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Interaction of vortices in uniaxial superconductors

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Parallel vortices in uniaxial superconductors may attract each other provided they are tilted with respect to the principal crystal directions and the intervortex spacing is on the order of the penetration depth. The vortex-vortex interaction potential and the line energy are evaluated within London theory.

In the isotropic case, the interaction of two parallel vortices separated by r is proportional to the field h of the first at the core location of the second: $\varepsilon=(\phi_0/4\pi)h(r)$, ϕ_0 being the flux quantum; it is assumed that $r\gg\xi$, the temperature-dependent coherence length.[1] The field $\mathbf{h}$ has only one component parallel to the axis of the vortex it belongs to. When r is large with respect to the core size ξ, London theory applies and yields $\varepsilon=\phi_0^2K_0(r/\lambda)/8\pi^2\lambda^2$. Here λ is the penetration depth and K_0 is the modified Bessel function.

In the anisotropic situation, the direction of the field $\mathbf{h}$ of an isolated vortex is no longer parallel to the vortex axis.[2-5] Moreover, the $\mathbf{h}$ direction changes in space in a complicated way. It has been shown, however, that the interaction ε is still given by the isotropic expression, in which the total field $h(r)$ is replaced with the projection $\mathbf{h}_z(\mathbf{r})$ upon the direction of the vortex axes (z).[4,6] Thus, the problem of interaction is reduced to that of finding the spatial distribution $h_z(x,y)$ of an isolated vortex.

This work has been stimulated by the observation[7] that along certain directions the field h_z can change sign. This implies a more complicated vortex-vortex interaction than the well-known repulsion. Another indication of a "nontrivial" interaction came from the recent calculations of the energy for the formation of vortex chains in low fields that show that the energy per vortex in the chain may be lower than the energy of an isolated vortex.[8] The chain formation has been observed earlier in (unpublished) numerical calculations of the equilibrium flux-line latices by the authors of Ref. 5.

Due to linearity of the London equations, the problem of the field of an isolated vortex is readily solved in the Fourier space:[2-5]

$$\begin{aligned} h_x(\mathbf{k}) &= -h_y(\mathbf{k})k_x/k_y=\phi_0\lambda^2 m_{xz}k_y^2/d\,,\\ h_z(\mathbf{k}) &= \phi_0(1+\lambda^2 m_{zz}k^2)/d\,, \\ d &= (1+\lambda^2 m_{zz}k_x^2+\lambda^2 m_c k_y^2)(1+\lambda^2 m_a k^2)\,. \end{aligned} \tag{1}$$

Here $m_{zz}=m_a\sin^2\theta+m_c\cos^2\theta$ and $m_{xz}=(m_a-m_c)\times\sin\theta\cos\theta$; $m_{a,c}$ are eigenvalues of the dimensionless "mass tensor" m_{ik} along the $\hat{\mathbf{a}},\hat{\mathbf{c}}$ axes of the uniaxial crystal; θ is the angle between the $\hat{\mathbf{c}}$ and the vortex axis $\hat{\mathbf{z}}$; we choose $\hat{\mathbf{y}}=\hat{\mathbf{c}}\times\hat{\mathbf{z}}$. In the notation of Ref. 2, $m_a^2m_c=1$ and λ is the (geometric) average penetration depth. The inverse Fourier transform provides $\mathbf{h}(\mathbf{r})$. However, the integration in

$$\mathbf{h}(\mathbf{r})=\int\mathbf{h}(\mathbf{k})\exp(i\mathbf{k}\cdot\mathbf{r})d^2\mathbf{k}/4\pi^2$$

is difficult analytically. One can simplify the numerical problem using Feynman's method (see, e.g., Ref. 9) in reducing multiple Fourier integrals to integration over finite intervals.[10]

To simplify the notation we take λ as unit length and measure the field in units of $\phi_0/2\pi\lambda^2$. Next, we rewrite $h_z(\mathbf{k})=2\pi(1+m_{zz}k^2)/BC$ with $B=1+m_{zz}k_x^2+m_ck_y^2$ and $C=1+m_ak^2$ in the form

$$h_z(\mathbf{k})=\frac{2\pi}{m_a}\left(\frac{m_{zz}}{B}-\frac{m_{zz}-m_a}{BC}\right)=h_1(\mathbf{k})-h_2(\mathbf{k})\,. \tag{2}$$

The first term reduces to the Fourier transform of K_0 after the scale transformation $k_x'=k_x\sqrt{m_{zz}}$, $k_y'=k_y\sqrt{m_c}$:

$$h_1(\mathbf{r})=\sqrt{m_{zz}}K_0(\rho_0),\ \rho_0^2=\frac{x^2}{m_{zz}}+\frac{y^2}{m_c}\,. \tag{3}$$

