Tilted-magnetic-field measurements of activation energies and cyclotron resonance for $Al_xGa_{1-x}As$-GaAs heterojunctions

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The Landau-level splitting in $Al_xGa_{1-x}As$-GaAs heterojunctions has been studied in tilted magnetic fields by analyzing either the thermally activated conductance or the cyclotron-resonance absorption. For an increasing parallel magnetic field, a decrease of the activation energy is found for even filling factors. At odd filling factors the activation energy increases with the parallel magnetic field, corresponding to an increase of the spin splitting. The angular dependence of the Landau-level splitting obtained from the activated conduction at a filling factor of $\nu=2$ is found to be in excellent agreement with the results obtained from cyclotron-resonance experiments. For higher filling factors ($\nu=4$ and 6) only a qualitative agreement between the two methods is observed. The quantitative discrepancy is attributed to the existence of mobility edges, which result in a reduction of the gap observed in thermally activated transport.

I. INTRODUCTION

The energy spectrum of a two-dimensional electron gas (2D EG) in the presence of a magnetic field can be investigated by different methods. Cyclotron resonance (CR) experiments[1] were widely applied for the investigation of the Landau-Level splitting in Si metal-oxide-semiconductor field-effect-transistors (MOSFET's) and $Al_xGa_{1-x}As$-GaAs heterojunctions. From dc transport measurements of the activated conductance at the minima of the Shubnikov–de Haas (SdH) oscillations information was not only obtained about the Landau-level splitting[2] but also about the density of states in the regime of the quantum Hall effect,[3] the spin splitting,[4] and even the energy gaps of the fractional quantum Hall effect.[5] Recent luminescence experiments[6] showed that spin-enhancement- and fractional-quantum-Hall-effect-related features can also be observed by means of optical experiments. CR is expected to reveal a good description of the Landau-level splitting, whereas this is not necessarily the case for the measurements of the activated conduction. In the case of the latter exchange and correlation effects are known to be important, e.g., the spin splitting is found to be greatly enhanced in dc transport compared to the splitting given by the bare conduction band g factor as measured by microwave-induced spin resonance.[7] Similarly, the Landau-level splitting measured by CR may therefore differ from the energy gap obtained by corresponding transport measurements of activation energies.

Additional information about the electronic properties of the system can be obtained from investigations of the effects of an additional magnetic field parallel to the plane of the 2D EG. Tilting the field leads to an increase in the spin splitting as it depends on the total magnetic field. In contrast, in the first-order perturbation approximation the Landau-level splitting depends only on the perpendicular component of the magnetic field. Therefore, in the case of spin-split Landau levels the activation energy should decrease with increasing parallel magnetic field due to the increasing spin splitting.[8] The increase of the spin splitting in parallel magnetic field was studied in detail in Refs. 4 and 9. Investigations of the Landau-level splitting in a tilted magnetic field by CR experiments have also been compared with second-order perturbation theory.[10] Since Landau-level splitting can be investigated both by cyclotron resonance and by measurements of the activated conductance, the unique possibility exists to compare the results of these two methods and to study the influence of exchange and correlation effects on this splitting. Therefore we performed a detailed investigation of how both the temperature-dependent conductivity and the cyclotron resonance depended on an additional parallel magnetic field in an $Al_xGa_{1-x}As$-GaAs heterojunction. In both cases, we found a decreasing value of the Landau-level splitting with increasing strength of the parallel magnetic field component. We can show that this is the expected behavior for a quasi-two-dimensional electron system with weak but nevertheless essential coupling between different subbands in a tilted magnetic field. It is found that for large tilt angles second-order perturbation theory breaks down.

The experiments are presented in Sec. II. In Sec. II A we describe the experimental setup and give in Secs. II B and II C the experimental results obtained by the two applied techniques: activation-energy measurements and far-infrared (FIR) spectroscopy of the cyclotron resonance (CR), respectively. In Sec. III we compare and discuss the experimental data. Finally, the main conclusions are summarized in Sec. IV.

II. THE MEASUREMENTS

A. Experimental setup

A high-mobility molecular-beam-epitaxy (MBE) grown $Al_xGa_{1-x}As$-GaAs heterojunction with a spacer-layer thickness of 43 nm was used for the experiments. At a temperature of $T=4$ K the mobility of the 2D EG has a value of $\mu=6.1\times10^5$ cm^2/V s for a carrier concentration of $n_s=1.4\times10^{11}$ cm^{-2}. Two samples which differed by about 10% in carrier concentrations have been produced from this heterojunction. For the transport measurements a Corbino device with channel width of 200 μm and inner diameter of 1 mm was defined by evaporation and annealing of circular Au-Ge-Ni contacts. For the optical measurements a rectangular sample with In contacts at its edges was used. Additional transport and optical measurements have been performed on similar samples from a heterojunction with a spacer-layer thickness of 27 nm.

