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Fractional quantum Hall effect in a two-dimensional electron-hole fluid

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We show that a two-dimensional electron-hole fluid in a strong perpendicular magnetic field has a quantized Hall conductance equal to $e^2\nu_c/h$ at certain values of ν_c, where $\nu_c=\nu_e-\nu_h$ and ν_e and ν_h are the electron and hole filling factors. The ground states responsible for the quantized Hall effect consist of an incompressible fluid of excess charges and a Bose-condensed gas of excitons which do not interact either with each other or with the excess charge fluid. These results follow from an exact mapping between the electron-hole system and a two-component electron system.

The fractional quantum Hall effect[1] (FQHE) is a consequence of the strong correlations which occur in a two-dimensional electron system in a perpendicular magnetic field sufficiently strong that only the lowest Landau level is occupied. At certain electron densities the chemical potential jumps discontinuously and, at the point of discontinuity, the ground state has an excitation gap. This incompressibility leads[1] to dissipationless current flow and to a quantized value for the Hall conductance. Strong-correlation effects may also be expected to occur in the strong-field limit for an electron-hole system created, for example, by optically pumping a gas of e-h pairs in a semiconductor quantum well. The case in which the density of electrons and holes are equal has been studied previously by a number of authors.[2,3] Surprisingly, the ground state in this case can be found *exactly* and consists, in the ideal case (see below), of a Bose-condensed gas of noninteracting magnetic excitons. In this Rapid Communication we show that the ground state of an ideal charged two-dimensional electron-hole fluid in the strong-magnetic-field limit consists of a gas of electrons[4] and a Bose-condensed gas of excitons which do not interact either with each other or with the electron gas. This fact follows from an exact mapping between a spin-polarized electron-hole system and a spin-$\frac{1}{2}$ electron system which, in the ideal case, has a spin-rotationally-invariant interaction.[5] Dissipationless current flow and a quantized Hall conductivity will occur in the electron-hole fluid whenever the filling factor of excess electrons equals a filling factor for which the FQHE occurs in pure electron systems.

To establish the mapping we start by considering the Hamiltonian for a system of spin-$\frac{1}{2}$ electrons with zero g factor (i.e., no Zeeman energy) in the strong-magnetic-field limit where electrons are confined to a single Landau level and the kinetic energy is quenched. (The choice of zero g factor is a convenience and not a necessity.)

$$H_{ee}=\sum_{\sigma,\sigma'}\sum_{m_1,m_2}\sum_{m_1',m_2'}\langle m_1',m_2'|V_{\sigma,\sigma'}|m_1,m_2\rangle \times c^\dagger_{m_1',\sigma}c^\dagger_{m_2',\sigma'}c_{m_2,\sigma'}c_{m_1,\sigma}\,, \tag{1}$$

where $c^\dagger_{m,\sigma}$ creates an electron of spin σ and intra-Landau-level label m, and the two-particle matrix elements of the electron-electron interaction are to be taken between the single-particle kinetic-energy eigenstates for electrons.[1] Note that H_{ee} does not contain spin-flip interactions so that $N_\uparrow$ and $N_\downarrow$, the number of up- and down-spin electrons, are separately good quantum numbers. The mapping is based on a particle-hole transformation for the spin-down electrons. We define the electron creation operator by

$$c^\dagger_{m,\uparrow}\equiv\hat{e}^\dagger_m \tag{2}$$

and define the hole creation operator to be the down-spin annihilation operator,

$$c_{m,\downarrow}\equiv\hat{h}^\dagger_m\,. \tag{3}$$

An elementary calculation then shows that the Hamiltonian for a spin-polarized electron-hole fluid in the strong-field limit is equivalent to H_{ee} up to a constant:

$$H_{eh}=H_{ee}-(2\pi l^2)^{-1}[(N_\downarrow-N_L/2)V_{\downarrow,\downarrow}(\mathbf{q}=0)+N_\uparrow V_{\uparrow,\downarrow}(\mathbf{q}=0)]-(N_\downarrow-N_L/2)I_{\downarrow,\downarrow}\,, \tag{4}$$

