shortly) have $\Delta E\tau_{\rm in} \gg 2\pi$, so that $\tau_{\rm in}$ is indeed the appropriate lifetime. We have accounted, in part, for the ineffectiveness of low-energy-transfer events by introducing a low-energy cutoff, but the best fit to the data occurs when the cutoff vanishes, which is consistent with our estimated values for $\tau_{\rm in}$. The interactions are three dimensional in character, in the case of phasons because the important wavelengths are less than the sample diameter, and in the case of electron collisions because the interaction length $(\hbar D/k_B T)^{1/2}$, where D is the diffusion constant given by Eq. (2.4), is less than the diameter of the sample, as applies in Refs. 8 and 9.

A. Electron-phason scattering

In a charge-density-wave (CDW) ground state,[29] the electronic charge density varies sinusoidally in space, which causes the lattice to deform to ensure charge neutrality. This deformation produces diffraction satellites around each of the Bragg reflections, which have recently been discovered in potassium by neutron-diffraction experiments.[30] In screening out the electronic charge in a CDW structure, each positive ion is displaced from its ideal lattice site $\mathbf{L}$ according to

$$\mathbf{u}(\mathbf{L}) = \mathbf{A}\sin[\mathbf{Q}\cdot\mathbf{L} + \phi(\mathbf{L},t)] , \tag{4.1}$$

where $\mathbf{Q}$ is the CDW wave vector, $\mathbf{L}$ is a lattice vector, and $\phi(\mathbf{L},t)$ is the phase. The resulting normal modes of the lattice corresponding to spatial and temporal fluctuations of this phase are called phasons.

The scattering of electrons from phasons has been shown to be an important mechanism in the resistivity below about 1.3 K.[21,22] The presence of a CDW produces gaps in the energy spectrum at wave vectors $\mathbf{k}=\pm\mathbf{Q}/2$, with an accompanying distortion of the spherical Fermi surface into conical-point regions close to the gaps. Since the energy spectrum and wave functions are distorted most severely in the neighborhood of the gaps, electron-phason scattering is most intense in these conical-point regions of the Fermi surface.

Because phasons are actually a type of lattice vibration, many of their properties are similar to those of phonons. In fact, the scattering of electrons from phasons is treated similarly to the scattering of electrons from phonons and, within Boltzmann transport theory, the temperature dependence of the electron-phason resistivity has been shown[21] to be given very closely by the Bloch-Grüneisen formula, but with a phason temperature, analogous to the Debye temperature, of only a few degrees, i.e., $\Theta_\phi=3.25$ K. However, the magnitude of the resistivity, or of the "transport" rate thus calculated, is quite different from what one would calculate with a Bloch-Grüneisen formula, because (1) the electron-phason interaction is confined to the conical point regions; (2) the Fermi-surface distortion changes the electronic velocity; and (3) the phason spectrum is highly anisotropic.

These same considerations are of importance in the calculation of the inelastic quasiparticle scattering rate due to electron-phason scattering, which is of interest here. The main difference between this quasiparticle rate and the transport rate is that here we require energy relaxation, whereas in the case of electrical resistivity we required momentum relaxation. If we assume a lower energy cutoff E_c below which scattering events are ineffective in destroying phase coherence, then the rate of decay of a quasiparticle population $f_{\mathbf{k}}$ in state $\mathbf{k}$ is given by

$$(\partial f_{\mathbf{k}}/\partial t)_{e\phi}=\int \frac{d^3k'}{(2\pi)^3}(W_{\mathbf{k}'\to\mathbf{k}}-W_{\mathbf{k}\to\mathbf{k}'})\Theta(E_c-\hbar\omega_{\mathbf{q}}) , \tag{4.2a}$$

where $\omega_{\mathbf{q}}$ is the frequency of a phason with wave vector $\mathbf{q}$, and

$$W_{\mathbf{k}\to\mathbf{k}'}=(2\pi/\hbar)(2\hbar/\rho_m)\sum_{\mathbf{q}}\gamma^2(\mathbf{k},\mathbf{k}')(q^2/\omega_{\mathbf{q}}) \times(1-f_{\mathbf{k}'})f_{\mathbf{k}}M(\mathbf{k},\mathbf{k}',\mathbf{q}) . \tag{4.2b}$$