A superconducting magnet with fields up to 14 teslas was used for the experiments. The devices were mounted in sample holders equipped with a rotating gear. The tilt angles were determined from the SdH periodicity assuming that the electron concentration and the degeneracy of the levels stay constant. This procedure appeared to be in excellent agreement with angles displayed by the scales at the sample holder.

For the measurements of the activation energies the temperature-dependent conductivity could be determined by varying the temperature from 20 K down to 1.4 K in a ^{4}He cryostat. The temperature was stabilized with a capacitance sensor and measured with a carbon glass resistor where the weak magnetoresistance was taken into account. In order to avoid complications by anisotropic resistivities, which could occur in a Hall bar sample tilted in a magnetic field, Corbino devices have been used for the electrical measurements. The dc excitation voltage was always less than 1 meV in order to avoid electrical heating of the electron gas. The Corbino geometry allowed a direct determination of the conductivity σ_{xx} by taking into account the appropriate geometric factors of our Corbino sample. Typical experimental recordings of the dependence of σ_{xx} on the magnetic field are shown in Fig. 1 for three different temperatures. The deepening of the minima in the SdH oscillations with decreasing temperature is clearly seen. The temperature dependence of σ_{xx} has been analyzed from the absolute values of σ_{xx} in the minima of the SdH oscillations. Assuming thermal excitation from the localized states at the Fermi energy to the next available extended states we expect to observe an activated behavior of σ_{xx} with activation energy E_a which is comparable to half of the level splitting. We will compare, below, the values obtained with a comparison with the CR measurements yielding the Landau level separation.

For the CR experiments, a CO_2-laser-pumped molecular-gas-laser system was used as the far-infrared source. Figure 2 shows recorder traces of the transmission signal at fixed laser wavelength ($\lambda=302$ μm) and the simultaneously recorded SdH oscillations. These measurements were performed at a temperature of $T=1.2$ K. The definition of the tilt angle θ, which is used to characterize the relation between parallel ($B_\parallel$) and perpendicular ($B_\perp$) magnetic field throughout the text, can be seen in the inset of Fig. 2. From the analysis of the peak positions measured at different wavelengths λ and angle θ we

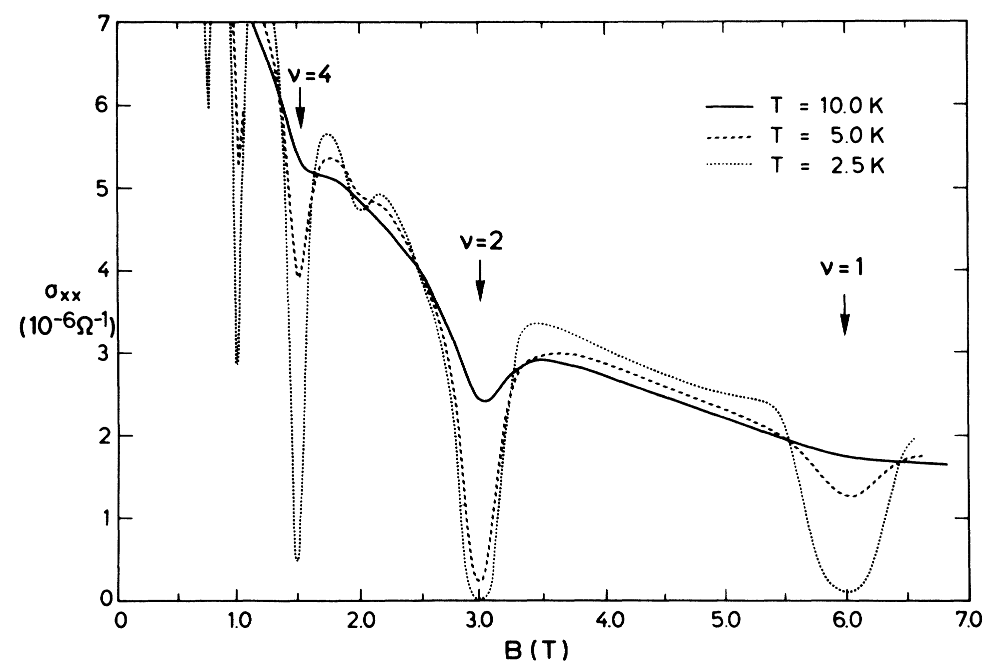

FIG. 1. The measured conductivity σ_{xx} of the Corbino sample vs magnetic field for three different temperatures T.