where $N_L=A/(2\pi l^2)$ is the number of single-particle states in a Landau level, H_{eh} is the Hamiltonian for electrons and holes confined to a single Landau level with $V_{ee}=V_{\uparrow,\uparrow}$, $V_{eh}=-V_{\uparrow,\downarrow}$, and $V_{hh}=V_{\downarrow,\downarrow}$, $[l\equiv(\hbar c/eB)^{1/2}]$. In H_{eh}, $\hat{e}^\dagger$ is the electron creation operator and $\hat{h}^\dagger$ is the hole creation operator so that the numbers of electrons and holes are related to the numbers of up- and down-spin electrons by $N_e=N_\uparrow$ and $N_h=N_L-N_\downarrow$. Equation (4) follows from the fact that the hole kinetic-energy eigenstates are the complex conjugates of the electron kinetic-energy eigenstates and from the following gauge-independent properties of the kinetic-energy eigenstates[1] in the lowest Landau level:

$$\sum_{m_1}\langle m_1,m_2|V_{\sigma,\sigma'}|m_1,m_2\rangle=(2\pi l^2)^{-1}V_{\sigma,\sigma'}(\mathbf{q}=0)\,, \tag{5}$$

$$\sum_{m_1}\langle m_1,m_2|V_{\sigma,\sigma'}|m_2,m_1\rangle=\int\frac{d^2\mathbf{q}}{(2\pi)^2}V_{\sigma,\sigma'}(\mathbf{q})\times\exp(-|\mathbf{q}l|^2/2)\equiv I_{\sigma,\sigma'}\,. \tag{6}$$

For an ideal two-dimensional electron-hole fluid (see below)

$$V_{e,e}(\mathbf{q}) = V_{h,h}(\mathbf{q}) = -V_{e,h}(\mathbf{q}) = 2\pi e^2/q \tag{7}$$

so that the corresponding electron system has spin-independent e^2/r interactions. Defining a net charge number and an exciton number by

$$N_c \equiv N_e - N_h \tag{8}$$

and

$$N_{ex} \equiv N_e - N_c = N_h , \tag{9}$$

Eq. (4) can be rewritten for this case as

$$H_{eh} - E_{eh}^H = H_{ee} - E_{ee}^H - E_0 - N_{ex} I , \tag{10}$$

where E_{eh}^H and E_{ee}^H are the infinite Hartree energies of the charged electron-hole and spin-$\frac{1}{2}$ electron systems which we may set to zero in the presence of neutralizing backgrounds. [The contribution coming from the right-hand side of Eq. (5) is canceled by the interaction with a uniform background.] $E_0 = -N_L I/2$ is the energy of the neutralized electron-electron system for $N_\uparrow = 0$ and $N_\downarrow = N_L$ (i.e., in the absence of electrons or holes).

We wish to consider what happens to the electron-hole system as N_{ex} is increased (i.e., as electron-hole pairs are added) at fixed total charge N_c. At $N_{ex} = 0$ the $\downarrow$ Landau level in the corresponding spin-$\frac{1}{2}$ electron system is full and $N_\uparrow = N_c$. As N_{ex} is increased in the electron-hole system electrons are flipped from down-spin to up-spin in the spin-$\frac{1}{2}$ system. The z component of total spin is related to N_c and N_{ex} by

$$S_z = (N_c - N_L)/2 + N_{ex} . \tag{11}$$

Since the interaction in the spin-$\frac{1}{2}$ system is spin rotationally invariant it follows that eigenstates of H_{ee} are eigenstates of total spin. All states with the $\downarrow$ Landau level full (i.e., no excitons) have spin quantum number[6] $S = (N_L - N_c)/2$ and, therefore, have degeneracy $N_L - N_c + 1$. One of these states occurs at each value of S_z. It follows that in the electron-hole problem as N_{ex} is increased there will be an exact many-body eigenstate at each value of N_{ex} with energies given by

$$E_{eh} = E_1(N_c) - N_{ex} I , \tag{12}$$

where $E_1(N_c)$ ground-state energy of a one-component plasma and I, defined in Eq. (6), may be recognized[7] as the binding energy of a single electron-hole pair in the strong-magnetic-field limit in a state of zero total momentum. The fact that E_{eh} is *exactly* linear in N_{ex} with coefficient $-I$ shows that the magnetoexcitons do not interact with each other and that all are in their zero-momentum state. The fact that E_{eh} is the sum of the excitonic contribution and $E_1(N_c)$ shows that the gas of $\mathbf{k} = 0$ excitons do not interact with the excess charges. We conclude from Eq. (12) that these exact eigenstates consist of a gas of noninteracting magnetoexcitons Bose condensed in a common $(\mathbf{k} = 0)$ state which do not interact with the excess electrons.