Here $\gamma^2(\mathbf{k},\mathbf{k}')$ is a function that governs the strength of the electron-phason interaction; ρ_m is the mass density; and

$$M(\mathbf{k},\mathbf{k}',\mathbf{q})=[n_{\mathbf{q}}\delta(E_{\mathbf{k}'}-E_{\mathbf{k}}-\hbar\omega_{\mathbf{q}})\delta_{\mathbf{k}',\mathbf{k}+\mathbf{q}} +(n_{\mathbf{q}}+1)\delta(E_{\mathbf{k}'}-E_{\mathbf{k}}+\hbar\omega_{\mathbf{q}})\delta_{\mathbf{k}',\mathbf{k}-\mathbf{q}}] , \tag{4.2c}$$

with $n_{\mathbf{q}}$ the phonon distribution function, $E_{\mathbf{k}}$ the electronic energy in state $\mathbf{k}$, and $\delta(E)$ the δ function representing energy conservation in either absorption or, emission.

In computing the quasiparticle rate one assumes that $f_{\mathbf{k}}$ differs from $f^0_{\mathbf{k}}$ only at a single point $\mathbf{k}$. The resulting rate $\tau^{-1}_{\mathbf{k}}$ is then averaged over the Fermi surface, as seems appropriate because of the diffusion process, and over energies with weighting factor $(\partial f^0_{\mathbf{k}}/\partial E_{\mathbf{k}})$. The averaged inelastic quasiparticle scattering rate can then be written approximately in the form

$$\overline{\tau^{-1}_{e\phi,\mathrm{qp}}}=\frac{4\pi}{\hbar}\Gamma_{e\phi}(k_BT)^3J_3(\Theta_c/T,\Theta^{\mathrm{qp}}_\phi/T) , \tag{4.3}$$

where $\Gamma_{e\phi}$ is approximately constant and where we have defined the Bloch-Grüneisen function with a low-energy cutoff as

$$J_n(a,b)=\int_a^b dz\, z^n/[(e^z-1)(1-e^{-z})] . \tag{4.4}$$

In Eq. (4.3), the parameter $\Theta_c=E_c/k_B$ may be regarded as adjustable for the purpose of comparing with the data on thin samples. The phason temperature $\Theta^{\mathrm{qp}}_\phi$ that enters this expression is not necessarily the same as the phason temperature $\Theta^{\mathrm{tr}}_\phi$ that explains the low-temperature resistivity data. We have therefore chosen to assume $\Theta^{\mathrm{qp}}_\phi$ to be an adjustable parameter for our purposes here. However, since the difference between these two Θ_ϕ arises only because different weighting factors enter into the calculation of the transition rate in the two cases, one would not expect them to differ greatly, which imposes a constraint on the fitting procedure. The parameter $\Gamma_{e\phi}$, which fixes the magnitude of the quasiparticle-scattering time in Eq. (4.3), must be obtained from a full numerical calculation as in the theory of the electron-phason resistivity of Ref. 21, and we have used the same parameters in calculating $\Gamma_{e\phi}$ as allowed fits to the temperature-dependent resistivity of large diameter samples near 1 K. The resulting numerical values for $\Gamma_{e\phi}$ will be discussed in Sec. V.

B. Electron-electron scattering

Studies done by Apel and Rice,[31] which focus on one-dimensional systems having only static impurities and interacting electrons, result quantum-mechanically in the expression

$$l_{\mathrm{in}}(T)/l_{\mathrm{el}}\sim(T/E_F)^{\gamma-2} , \tag{4.5}$$

where $\gamma\geq 1$. If one tries to use this form of $l_{\mathrm{in}}(T)$ to reproduce the measured resistivity values, one obtains a temperature dependence that varies even more slowly than T^2 and the resulting curve possesses the wrong concavity for explaining the distribution of the experimental data points. The dependence given in Eq. (4.5) contrasts

with one-dimensional Landau-Baber scattering, which varies linearly in T.