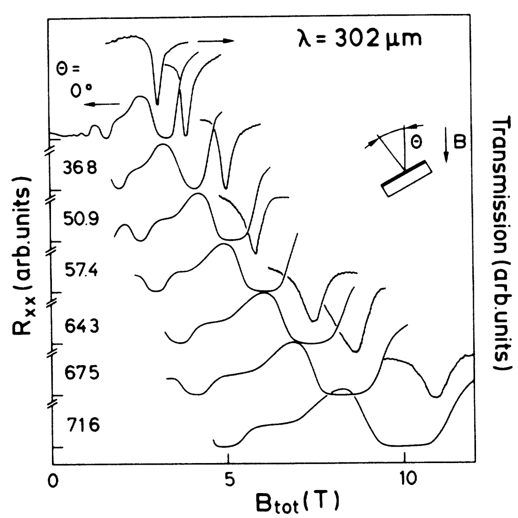

FIG. 2. The resistance R_{xx} and the simultaneously measured cyclotron resonance at a wavelength of 302 μm vs the total magnetic field for seven different tilt angles θ. The definition of the angle θ is shown in the inset.

obtained the dependence of the Landau-level splitting on both the parallel and perpendicular field component. However, for comparison with the activated transport results we had to deduce the Landau-level splitting at constant filling factor, i.e., at constant perpendicular magnetic field.

B. Measurements of the activation energies

In Fig. 1 we have indicated the position of the filling factors $\nu=1$, 2, and 4, where the filling factor is defined by $\nu=n_s/(eB_\perp/h)$, where n_s is the carrier concentration of the 2D EG, e the elementary charge, h Planck's constant, and $B_\perp$ the magnetic field perpendicular to the interface layer. An odd filling factor corresponds to the spin splitting of the Landau levels, i.e., the Fermi energy E_F then lies in the middle between the two spin levels of one Landau level, whereas an even filling factor indicates a Landau level splitting, i.e., with the Fermi energy E_F lying in the middle between two different Landau levels. Figure 3 shows the measured temperature dependence of σ_{xx} at a filling factor of $\nu=2$ for different angles θ.

For the temperature dependence of the absolute values of σ_{xx} in the minima of the SdH oscillation, an activated behavior was found experimentally,[11] which agrees with the following expression:

$$\sigma_{xx}(T)=\sigma_0\exp\left(-\frac{E_a}{kT}\right), \quad (1)$$

where σ_0 is some constant prefactor, E_a the activation energy, and T the temperature. This behavior is usually explained by the existence of a mobility edge which separates localized and extended states near the center of a Landau level. Thermal excitation from the localized states at the Fermi energy to extended states results in an activated conductance under the condition that kT is much smaller than the gap energy to be measured. If the width in energy of the extended states which determine

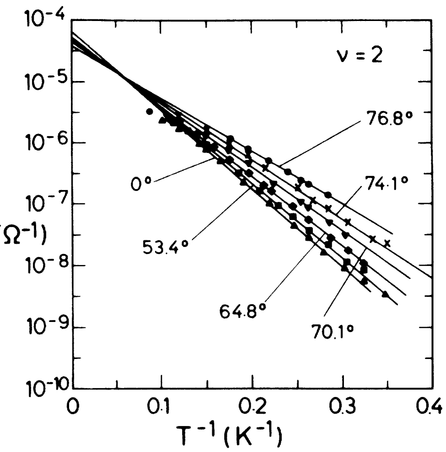

FIG. 3. The temperature dependence of the conductivity at a filling factor of $\nu=2$ for six different tilt angles. The straight lines represent fits to the expression $\sigma_{xx}(T)=\sigma_0\exp(-E_a/k_BT)$.

the mobility edge is much smaller than this gap energy than this method allows a direct determination of the level splittings in the system.

The straight lines in Fig. 3 indicate a fit of the measured conductivity values to expression (1). A clear activated behavior can be seen for a range of more than two orders of magnitude in the conductivity. Similar measurements for filling factors $\nu=4$ and 6 are shown in Fig. 4. The activation energies, deduced from Figs. 3 and 4,