If the absolute ground state of the two-spin system is maximally polarized [i.e., has $S = (N_L - N_e)/2$], Eq. (12) gives the ground-state energy of the electron-hole system from $N_{ex} = 0$ to $N_{ex} = N_L - N_c$, at which point the electron Landau level is full. Equation (12) shows that as electron-hole pairs are added they bind to form excitons which, in the many-body ground state, do not interact either with the original system or with each other. This fact is not at all obvious and is our principal result. If the absolute ground state of the two-spin system at $N_\uparrow + N_\downarrow = N_L + N_c$ is not maximally polarized the state whose energy is given by Eq. (12) will be preempted for at least one value of N_{ex}. Electron-hole states corresponding to spin-$\frac{1}{2}$ states with spin quantum number S will occur for

$$(N_L - N_c)/2 - S \le N_{ex} \le (N_L - N_c)/2 + S . \tag{13}$$

For example, electron-hole states corresponding to singlet two-spin states, expected[8] to be absolute ground states for $\nu_c \equiv \lim_{N_L \to \infty}(N_c/N_L) = \frac{2}{5}$ and other filling factors, occur at only one value of N_{ex}.

In terms of the above discussion, previous[2,3] results for $N_c = 0$ follow directly from the fact that the ground state of the two-spin system at $\nu = 1$ is fully spin polarized. The surprising properties predicted theoretically for the *neutral* electron-hole fluids have been known for some time but, as far as we are aware, no practical way has been found to study them experimentally. (There is, however, at least in principle, a possibility of detecting superflow of the exciton condensate in an electric field.[3,9]) On the other hand, as we discuss below, the quantum Hall effect can be exploited to study the analogous properties of *charged* electron-hole fluids. Furthermore, transport studies of charged electron-hole fluids offer the possibility of obtaining information on two-spin electron systems which would not be available from direct studies. Our remarks here are based on the assumption that the electron-hole recombination time is long compared with both the time required for the photoexcited electron-hole pairs to reach equilibrium and the time required to measure the electrical transport coefficients of the two-dimensional electron-hole layer. The quantum Hall effect occurs when[1] the energy to add charges to the system differs from the energy to remove charges. With N_{ex} fixed, the quantum Hall effect in the electron-hole fluid will occur at electron densities where the energy to increase N_c by one changes discontinuously. In this circumstance, the dissipation in the system will be activated with an activation energy given by the energy necessary to create independent electron-hole pairs in the system, i.e., by the discontinuity in the chemical potential associated with changes in N_c. Equation (12) then shows that the fractional quantum Hall effect will occur in the electron-hole fluid when ν_c equals a filling factor at which the effect occurs in a spin-polarized electron system. Moreover, the activation energy will be independent of N_{ex}, provided that the two-spin ground state at filling factor $\nu = 1 + \nu_c$ is maximally spin polarized. This is true despite the presence of many low-lying excitations of the exciton condensate as illustrated in Figs. 1 and 2, where we present the results of finite-size exact diagonalization studies of the electron-hole system. When the two-spin state is not maximally spin polarized the activation energy will be changed over the range of

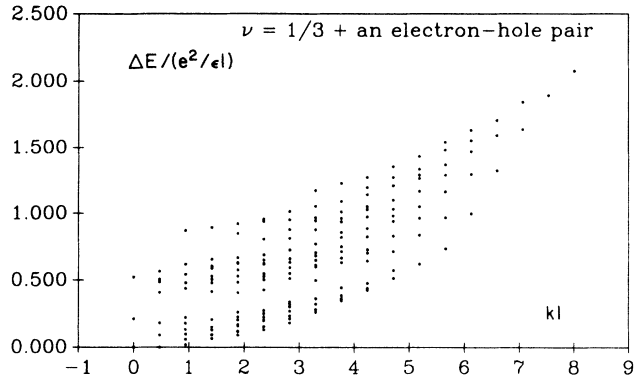

FIG. 1. Spectrum for a small electron-hole system with $N_c = 4$ and $N_{ex} = 1$ on the surface of a sphere at charge filling factor $\frac{1}{3}$. The low-lying excitations at the smallest values of wave vector k are magnetoexciton excitations. Energies are in units of $e^2/\epsilon l$ and lengths are in units of l.