Our approach is simply to infer the electron-electron quasiparticle rate following a procedure similar to that for electron-phason scattering, again assuming three-dimensional character and introducing a low-energy cutoff E_c in the energy transferred between electrons during collisions. Again we expect this cutoff to be small or zero since, in our case, essentially all collisions are effective in destroying phase coherence ($\Delta E \tau_{\rm in} \gg 2\pi$). The cutoff is incorporated in the expression for the decay rate of the distribution function[32]

$$[\partial f(\mathbf{k})/\partial t]_{ee} = \frac{2}{(2\pi)^6}\int d^3k_2 \int d^3k_3 \int d^3\mathbf{k}_4 \{ W(\mathbf{k}_3,\mathbf{k}_4 \rightarrow \mathbf{k}_1,\mathbf{k}_2) f(\mathbf{k}_3) f(\mathbf{k}_4)[1-f(\mathbf{k}_1)][1-f(\mathbf{k}_2)] - W(\mathbf{k}_1,\mathbf{k}_2 \rightarrow \mathbf{k}_3,\mathbf{k}_4) f(\mathbf{k}_1) f(\mathbf{k}_2)[1-f(\mathbf{k}_3)][1-f(\mathbf{k}_4)]\} \times \delta(E_3+E_4-E_1-E_2)\Theta(E_c-|E_3-E_1|) , \quad (4.6)$$

where

$$W(\mathbf{k}_3,\mathbf{k}_4 \rightarrow \mathbf{k}_1,\mathbf{k}_2) = (2\pi/\hbar)|\langle \mathbf{k}_1,\mathbf{k}_2|H_{ee}|\mathbf{k}_3,\mathbf{k}_4\rangle|^2 , \quad (4.7)$$

where H_{ee} is the effective electron-electron interaction Hamiltonian. For computing the quasiparticle rate, $f(\mathbf{k})$ differs from equilibrium only at a single point $\mathbf{k}$, and the resulting rate is averaged over Fermi surface positions and energies (weighted as before) to give[33]

$$\overline{\tau_{ee,\rm qp}^{-1}} = [\pi \Gamma_{ee}(k_B T)^2/(4\hbar E_F)] J_2(\Theta_c/T, \infty) , \quad (4.8)$$

where $J_n(a,b)$ is defined by Eq. (4.4), $\Theta_c = E_c/k_B$ as before, and

$$\Gamma_{ee} = 2N_0 \overline{|\langle \mathbf{k}_1,\mathbf{k}_2|H_{ee}|\mathbf{k}_3,\mathbf{k}_4\rangle|^2} \quad (4.9)$$

is a dimensionless number of order unity characterizing the collision strength, with $2N_0$ the density of states at E_F. The transport rate appearing in the classical electrical resistivity $\rho_{ee} = (m/ne^2)\tau_{ee,\rm tr}^{-1}$ is computed using the appropriate (current-carrying) form for $f(\mathbf{k})$, and of course no low-energy cutoff, with the result[33]

$$\overline{\tau_{ee,\rm tr}^{-1}} = [\pi^3 \Gamma_{ee}(k_B T)^2/(12\hbar E_F)]\Delta , \quad (4.10)$$

where $\Delta < 1$ is a measure of the relative amount of umklapp character in the scattering. In fact, Δ is precisely the ratio of the above two rates when there is no low-energy cutoff, since $J_2(0,\infty) = \pi^2/3$. Umklapp scattering is greatly enhanced in the direction of the CDW wave vector,[22] with $\Delta \approx 0.6$.

Finally, we may express the inelastic quasiparticle scattering rate parameter in terms of the measured temperature-dependent electrical resistivity at $T = 1$ K (assuming the dominant mechanism is electron-electron scattering)

$$\overline{\tau_{ee,\rm qp}^{-1}} = (ne^2/m)\rho_{ee}(T=1\ {\rm K}) T^2 [3J_2(\Theta_c/T,\infty)/\pi^2\Delta] , \quad (4.11)$$

which will be used in Sec. V to calculate the inelastic-scattering length $l_{\rm in}$.

