PHYSICAL REVIEW B VOLUME 42, NUMBER 5 15 AUGUST 1990-I

Phonon-assisted cyclotron resonance in *n*-type quantum-well structures

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(Received 21 March 1990)

We have calculated the phonon-assisted cyclotron resonance in *n*-type two-dimensional systems where absorption or emission of an optical phonon accompanies a Landau transition. The expression for the phonon-assisted cyclotron resonance absorption coefficient is obtained using the linear-response theory and Green's-function method. A canonical transformation is used to eliminate the electron–optical-phonon interaction to first order. From the above formulation we have calculated the line shape for usual cyclotron resonance and a phonon-assisted sideband in the Faraday configuration for left- and right-circularly-polarized light. An additional peak in the absorption coefficient due to phonon-assisted transition is predicted $GaAs/Ga_{1-x}Al_xAs$ quantum well.

I. INTRODUCTION

Two-dimensional (2D) electrons formed at metal-oxide-semiconductor interfaces (i.e., inversion layers), semiconductor heterojunctions, and superlattices have attracted attention not only from the viewpoint of device applications, but also from the viewpoint of pure physics. In device applications the high-electron-mobility transistor,[1] quantum-well laser diode,[2] and resonant-tunneling hot-electron transistor[3] are being pursued. In pure physics, 2D confinement of the electron has revealed a number of interesting physical properties. In particular, an external magnetic field applied perpendicular to the interface yields pronounced oscillatory behavior in various quantities related to electrical, magnetic, and thermal properties.[4]

The effect of electron-phonon interactions on the transport and optical properties of 2D systems has been investigated extensively.[5–11] Understanding of phonons (especially longitudinal-optical phonons) is essential in estimating the strength of electron-phonon interactions, which are one of the most important factors having great influence on device performance through electron velocity and mobility. Recently there has been a considerable interest in the study of phonon-assisted transport and optical experiments and theories. These include the phonon-assisted electron tunneling,[12–16] phonon-assisted hot-electron relaxation,[17] phonon-assisted emission of free excitons,[18] and phonon-assisted cyclotron resonance (CR) in 2D systems. However, the phonon-assisted cyclotron resonance in 2D systems has attracted less attention in contrast to 3D systems.[19] Recently, we have studied the phonon-assisted dielectric response of *p*-type 2D single-quantum-well structures in the presence of a strong magnetic field.[20] We have calculated the phonon-assisted cyclotron resonance in which absorption or emission of a phonon accompanies the cyclotron transition. In this paper we apply our theory to *n*-type systems to calculate their absorption spectrum. Our main finding is that the phonon-assisted CR peak will have an intensity of $\sim 10\%$ of that of the usual CR, and that it should be observable experimentally. In the rest of the paper we outline briefly the theory for the calculation of conductivity tensor, present our results of phonon-assisted cyclotron resonance, and compare them with other approaches.

II. THEORY

We consider *n*-type single-quantum-well structure where 2D electrons move in the *x*-*y* plane. A uniform, static magnetic field **B** is applied externally in the *z* direction. The Hamiltonian of the system is

$$H_p=\sum_{n,p}E_{np}c^\dagger_{np}c_{np}+\sum_{\mathbf{Q}}\hbar\omega_{\mathbf{Q}}b^\dagger_{\mathbf{Q}}b_{\mathbf{Q}} -i\sum_{\mathbf{Q}}\sum_{\substack{n,n'\\p,p'}}\gamma(\mathbf{Q})D^{p'p}_{n'n}(\mathbf{Q})c^\dagger_{n'p'}c_{np}(b_{\mathbf{Q}}-b^\dagger_{-\mathbf{Q}}) , \quad (1)$$

where

$$D^{p'p}_{n'n}(\mathbf{Q})=F_{p'p}(Q_z)J_{n'n}(Q_\perp) ,$$
$$F_{p'p}(Q_z)=\langle p'|e^{iQ_zz}|p\rangle , \quad (2)$$
$$J_{n'n}(Q_\perp)=(-1)^n\left[\frac{n'!}{n!}\right]^{1/2}x^{(n'-n)/2}\times L_n^{n'-n}(x^2)e^{-x^2/2},\quad x^2=\frac{Q_\perp^2l^2}{2} .$$