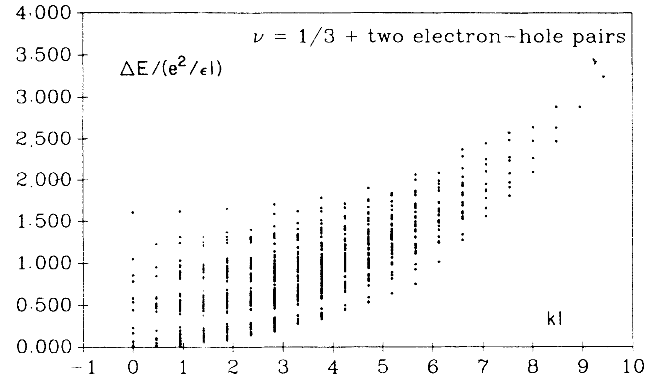

FIG. 2. Spectrum for a small electron-hole system with $N_c = 4$ and $N_{ex} = 2$ on the surface of a sphere at charge filling factor $\frac{1}{2}$. The new low-lying $k = 0$ excitons are two-magnetoexciton states. Energies are in units of $e^2/\epsilon l$ and lengths are in units of l.

N_{ex} specified by Eq. (13). Thus transport experiments on charged electron-hole fluids as a function of N_{ex} at fixed N_c can unambiguously determine the spin quantum number of the absolute ground state in the corresponding spin-$\frac{1}{2}$ electron system.

An electron-hole fluid confined to a quantum well differs from an ideal two-dimensional fluid because of the finite thicknesses and possible spatial separation of the electron and hole layers and because of heavy-hole–light-hole mixing in the valence band. In the strong-field limit it is possible to account for these effects by changing the effective interactions among the electrons and holes.[3] Realistic effective interactions will not lead to spin-rotationally-invariant interactions in the corresponding two-component electron systems so that the separation between charge and excitonic subsystems in the ground state will no longer be perfect. The finite gap for charged excitations responsible for the FQHE discussed above cannot be removed by a infinitesimal change in the effective interactions. The FQHE therefore can survive in the realistic case, and its occurrence or nonoccurrence may depend on the details of the electron and hole wave functions. The degree of spin-rotational symmetry breaking in the equivalent spin-$\frac{1}{2}$ system can be altered by applying a bias across the quantum well containing the electron-hole system. (This may be useful if a lengthening of the electron-hole recombination time is desired.) In the limit of large biases the electron and hole systems will be widely separated and we may expect similarities between the electron-hole[10] system and two-layered electron systems.[11,12] (We emphasize that the mapping between electron-hole systems and spin-$\frac{1}{2}$ electron systems is still valid.)

In summary, we have found that an exact mapping exists between a spin-polarized electron-hole fluid and a two-component electron fluid in the strong-magnetic-field limit. For an ideal electron-hole fluid the corresponding spin-$\frac{1}{2}$ system has a spin-rotationally-invariant interaction. On the basis of this mapping we are able to conclude that an ideal two-dimensional electron-hole fluid has a ground state which consists of a Bose condensate of excitons which do not interact either with each other or with the fluid formed by the excess charges. We predict that the charged electron-hole fluid exhibits a fractional quantum Hall effect when the filling factor of excess carriers is a fraction with an odd denominator and that the activation energy for dissipation is independent of the exciton-condensate density. We believe that these predictions can be tested experimentally by measuring transport coefficients and that, since fixed exciton number corresponds to fixed polarization in the two-component electron system, transport studies of the electron-hole system will provide information which cannot be obtained by direct study of two-component electron systems.

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[4]We discuss the case where the excess carriers are electrons. The situation when the excess carriers are holes is equivalent.

[5]It has recently been emphasized by A. B. Dzyubenko and Yu. E. Lozovik (unpublished) that the exact ground state can be

found for the neutral electron-hole system *because* of a hidden symmetry in the Hamiltonian of the system. In the corresponding two-spin system this symmetry is simply the invariance of the Hamiltonian under rotations in spin space.

[6]S must equal at least $|S_z| = N_L - N_c$ and cannot exceed $N_L - N_c$ since there are no states in the Hilbert space with larger values of $|S_z|$.

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