V. NUMERICAL RESULTS

Our results are summarized by Eq. (3.24) for the temperature-dependent resistivity, with $\tau_{\rm in}$ given by Eqs. (4.3) and (4.11) for electron-phason and electron-electron scattering, respectively. In this section we compare these results with the experimental data for two of the thinnest samples shown in Fig. 1. The thinnest sample (the open diamonds in Fig. 1), which has a diameter of 0.09 mm and a length of 5 cm, has an estimated residual resistivity $\rho_0 = 127.2$ pΩ m and $\rho_{4.2\ \rm K} = 130$ pΩ m. The classical resistance corresponding to the residual resistance is $r_c(L) = 7.75\times10^{-8}\pi\hbar/e^2 = 1.00\times10^{-3}$ Ω. The value $\alpha \approx 3\times10^{-11}$ is determined from the classical resistance formula $r_c(L) = \alpha L/l_{\rm el}$, with $l_{\rm el} \approx 0.02$ mm. Because of the high precision of the resistivity measurements, we consider only the uncertainty in the temperature scale. The values of ΔT were designated uncertain to within 1% for $0.09 \le T < 0.8$ K and about 3% for both $T < 0.09$ K and $0.8 \le T \le 1.3$ K.[22]

We make the comparison to the data as measured by fitting the temperature derivative of resistivity using the theoretical functions as follows: First, noting that the temperature-dependent resistivity is proportional to $\tau_{\rm in}^{-2}$, according to Eq. (3.24), our predicted temperature dependences for the resistivity take the form $\rho_{e\phi}^{(\rm loc)}(T) = B_{e\phi}[T^3 J_3(\Theta_c/T, \Theta_\phi^{\rm qp}/T)/J_3(\Theta_c,\Theta_\phi^{\rm qp})]^2$ for electron-phason scattering, and $\rho_{ee}^{(\rm loc)}(T) = B_{ee}[T^2 J_2(\Theta_c/T,\infty)/J_2(\Theta_c,\infty)]^2$ for electron-electron scattering. Regarding Θ_c, $\Theta_\phi^{\rm qp}$, and the coefficients B as free parameters, we fit the derivatives of these functions to the data. Fits for the open diamond data of Fig. 1 for electron-electron and electron-phason scattering are shown in Fig. 3 as the solid and dashed curves. For both fits, the value found for Θ_c is indistinguishable from zero. This means that the electron-electron function is $\rho_{ee}^{(\rm loc)}(T) = B_{ee}T^4$, with $B_{ee} = -0.55$ fΩ m/K^4. The phason temperature is found to be $\Theta_\phi^{\rm qp} = 5.9$ K and the coefficient $B_{e\phi} = -0.46$ fΩ m/K^6. For comparison, another sample (the solid diamonds in Fig. 1) was fit to the electron-electron mechanism with $B_{ee} = -0.46$ fΩ m/K^4 and the result is shown in Fig. 3 by the dotted-dashed curve.