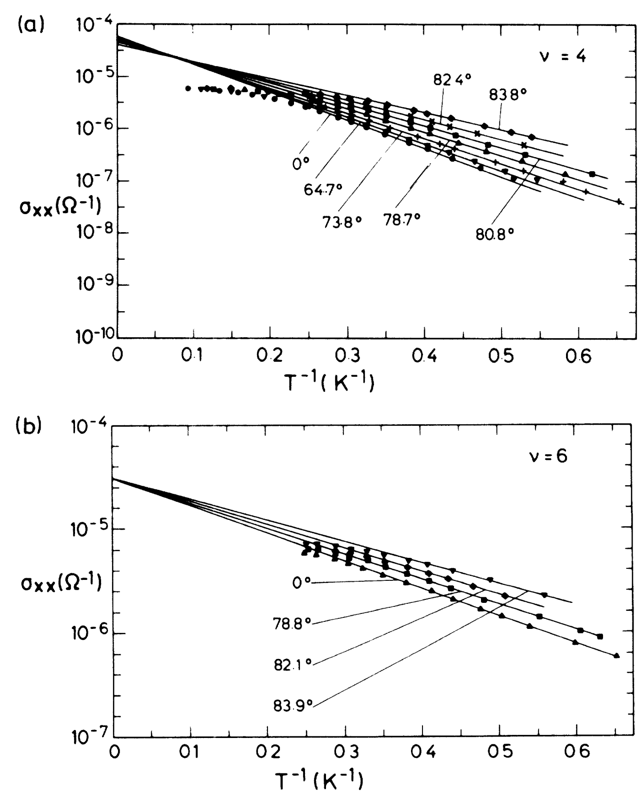

FIG. 4. (a) Temperature dependence of the conductivity at a filling factor of $\nu=4$ for seven different tilt angles. (b) Temperature dependence of σ_{xx} at a filling factor of $\nu=6$.

are plotted in Fig. 5 versus the parallel magnetic field component. The lines connecting the data points are just guides to the eye. A decrease of the activation energy for an increasing parallel magnetic field can be observed for all three filling factors. Similar results are observed for a second heterojunction which has a carrier concentration of $n_s = 2.6\times10^{11}$ cm^{-2} and a mobility of 9.3×10^5 $\mathrm{cm^2/V\,s}$. On application of a parallel magnetic field of 12.6 T the activation energy kT_0 measured at a filling factor of $\nu=2$ dropped from a value of $T_0=45.7$ K to a value of $T_0=36.2$ K in this sample. For a filling factor of $\nu=4$ the observed decrease was from 16.6 to 12 K at a parallel magnetic field of 13.6 T. So we obtained qualitatively the same results for both heterojunctions.

Up to now we discussed the results for even filling factors, which refers to the Fermi energy lying between two different Landau levels. We will now change to the results for odd filling factors, i.e., for the Fermi energy between the spin split levels of one Landau level. Figure 6 gives the measured temperature dependence at a filling factor of $\nu=1$ for different tilt angles. Figure 7 shows the dependence of the deduced activation energies on the parallel magnetic field component. In contrast to the results obtained for even filling factors the activation energy increases with increasing parallel magnetic field for this odd filling factor.

Due to the low filling factor and the relatively high magnetic field the above mentioned assumption for the extended states is expected to hold also for the spin levels and therefore the measured activation energy should be directly related to the spin splitting

$$E_a = \tfrac{1}{2} g^* \mu_B B_{\mathrm{tot}} , \tag{2}$$

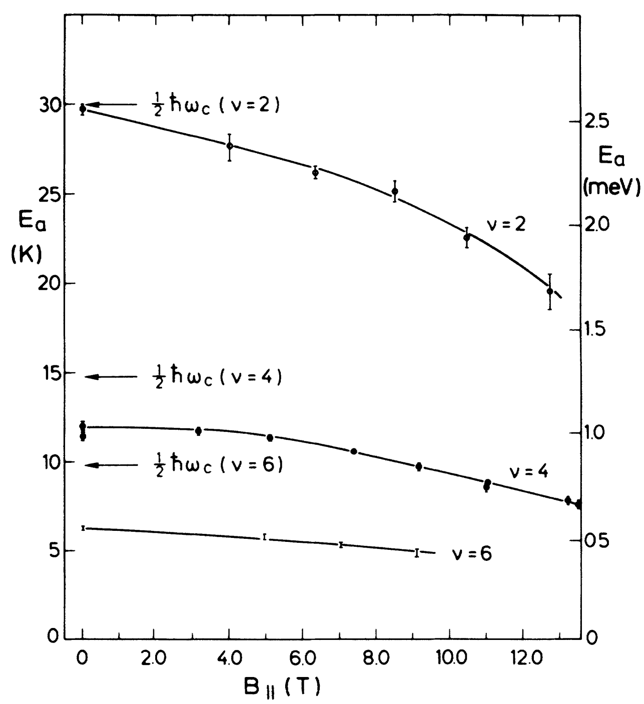

FIG. 5. The measured activation energies for the filling factors $\nu=2$, 4, and 6 as a function of the parallel magnetic field component. The lines are just guides to the eye.

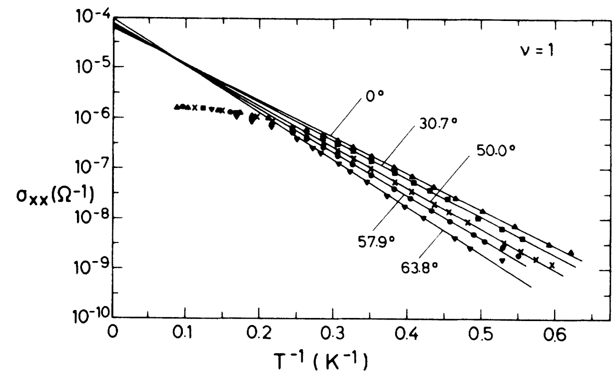

FIG. 6. The temperature dependence of the conductivity at a filling factor of $\nu=1$ for different tilt angles.