In the above equation, E_{np} is the energy of the *n*th Landau level in the *p*th subband; $c^\dagger_{np}$ ($b^\dagger_{\mathbf{Q}}$) and c_{np} ($b_{\mathbf{Q}}$) are the Landau (phonon) creation and annihilation operators, respectively. The phonon wave vector **Q** has components along the *x*-*y* plane and the *z* direction which are denoted $Q_\perp$ and Q_z, respectively. $\omega_{\mathbf{Q}}=\omega_{\mathrm{L}}$ is the frequency of the optical phonons which we consider to be dispersionless, and $l=(\hbar c/eB)^{1/2}$ is the magnetic radius. The electron–optical-phonon coupling is given by[11]

$$|\gamma(\mathbf{Q})|^2=\frac{4\pi\alpha}{Q^2}\left[\frac{\omega_{\mathrm{L}}^3}{2m}\right]^{1/2} , \quad (3)$$

where α is the dimensionless coupling constant whose value for GaAs is 0.07.

The interaction between the electrons and the electromagnetic radiation is written

$$H_{\rm int}=\frac{i}{\omega}e^{-i\omega t}\mathbf{E}_0\cdot\mathbf{j}(\mathbf{q}) , \tag{4}$$

where $\mathbf{j}(\mathbf{q})=\frac{1}{2}[\mathbf{j}\exp(i\mathbf{q}\cdot\mathbf{r})+\exp(i\mathbf{q}\cdot\mathbf{r})\mathbf{j}]$. $\mathbf{E}_0$ is the electric field vector of the electromagnetic wave, $\mathbf{q}=(q_\perp,q_z)$; $q_\perp$ and q_z are the photon wave vectors parallel and perpendicular to the 2D plane, respectively. We use a circular-polarization representation whose basis vectors are $\hat{\mathbf{e}}_\pm=(\hat{\mathbf{e}}_x\pm i\hat{\mathbf{e}}_y)/\sqrt{2}$ and $\hat{\mathbf{e}}_z$. Here the unit vectors $\hat{\mathbf{e}}_x$, $\hat{\mathbf{e}}_y$, and $\hat{\mathbf{e}}_z$ are taken in the x, y, and z directions, respectively. The components of the current operator relative to this basis become $j_-=el(\partial H/\partial c_{np})$ and $j_+=el(\partial H/\partial c^\dagger_{np})$.

We are interested in the phonon-assisted cyclotron resonance at low temperature, where Landau-level broadening is mainly due to impurity scattering and electron–acoustic-phonon interactions. Therefore we consider only one-phonon processes, and neglect higher-order effects. The electron–optical-phonon interaction term is removed from the Hamiltonian to first order in the coupling $\gamma(\mathbf{Q})$ by the canonical transformation $\tilde{H}=\exp(iS)H\exp(-iS)$, in which

$$S=\sum_{\substack{n',n,\\ p',p}}\sum_{\mathbf{Q}}\gamma(\mathbf{Q})D^{p'p}_{n'n}c^\dagger_{n'p'}c_{np}\left[\frac{b_{\mathbf{Q}}}{E_{n'p'}-E_{np}-\omega_{\rm L}}-\frac{b^\dagger_{-\mathbf{Q}}}{E_{n'p'}-E_{np}+\omega_{\rm L}}\right] . \tag{5}$$

The transformed current operator is obtained from $\tilde{H}$,

$$\tilde{J}_\pm(\mathbf{q})=j^0_\pm(\mathbf{q})+i[S,j^0_\pm(\mathbf{q})]\equiv j^0_\pm(\mathbf{q})+j^1_\pm(\mathbf{q}) . \tag{6}$$

If we now employ the Kubo formula,

$$\sigma_{-+}(\omega)=\frac{1}{2\omega}\int_0^\infty dt\, e^{i(\omega+i\eta)t}\langle[\tilde{J}_-(\mathbf{q},t)\tilde{J}_+(\mathbf{q},0)]\rangle , \tag{7}$$

to calculate the conductivity tensor, the current-current correlation function will consist of four terms:

$$\langle[\tilde{J}_-(\mathbf{q},t),\tilde{J}_+(\mathbf{q},0)]\rangle=\langle[j^0_-,j^0_+]\rangle+\langle[j^0_-,j^1_+]\rangle+\langle[j^1_-,j^0_+]\rangle+\langle[j^1_-,j^1_+]\rangle . \tag{8}$$