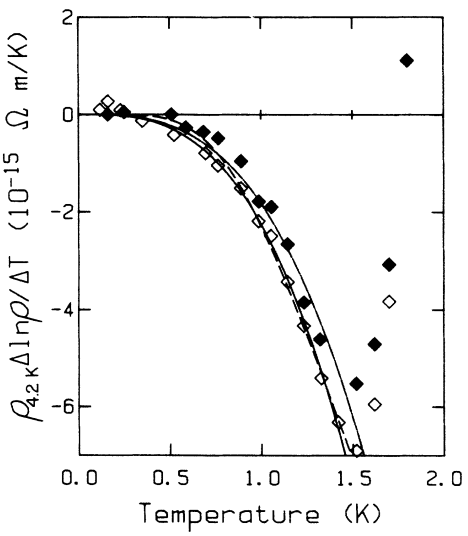

FIG. 3. Plot of $\rho_{4.2\text{ K}}\Delta\ln\rho/\Delta T$ vs temperature for the samples of Ref. 1 corresponding to the open and solid diamonds in Fig. 1. The solid and dashed curves, respectively, corresponding to electron-electron and electron-phason scattering, are fits to the open diamonds using temperature derivatives of Eq. (3.24), i.e., of $\rho_{ee}^{(\mathrm{loc})}(T)=B_{ee}[T^2J_2(\Theta_c/T,\infty)/J_2(\Theta_c,\infty)]^2$ for electron-electron scattering and of $\rho_{e\phi}^{(\mathrm{loc})}(T)=B_{e\phi}[T^3J_3(\Theta_c/T,\Theta_\phi^{\mathrm{qp}}/T)/J_3(\Theta_c,\Theta_\phi^{\mathrm{qp}})]^2$ for electron-phason scattering. The best fit parameters were $\Theta_c=0$ for both mechanisms, $B_{ee}=-0.55$ fΩ m/K^4, $\Theta_\phi^{\mathrm{qp}}=5.9$ K, and $B_{e\phi}=-0.46$ fΩ m/K^6. The dotted-dashed curve is a fit of electron-electron scattering to the solid diamonds with $\Theta_c=0$ and $B_{ee}=-0.46$ fΩ m/K^4.

While the "temperature" parameters Θ_c and $\Theta_\phi^{\mathrm{qp}}$ determine the functional forms of τ_{in}, the strength parameters $\Gamma_{e\phi}$ and Γ_{ee} in Eqs. (4.3) and (4.11), respectively, are determined by fitting the measured electrical resistivities with expressions involving the analogous transport times. The coefficients $B_{e\phi}$ and B_{ee} may thus be used to determine the number of sets of intimate channels (or the "channel number" parameter) ν. This channel parameter and the inelastic-scattering lifetime τ_{in} are determined for each mechanism as follows.

For electron-electron scattering, in Eq. (4.11), we use the maximum estimated value for the CDW umklapp fraction $\Delta=0.6$, and the largest value of the temperature-dependent resistivity observed in large samples[23] $\rho_{ee}(1\text{ K})=7.5\times10^{-15}$ Ω m, assuming that for that measurement, the sample essentially consisted of a single CDW Q-domain oriented along the length of the sample.[22] Quoting the inelastic-scattering lifetime in length units, we find as a result that $v_F\tau_{\mathrm{in},ee}\approx0.29$ m at 1 K, and $\nu=2.3\times10^4$.

For electron-phason scattering, with $\Theta_\phi^{\mathrm{qp}}=5.9$ K, $\Theta_c=0$, we have calculated $\Gamma_{e\phi}$ numerically using the procedure of Ref. 21, with the modifications discussed in Sec. IV A, using parameters that provide agreement with the low-temperature resistivity data for samples of large diameter. For a transverse phason velocity of $v_\perp=1.4\times10^5$ cm/sec and the phason anisotropy parameter $\eta=\frac{1}{9}$, we compute that $v_F\tau_{\mathrm{in},e\phi}\approx0.50$ cm at 1 K and $\nu=5.6$.

Unfortunately, $v_F\tau_{\mathrm{in},e\phi}$ is too short to satisfy $v_F\tau_{\mathrm{in}}\gg L$, under which Eq. (3.24) is valid. Furthermore, although $v_F\tau_{\mathrm{in},ee}$ satisfies this condition, since we must add the two contributions to the *rates*, $\tau_{\mathrm{in}}^{-1}=\tau_{ee,\mathrm{qp}}^{-1}+\tau_{e\phi,\mathrm{qp}}^{-1}$, the *net* value of $v_F\tau_{\mathrm{in}}$ is also too short. Since electron-electron scattering by itself is consistent with the data, perhaps for some reason electron-phason scattering becomes ineffective in the thinner samples. CDW domains are believed to orient near surfaces so that the CDW wave vector is perpendicular to the surface. This means that electrons are most strongly scattered by phasons when directed perpendicular to the wire axis. In classical transport theory this would reduce the effectiveness significantly, but in the localization regime, an analogous argument would probably not apply. For example, scattering by phasons would seem to retain its effectiveness in coupling transverse quantum numbers (causing transitions between channels), which would degrade localization effects. Phason ineffectiveness would more likely result from distortion of the CDW itself in the presence of a surface, which would be accompanied by a change in the phason spectrum. More significantly, the conical points of the Fermi surface could be significantly altered by a small CDW distortion, changing the electron-phason coupling.