where g^* is the effective g factor and μ_B the Bohr magneton. The deduced effective g factor for a vanishing parallel magnetic field has a value of $g^*=8.4$, which is a little bit higher than the g-factor values given in Ref. 4 for comparable measurements on a different sample. These sample-dependent values may be explained by the quality of the samples, i.e., an increased broadening of the levels leading to a reduced spin enhancement, or by considering the influence of the specific sample structure on the bare conduction band g-factor.[12]

However, the effective g-factor is about a factor of 20 larger than the bare g factor g_0 expected in these samples.[7] Large spin splittings measured originally by Fang and Stiles for the 2D EG in a Si MOSFET,[13] have been explained by Janak[14] by including an additional many-particle exchange interaction E_{ex}. We expect

$$g^* \mu_B B_{\mathrm{tot}} = g_0 \mu_B B_{\mathrm{tot}} + E_{\mathrm{ex}} . \tag{3}$$

Ando and Uemura[15] pointed out that the exchange interaction depends linearly on the occupation difference of the spin polarized levels. By converting the activation

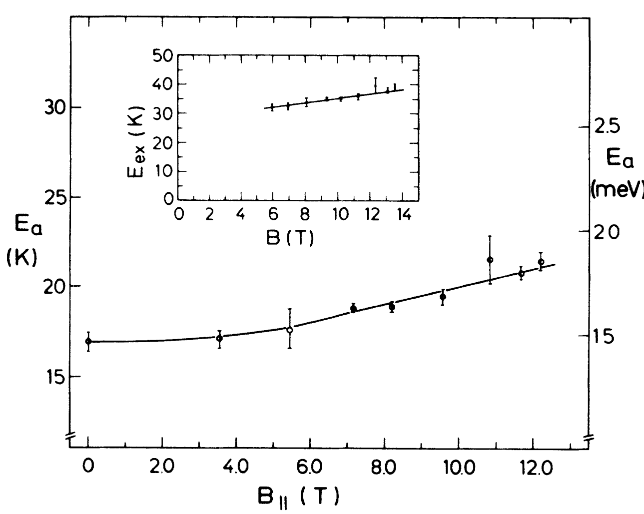

FIG. 7. The activation energies obtained from Fig. 6 as a function of the parallel magnetic field component. The inset shows the deduced exchange energy vs the total magnetic field.

energies given in Fig. 7 with the help of an assumed bare g factor of $g_0=0.4$ one gets a linear dependence of the exchange energy on the total magnetic field (see inset of Fig. 7). Such a linear increase of the exchange energy was also found in Ref. 4 for smaller total magnetic fields, whereas at high tilt angles a saturation of the exchange energy was observed. The increasing exchange energy can be explained by a change in the spin-population difference caused by a decreasing overlap of the two spin levels. This change in the population difference has to saturate for the case of all electrons being in one spin state. Apparently in our case this situation could still not be reached even at the highest available magnetic field.

Some additional remarks are necessary with regard to the activation plots in Figs. 3, 4, and 6. The straight lines obtained from a fit of the measured data points to expression (1) have an intercept with the y axes at a conductivity value σ_0. Apparently this intercept changes on tilting the sample at a given filling factor, i.e., on adding a parallel field component. The obtained prefactors σ_0 are all in the conductivity range between 3×10^{-5} to $1\times10^{-4}\ \Omega^{-1}$, which is about equivalent to the range e^2/h to $3e^2/h$. This gives some confirmation to the results of Clark *et al.*[16] that there could exist a universal minimal conductivity value, first proposed by Mott[17] for the case of zero magnetic field. From our experiments, however, it is clear that this unknown prefactor depends also on some additional effects like the component of the parallel magnetic field.

C. Cyclotron resonance

Recorded traces of the CR transmission signal at fixed laser wavelength ($\lambda=302\ \mu$m) and the simultaneously recorded SdH oscillations have been shown in Fig. 2. At zero angle θ (sample interface perpendicular to the direction of the magnetic field) the CR peak occurs at the magnetic field corresponding to a filling factor slightly larger than $\nu=2$. For increasing angle, both the CR peak and SdH minima occur at increasingly higher total magnetic field B_{tot}. While, however, the SdH minima shift according to the magnetic field component perpendicular to the layers $B_\perp=(\cos\theta)B_{\text{tot}}$, the CR peak position is observed for increasing angles at decreasing values of ν. At $\theta=64°$, for example, the CR position shown in Fig. 2 coincides with $\nu=2$, i.e., at the center of the plateau in R_{xx} and for even larger angles the CR appears at $\nu\leq2$. From our measurements we may define the cyclotron effective mass as $m_c(\theta)=(\cos\theta)B_{\text{tot}}^c e/\omega_L$, where B_{tot}^c is the peak resonance position and ω_L is the laser frequence $2\pi c/\lambda$. The result for m_c at angle $\theta=0$ for the measurement shown in Fig. 2 is $m_c=0.0679m_e$, a reasonable value for a CR measurement in an $Al_xGa_{1-x}As$-GaAs heterojunction. By comparison, the CR measurements at $\theta=64°$ of Fig. 2 yields an effective mass of $m_c=0.0719m_e$, which is an increase of about 5% and therefore implies a 5% decrease in the Landau-level splitting.