The first term gives rise to $\sigma^0(\omega)$, the conductivity tensor in the absence of phonons; the second and third terms have a vanishing contribution since phonon operators $b^\dagger_{\mathbf{Q}}$ and $b_{\mathbf{Q}}$ have zero trace in the noninteracting basis representation.[20] The last term, $\sigma^1(\omega)$, introduces a new contribution to the conductivity tensor due to the phonon-assisted transition between two Landau levels.

We can now calculate the absorption coefficient for normal and phonon-assisted cyclotron resonance by $\alpha\sim{\rm Re}\sigma(\omega)$. If we choose the electromagnetic wave vector $\mathbf{q}$ parallel to the magnetic field direction (viz., Faraday configuration), the expression for the absorption coefficient α^1 simplifies to (apart from some constants)

$$\alpha^1_-(\omega)=\frac{1}{4\omega\omega_{\rm L}^2}\sum_{\mathbf{Q}}\sum_{\substack{n,n'\\ p,p'}}\frac{|\gamma(\mathbf{Q})|^2\,\Gamma_{n'p'}N^Q_{np,n'p'}D^{p'p}_{n',n-1}(\mathbf{Q})D^{p'p}_{n',n+1}(\mathbf{Q})}{(E_{np}-E_{n'p'}\pm\omega_{\rm L}+\omega)^2+\Gamma^2_{n'p'}} , \tag{9}$$

where

$$N^Q_{np,n'p'}=N_{\mathbf{Q}}(f_{np}-f_{n'p'})+f_{np}(1-f_{n'p'}) \tag{10}$$

is a thermal factor, in which $f_{np}=f(E_{np})$ and $N_{\mathbf{Q}}=N_0(\omega_{\rm L})$ are the Fermi and Bose distribution functions. The Landau-level broadening Γ_{np} is the imaginary part of the electron self-energy in our formulation.[20,21] In comparison, the absorption coefficient when phonon modes are absent reads

$$\alpha^0_-(\omega)=\frac{1}{4\omega}\sum_{\substack{n,n'\\ p,p'}}\frac{f(E_{n'p'})\Gamma_{np}}{(E_{n'p'}-E_{np}+\omega)^2+\Gamma^2_{np}} . \tag{11}$$

Similar results for the absorption coefficient in p-type GaAs/$Ga_{1-x}Al_xAs$ were obtained earlier.[20] The expressions we have given for $\alpha(\omega)$ are for the left-circularly-polarized light; $\alpha_+(\omega)=\alpha_-(-\omega)$ yields results for the right-circularly-polarized light.

We can see directly from Eq. (9) that the absorption peaks will appear at $\omega=E_{n'p'}-E_{np}\pm\omega_{\rm L}$. When a transition takes place within one subband (viz., $p\to p$) and different Landau levels (i.e., $n\to n+1$) according to the above energy conservation, it is called phonon-assisted cyclotron resonance. At low temperatures, the term with the plus sign will dominate, corresponding to the emission of a phonon. When $p\neq p'$ and one has $\Delta n=\pm1$, it is termed combined phonon-assisted cyclotron resonance.

III. RESULTS AND DISCUSSION

We have numerically evaluated the absorption coefficients $\alpha^1(\omega)$ and $\alpha^0(\omega)$ in Eqs. (9) and (11), respectively. For the matrix element $F_{pp'}$ we assumed a square-well potential with wave functions in the z direction, $|p\rangle\sim\sin(2\pi pz/L)$, of width $L=100$ Å, appropriate to

$GaAs/Ga_{1-x}Al_xAs$ superlattices. Use of a variational wave function $|p=1\rangle \sim \exp(-bz)$, where b is a variational parameter, does not change the main conclusions reached in this work. We use 3D optical phonons [see Eq. (3)], since we include the extension of the electron system in the z direction; hence the summation in Eq. (9) is over 3D wave vector $\mathbf{Q}$. Our results are presented and discussed in the following.