Deviation of the theoretical curves from the data below 0.35 K is consistent with anomalous behavior seen in thicker samples by Lee *et al.*[26] These anomalies could indicate that a new phenomenon is appearing below 0.35 K. The rise in the data near 1.3 K is due to the exponential rise of electron-phonon umklapp scattering. When this (or presumably any other inelastic mechanism) becomes sufficiently strong, localization effects are destroyed, and the usual temperature-dependent increase in resistivity becomes visible. This effect is excluded in the present theory.

VI. CONCLUSIONS

We have found that the destruction of the coherence of one-dimensional localized electronic states by inelastic collisions can explain the decreasing resistivity with increasing temperature seen experimentally in thin potassium wires below 1.3 K. The decrease comes about because each inelastic event destroys the phase memory of the electrons, effectively breaking the wire up into a series of segments, such that each segment has a resistance that increases exponentially along its length, while the resistance of the whole wire is just the simple sum of resistances of the segments.

In order to compare our results with experiment, we considered the two inelastic mechanisms that dominate the temperature-dependent resistivity of large diameter samples below 1.3 K, namely electron-electron scattering and electron-phason scattering. The results are summarized in Eqs. (3.24), (4.3), and (4.11). A nonlinear least-squares fit to the data for each mechanism results in $\Theta_c=0$ for both mechanisms. For electron-electron scattering we determine an acceptable value of ν and show that $v_F\tau_{\mathrm{in}}>L$, consistent with our assumption lead-

ing to Eq. (3.24). Similar considerations for electron-phason scattering lead to $v_F\tau_{in} < L$, which is inconsistent.

If our estimate for the electron-phason τ_{in} is correct, this invalidates our simple expression $R(L,\tau_{in}) \sim \tau_{in}^{-2}$ [Eq. (3.24)] but not the more general result [Eq. (3.17)], since τ_{in} still satisfies the condition for one dimensionality $l_T > d$, where d is the diameter of the sample. Thus we would still predict a decrease in resistivity with increasing temperature, but the significant effect would be completed at temperatures below the range of interest in the experiment, leaving these data unexplained.

If, on the other hand, electron-phason scattering somehow becomes ineffective in degrading phase coherence, the remaining electron-electron mechanism provides agreement with the data. The most plausible route to such ineffectiveness may lie in a possible distortion of the CDW state as the wire becomes thinner. Such a distortion would probably be related to the tendency of the CDW wave vector to orient perpendicular to the nearest surface.

While efforts to confirm localization effects in one-dimensional systems have in the past been concentrated on very impure samples, with residual resistances ~ 10 kΩ, we have presented evidence for localization in the very pure samples of potassium. It is in fact the high purity that makes this possible, since this allows the elastic mean free path to be comparable, and the Thouless length to be long compared to the diameter of the wire.

The steep temperature dependence of the data, which conforms to our result $R(L,\tau_{in}) \sim \tau_{in}^{-2}$, is appropriate to the regime $\tau_{in} > L/v_F$. This regime applies if the simple estimate of electron-electron scattering, based on bulk resistivity data, is the correct one. However, this also requires that somehow electron-phason scattering does not contribute in a similar way to localization effects. In fact, the central prediction of this paper is that the temperature-dependent part of the resistivity should vary as τ_{in}^{-2}, or as $(1/l_T^4)$.

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*Present address: Acoustics Processing Division (Code 5032), U.S. Naval Air Development Center, Warminster, Pennsylvania 18974-5000.

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