Similar values for m_c ($\theta=0$) were found for the measurements at wavelengths $\lambda=513\ \mu$m and 251 μm, where an investigation of the angular-dependent CR has also been performed. Therefore, the range of field covered in these cases was $1.4\ \text{T}<B_\perp<3.8$ T. An immediate comparison of the CR experiment and activated transport measurement is possible from Fig. 5. The arrows indicate the Landau level gap of $\hbar\omega_c/2$ at $\nu=2$, 4, and 6 deduced from the CR performed at angle $\theta=0$. We find that the activation energy measured at $\nu=2$ is very close to $\hbar\omega_c/2$. For $\nu=4$ and 6 the activation energy is less than the value of half of the Landau-level splitting. This will be discussed in Sec. III, where we will compare the angular dependence and correspondingly the $B_\parallel$ dependence, of the activation energy and CR measurements.

III. COMPARISON

In the above, we have determined the angular and magnetic field dependence of the mobility gap between Landau levels and the Landau-level transition energy from measurements of the activation energy and the CR, respectively. Now, we would like to make a comparison between these two experimental values which, however, have been obtained under slightly different conditions. Whereas E_a was obtained at a given filling factor at exactly constant $B_\perp$, the CR measurements were performed at a constant laser frequency and therefore at slightly different $B_\perp$. Because of this difference we will look for an analytic expression of the angular-field dependence that allows an appropriate description and convenient comparison of the two measurements.

In the approximation of an isotropic effective mass m^* the Hamiltonian of an interface electron in an arbitrarily oriented magnetic field $\mathbf{B}=(0,B_y,B_z)$ reads[18]

$$H=\frac{1}{2m^*}[(p_x+eB_yz)^2+(p_y+eB_zx)^2+p_z^2]+V(z)\ , \qquad (4)$$

where $V(z)$ is the interface potential. In Eq. (4) spin terms have been neglected because they are not important for our discussion. Following Ando's calculation,[18] an approximative solution of Eq. (4) can be obtained by taking the coupling between the z and x-dependent part of the motion, given by $H'=(e/m^*)zp_xB_y$, as a small perturbation. An expression for the angular-field dependence of the Landau-level transition energy obtained from the second-order perturbation calculation of Eq. (4) has been given recently by Oelting *et al.*[9]

$$\hbar\omega_c(\theta)=\hbar\omega_c^\perp\left[1-\tan^2\theta\sum_{i=1}^{\infty}\alpha_i\frac{(\hbar\omega_c^\perp)^2}{1-(\hbar\omega_c^\perp/E_{i0})^2}\right]\ , \qquad (5)$$

where $\hbar\omega_c(\theta)$ is the Landau-level transition energy, $\hbar\omega_c^\perp=(\hbar e/m^*)B_\perp$, $\tan\theta=B_\parallel/B_\perp,\alpha_i$ are the coupling parameters, and E_{i0} the subband transition energies to all the excited 2D subbands. For the case of a 2D electron system with $\alpha_i\to0$ and $E_{i0}\to\infty$, we immediately obtain from Eq. (5) the well-known relation for an ideal 2D EG:

$$\hbar\omega_c(\theta)=\frac{\hbar e}{m^*}B_\perp\ .$$

In $Al_xGa_{1-x}As$-GaAs heterojunctions of a similar type to ours, it was found that Eq. (5) with $\alpha_i=0$ for $i\geq2$ provides a reasonable fit to the CR data for relatively small angles θ. From the resonance condition

$\hbar\omega_L = \hbar\omega_c(\theta)$ (where $\hbar\omega_L$ is the constant energy of the incident radiation) one obtains from this approximation of Eq. (5)

$$(1-b) \simeq \alpha_1 \frac{(\hbar\omega_L)^2}{b^2} \tan^2\theta \ , \tag{6}$$

with b being defined as the ratio of the resonance fields at angle zero and θ, respectively,

$$b = \frac{B_\perp^c(\theta=0)}{B_\perp^c(\theta)} = \left[\frac{m_c(\theta)}{m_c(\theta=0)} \right]^{-1} . \tag{7}$$