Figure 1 shows the calculated absorption coefficients at $B=15$ T and $T=2$ K for a constant value of the broadening parameter Γ_{01}. The first peak positioned at $\omega=\omega_c$ is the main cyclotron resonance; the second peak is due to the phonon emission in the transition from an $n=1$ to an $n=0$ state. The latter occurs at $\omega=\omega_c+\omega_L$, where $\omega_L=36.2$ meV. We observe that the intensity of the phonon-assisted transition peak is $\sim 10\%$ of that of the cyclotron-resonance peak. This is to be compared with the case in p-type single-quantum-well structures, where we have found that the phonon-assisted transition-peak intensity to be 20–30% of the CR intensity.[20] Similar to the p-type case, in n-type systems such an effect is expected to be observed experimentally as well. The Landau-level broadening Γ_{01} has no effect on the locations or the relative intensities of the peaks. We have used $\Gamma_{01}=0.015\omega_L \approx 0.54$ meV in Fig. 1, and, in general, as Γ_{01} increases the cyclotron-resonance linewidth increases.

We display in Fig. 2 the magnetic field dependence of the ratio of the peak intensities of the cyclotron resonance and phonon-assisted transition. The ratio of the peak intensities is an increasing function of the applied magnetic field. Curves A, B, and C in Fig. 2 refer to $T=2, 100$, and 200 K, respectively. It is interesting to

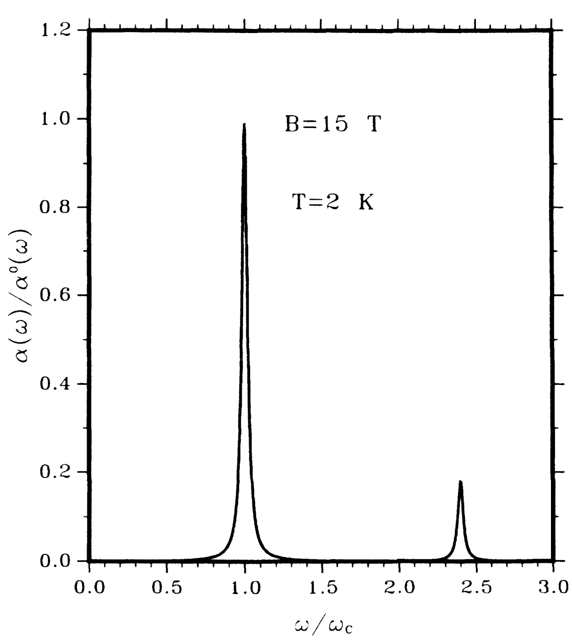

FIG. 1. Absorption coefficient $\alpha(\omega)$ divided by $\alpha^0(\omega)$ vs ω/ω_c for electrons in the $GaAs/Ga_{1-x}Al_xAs$ superlattice, at $T=2$ K and $B=10$ T. The first peak originates from $\alpha^0(\omega)$; the second peak is the phonon-assisted resonance coming from $\alpha^1(\omega)$.

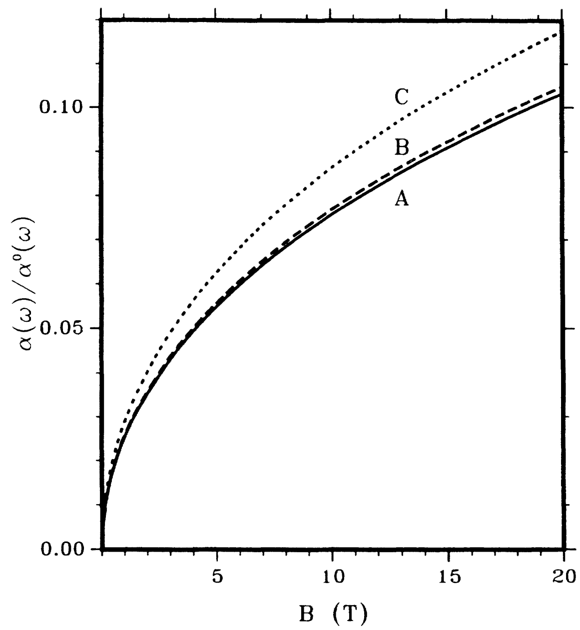

FIG. 2. Ratio of the peak values of the absorption coefficients $\alpha^1(\omega_L+\omega_c)$ and $\alpha^0(\omega_c)$ vs the magnetic field B, at $T=2$ K (solid line), $T=100$ K (dashed line) and $T=200$ K (dotted line).

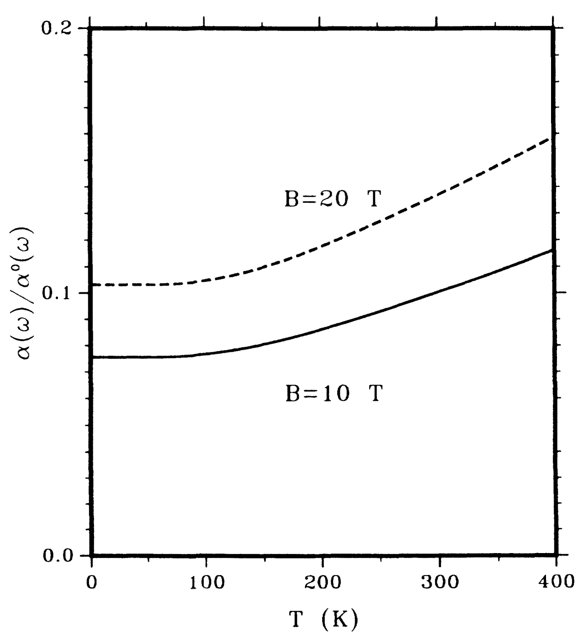

FIG. 3. Ratio of the peak values of the absorption coefficients $\alpha^1(\omega_L+\omega_c)$ and $\alpha^0(\omega_c)$ vs temperature T, at $B=10$ T (solid line) and $B=20$ T (dashed line).

note from Fig. 2 that the difference in the ratio of the peaks between two temperatures (i.e., $T=100$ and 200 K) increases with increasing magnetic field. We have used Eq. (9) to calculate curves B and C in Fig. 2 at high temperatures. The optical-phonon energy ω_L is 36.2 meV, which corresponds to ~ 420 K; hence we expect our one-phonon– process approximation to be valid in the temperature range of interest here. In Fig. 3 we show the temperature dependence of the peak ratio at fixed magnetic fields. We observe that according to Eqs. (9) and (11) the ratio of the peaks at a given magnetic field remains essentially constant until around 100 K, and then increases with T.

Clearly, it follows from Eq. (9) that the intensity of the phonon-assisted CR peak in the absorption spectrum is directly proportional to the electron-phonon– interaction strength. The larger the coupling constant α, the more pronounced the phonon-assisted CR peak will be. This, in turn, will have experimental significance for semiconductor heterojunctions and superlattices—namely, in the CR experiments one can determine the strength of the electron–optical-phonon interaction (i.e., coupling constant α) directly by measuring the intensity of the phonon-assisted CR peak and by using Eq. (9).

Recently, Goldman *et al.*[13] have observed phonon-assisted resonant tunneling (PART) due to optical phonons in a double-barrier structure, and Cai *et al.*[15] have reported a model calculation of the transmission coefficient for the phonon-assisted electron tunneling through a semiconductor double barrier. They found an extra peak corresponding to phonon-assisted resonance in the transmission coefficient as a function of the incident energy. This is quite suggestive in view of our calculation, since it is essentially the same phenomenon we are describing in this work. According to the calculation of Cai *et al.*,[15] the phonon-assisted resonant-tunneling peak intensity is ~ 15–20% of the main transmission peak, which is about the same as ours. Furthermore, they found that the PART peak intensity increases with temperature as well as with electron-phonon–interaction strength. Our findings are consistent with the results of Cai *et al.*,[15] and we feel that the phonon-assisted CR peak should be observed in a low-temperature, high-magnetic-field, CR experiment.