From Eq. (6) we expect, therefore, to find a linear dependence of our CR data when plotted as $b^2(1-b)$ versus $(\hbar\omega_L)^2\tan^2\theta$. This plot is shown in the inset of Fig. (8) with the data points obtained from the CR measurement at wavelengths $\lambda = 513$, 302, and 251 μm. We indeed find that all the data points fall closely on an universal curve, which, however, does not have the simple linear dependence as suggested by Eq. (6). This result implies that the Landau-level transition energy $\hbar\omega_c(\theta)$ depends more weakly on θ than is predicted by the second-order perturbation approach to Eq. (4). The approximation made in Eq. (5) is of minor importance. Terms higher than second order, however, become important at large angles, an example is the influence of the diamagnetic shifted subband levels on the Landau level splitting. Instead of going through such a higher-order perturbation calculation we will fit our experimental result to an exact solution of Eq. (4) which can be obtained in an analytic form for the case of a harmonic potential $V(z)$,[19]

$$\hbar\omega_c(\theta) = \frac{1}{\sqrt{2}} \{ \varepsilon^2 - [\varepsilon^4 - 4(\hbar\omega_c^\perp)^2 E_{10}^2]^{1/2} \}^{1/2} \ , \tag{8}$$

with

$$\varepsilon^2 = E_{10}^2 + (\hbar\omega_c^\perp)^2 + (\hbar\omega_c^\parallel)^2 \ . \tag{9}$$

From the resonance condition $\hbar\omega_L = \hbar\omega_c(\theta)$ we obtain now for the resonance field ratio

$$1 - b^2 = 2\gamma \frac{(\hbar\omega_L)^2}{1 - \dfrac{(\hbar\omega_L)^2}{E_{10}^2}} \tan^2\theta \ , \tag{10}$$

where the coupling parameter γ is related to the harmonic potential subband energy $2\gamma = E_{10}^{-2}$. For $\gamma = \alpha_1$ and $\tan\theta \to 0$. Equation (10) becomes identical to the small-angle approximation given by Eq. (6). From the comparison with our experimental results shown in Fig. 8, we find that Eq. (10) indeed describes very well the measured angular-field dependence up to the highest angles. From the slope of the straight line shown in Fig. 8 we obtain $\gamma = 7.35 \times 10^{-4}$ (meV)$^{-2}$ for all wavelengths. The accuracy of our measurements does not allow us to observe the very small correction due to the denominator $1-(\hbar\omega_L)^2/E_{10}^2$ in Eq. (10). With $E_{10} = 26.1$ meV, which is the value obtained from γ, these corrections are expected to be smaller than 2%. By means of expressions (8) and (9) and the values for γ and E_{10}, it is now straightforward to calculate the angular-field dependence

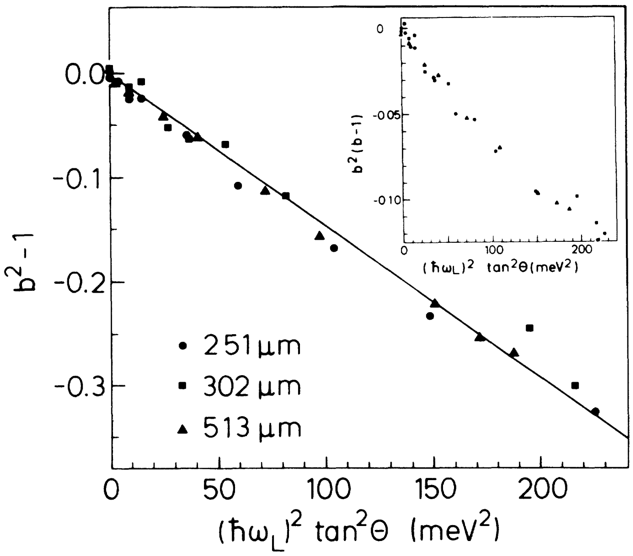

FIG. 8. Compilation of the angular dependence of the CR measured at $\lambda = 205$, 305, and 513 μm. The CR data are plotted as b^2-1 vs $(\hbar\omega_L)^2\tan^2\theta$ in accordance with the harmonic potential calculation (see text). b is the normalized peak position of the perpendicular field component $B_\perp(\theta=0)/B_\perp(\theta)$ and $\hbar\omega_L$ is the laser energy of the incident radiation. The inset shows a compilation of the same CR data in a plot according to the second-order perturbation approach, i.e., the y axis is $b^2(b-1)$. The harmonic type of data representation can be nicely fitted by a straight line of slope 7.35×10^{-4} meV^{-2}.

of $\hbar\omega_c(\theta)$ for the case of the activation energy measurements, which have been performed at constant $B_\perp$.