Xiaoguang *et al.*[9] have studied the magneto-optical-absorption spectrum of a 2D polaron using the memory-function formalism and have investigated the polaron effective mass in the weak-electron-phonon-coupling limit and the phonon-assisted harmonics. They have calculated the intensities of the different harmonics positioned at $\omega=\omega_L+n\omega_c$ ($n=1,2,\ldots$). Our calculation also predicts phonon-assisted harmonics [see Eq. (9)] at the same positions in the absorption spectrum, but here we report only the relative intensity of the phonon-assisted peak (first harmonic) to the main CR peak. In Ref. 9 the intensity and peak position of the phonon-assisted CR peaks depend on the Landau-level-broadening parameter Γ. We have found that Γ_{01} does not affect the peak positions and relative intensity. They have reported that for $\Gamma=0.1\omega_L$ the intensity of the phonon-assisted peak more than doubles between magnetic field values of $B=5.0$ and 10.0 T. Our calculation (see Fig. 2) indicates a similar trend at low magnetic fields, perhaps less pronounced, but generally consistent with their results.

In the present calculation the Landau-level broadening Γ_{01} is taken as an input parameter, since our aim was to obtain the relative intensities of cyclotron resonance and phonon-assisted transition peaks rather than to employ a first-principles calculation of the CR line shape. Our results are independent of Γ_{01}, as it cancels out in the ratio of the peaks. However, the actual line shape of the absorption spectrum and the cyclotron-resonance linewidth will depend on the Landau-level broadening. We have presented our results at a value of $\Gamma_{01}=0.015\omega_L$, which is a realistic one. In the low-temperature regime the broadening will be mainly due to impurity scattering and electron–acoustic-phonon interactions. It appears necessary to calculate the level broadening self-consistently, including screening effects to obtain reasonable agreement with the experimental data. Our self-consistent calculations[22] yield $\Gamma\approx 0.10$–0.20 meV in the temperature range $0<T<10$ K, consistent with the value used for Γ_{01} here.

Although we have not presently investigated the effect of the temperature dependence of the Landau-level broadening Γ on the CR peak intensities, we can embark on a qualitative discussion. The temperature dependence of Γ will have an observable influence on the absolute magnitude of the CR peak intensities. In general, Γ increases as temperature increases and, since $\alpha^0(\omega_c)\sim 1/\Gamma$, at the peak position, the intensity of the CR peak will decrease with temperature. Similarly $\alpha^1(\omega_c+\omega_L) \sim N^Q_{01,11}/\Gamma$ at the peak position where the thermal factor $N^Q_{np,n'p'}$ is defined in Eq. (10). Both $N^Q_{01,11}$ and Γ increase with T, making the decrease in the peak intensity of α^1 somewhat slower. Therefore the rate of decrease of α^0 will be faster than that of α^1 as a function of temperature, and the ratio of the peak intensities, α^1/α^0, will increase with T, similar to the curves shown in Fig. 3. Note, however, in Fig. 3 we have used a constant Γ, so the temperature dependence comes solely from the thermal factor $N^Q_{01,11}$.

At high temperatures one needs to include the contributions from the optical phonons to Landau-level broadening. Strictly speaking, the expression derived for the absorption coefficient is valid only at low temperatures, since we have neglected effects other than the one-phonon process. A full calculation including higher-order phonon contributions, presumably will change the temperature dependence of the curves in Fig. 3 in the high-T region. We have further ignored nonparabolicity effects such as the magnetic field dependence of the effective mass. Inclusion of these details will not present any difficulty when comparing our results with an experiment in which phonon-assisted CR is measured. At this time, we are not aware of any such experiments.

In summary, we have presented calculations on the conductivity tensor for the electrons in $GaAs/Ga_{1-x}Al_xAs$ quantum-well structures. In particular, we have evaluated the absorption coefficient $\alpha(\omega)$ which displays the usual cyclotron resonance and an additional peak due to phonon-assisted transitions. The intensity of the phonon-assisted peak is resolvable ($\sim 10\%$ of the main CR peak), and observation of this effect in cyclotron-resonance experiments is predicted. Extension of the present theory to multiple-quantum-well structures would be interesting.

ACKNOWLEDGMENTS

One of us (M.S.) gratefully acknowledges the financial support of the Natural Sciences and Engineering Research Council of Canada.

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