Figure 9 shows the Landau-level splitting $\hbar\omega_c(\theta)$ deduced from the activation energy measurements and the cyclotron resonance experiments as a function of the square of the parallel magnetic field. To get the real Landau level splitting one has to take into account the spin splitting. Therefore, the Landau-level splitting $\hbar\omega_c(\theta)$ is calculated from the measured activation energies E_a by

$$\hbar\omega_c(\theta) = 2E_a(\theta) + g_0\mu_B B_{\text{tot}} \ . \tag{11}$$

Here the assumption was made that there is no exchange enhancement for a fully filled or totally empty Landau level. The values obtained in this way are marked with the corresponding error bars in Fig. 9. The results of the analysis of the cyclotron resonance experiments are shown by full lines calculated using expressions (8) and (9). The regime in which expressions (8) to (10) have been directly verified by the angular dependence of the CR is shown by the box which extends from $B_\parallel = 0$ T, $\hbar\omega_c = 5.1$ meV to $B_\parallel = 10$ T, $\hbar\omega_c = 2.4$ meV.

At filling factor $\nu = 2$ one observes a remarkable agreement between the results obtained in using these two different methods. The assumption of using the bare spin splitting of the filled Landau levels for analyzing the measurements of the activation energy at even filling factors is thus vindicated. Due to the low magnetic field value we find no evidence for an exchange enhancement of the Landau level splitting as observed at higher magnetic fields in Ref. 20. In our measurements, the difference be-

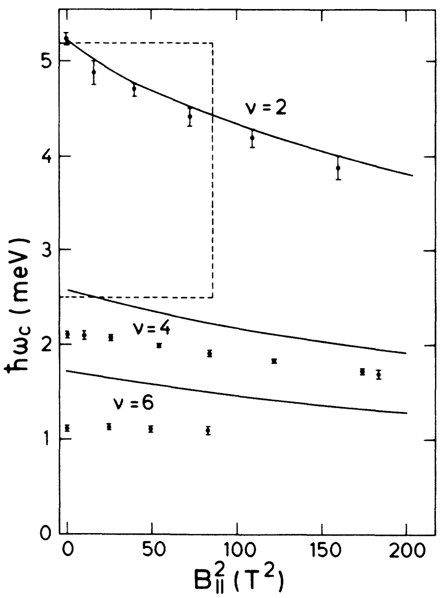

FIG. 9. The results obtained for the Landau level splitting $\hbar\omega_c$ by both methods as a function of the square of the parallel magnetic field component. The solid lines represent the results of the fit of the cyclotron resonance to the theory, whereas the points with error bars show the values obtained from the activation energies. For the fit to the theory, CR data points are used which have been measured in the energy field regime indicated by the dashed box.

tween the values obtained from CR and from activated conductance for an increasing parallel magnetic field is always less than 0.1 meV. Again the dependence on the square of the parallel magnetic field is sublinear, i.e., the decrease of the Landau-level splitting with increasing parallel magnetic field is less than expected from second-order perturbation theory. For the filling factors $\nu=4$ and 6 the agreement between the two methods is not as good as for the filling factor $\nu=2$. Even for a vanishing parallel magentic field the Landau-level splitting deduced from the activation energy for a filling factor of $\nu=4$ is about 0.45 meV smaller than the result from cyclotron resonance and for the filling factor $\nu=6$ the difference is as much as 0.6 meV. We explain these differences as due to the occurrence of mobility edges which are influencing electrical transport measurements but not optical measurements. The observation of an activation energy is interpreted in a model using extended and localized states as mentioned above. For a width in energy of the extended states much less than the Landau-level splitting one would expect to obtain activation energies comparable with cyclotron resonance frequencies as is the case for a filling factor of $\nu=2$ in our measurements. A finite energy range of the extended states will result in a decrease of the measured activation energy. Thus, this reduction is greater for the higher filling factors, since the localization should be weaker for higher Landau levels, where the hopping distance of the cyclotron motion is increasing with the Landau level index and decreasing with the magnetic field. We do indeed observe this behavior in our measurements. For a filling factor of $\nu=4$ the decrease is only 17%, whereas for a filling factor of $\nu=6$ it is 35%. For these filling factors the dependence on the parallel magnetic field is also weaker than expected from the CR data. One could explain this effect as due to narrowing of the extended states region, i.e., as an increase of localization, due to shrinkage of the wave function in an increasing parallel magnetic field.

IV. CONCLUSION

For a filling factor of $\nu=2$ we found a decrease of the Landau-level splitting, deduced from the measured activation energies and the cyclotron-resonance positions, with increasing parallel magnetic field. For higher tilt angles the decrease is less than expected from a second-order perturbation calculation, it can, however, be fitted with the results of an exact calculation for a parabolic potential. For the filling factors $\nu=4$ and 6 the activation energies are smaller than expected from cyclotron resonance. The differences are explained by the existence of finite energy ranges of extended states in the centers of the Landau levels. Due to the observed decrease of the Landau-level splitting the g-factor values obtained by the, so-called, coincidence method[10] can be only interpreted as upper limits. Since the decrease is less for the higher Landau levels, the error in the g-factor values should be also less in these cases. The increase of the activation energy obtained at a filling factor of $\nu=1$ for an increasing parallel magnetic field is explained by an increasing spin population difference.

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