To transform the second term, we use the identity[9]

$$(BC)^{-1}=\int_0^1 du/[uB+(1-u)C]^2\,.$$

The denominator here is $[1+a^2(u)k_x^2+b^2(u)k_y^2]^2$ with

$$a^2=m_{zz}-(m_{zz}-m_a)u,\ b^2=m_c-(m_c-m_a)u\,. \tag{4}$$

Therefore,

$$h_2(\mathbf{r})=2\pi\frac{m_{zz}-m_a}{m_a}\int_0^1du\int\frac{d^2\mathbf{k}}{(2\pi)^2}\frac{e^{i\mathbf{k}\cdot\mathbf{r}}}{[1+a^2(u)k_x^2+b^2(u)k_y^2]^2}=\frac{m_{zz}-m_a}{2m_a}\int_0^1du\frac{\rho}{ab}K_1(\rho),\ \rho^2=\frac{x^2}{a^2(u)}+\frac{y^2}{b^2(u)}\,. \tag{5}$$

Equations (3) and (5) represent the field $h_z(\mathbf{r})=h_1(\mathbf{r})-h_2(\mathbf{r})$ exactly. In particular, one can verify that the total flux of h_z is ϕ_0. Note that neither $K_1(\rho)$ nor $a(u)$, $b(u)$ are singular in the integration domain of h_2; thus h_2 can be safely evaluated numerically.

At $\theta=\pi/2$ (the vortex axis in the a-b plane), $m_{zz}=m_a$ and h_2 vanishes, whereas h_1 gives the known result $h(\theta=\pi/2)=K_0[(x^2/m_a+y^2/m_c)^{1/2}]/(m_a m_c)^{1/2}$.

For the vortex parallel to $\mathbf{c}$ [$\theta=0$, $m_{zz}=m_c$, $a(u)=b(u)$], the integral in Eq. (5) is performed explicitly to give $h(\theta=0)=K_0(r/\sqrt{m_a})/m_a$, as it should.

Let us consider now the behavior of $h_z(x,0)$ at the axis x, which is the projection of $\mathbf{c}$ upon the x-y plane. After simple algebra we obtain from (3) and (5)

$$h_z(x,0)=\sqrt{m_{zz}}K_0\left(\frac{x}{\sqrt{m_{zz}}}\right)-x\sqrt{m_c}\cos\theta\int_{v_0}^{v_1}\frac{K_1(xv)dv}{(1-v^2m_a\sin^2\theta)^{1/2}}, \tag{6}$$

where $v_0=1/\sqrt{m_{zz}}$ and $v_1=1/\sqrt{m_a}$. At large distances with respect to λ ($x\gg 1$), the function $K_1(xv)$ decays exponentially when the change of v is on the order of $1/x$. One can expand the slowly varying function $(1-v^2m_a\sin^2\theta)^{-1/2}$ in the relevant domain $(v-v_0)\sim 1/x$ in powers of $(v-v_0)$ and replace the upper integration limit with ∞, provided

$$v_1-v_0=m_a^{-1/2}-m_{zz}^{-1/2}\gg 1/x\,. \tag{7}$$

Substitute now for K_1 its asymptotic expansion (up to the second order) and carry out the integration; combining the result with the asymptotic expansion for K_0, one obtains[11]

$$h_z(x\rightarrow\infty,0)\simeq-\frac{m_{zz}^{5/4}m_a}{4m_cx}\tan^2\theta\left(\frac{\pi}{2x}\right)^{1/2}\exp\left(-\frac{x}{\sqrt{m_{zz}}}\right). \tag{8}$$

The striking feature of this result is that the field h_z is negative at large distances in accordance with Ref. 7 (see Ref. 12). This means that the interaction energy goes through a minimum, which is likely to be situated at the x axis, because the ($\mathbf{c}$,$\mathbf{z}$) [or ($\mathbf{x}$,$\mathbf{z}$)] plane is the only symmetry plane of the problem,[8] the conjecture confirmed by numerical results. The position of this minimum x_m, an important feature of the interaction energy, is determined by

$$K_1\left(\frac{x_m}{\sqrt{m_{zz}}}\right)=x_m\sqrt{m_c}\cos\theta\int_{v_0}^{v_1}\frac{K_0(x_mv)v\,dv}{(1-v^2m_a\sin^2\theta)^{1/2}}\,. \tag{9}$$

One can easily see that at $\theta=0$ and $\theta=\pi/2$ [for a given anisotropy ratio $\gamma=(m_c/m_a)^{1/2}$], the only solution of this equation is $x_m=\infty$. Equation (9) can be solved numerically; examples are shown in Fig. 1(a) for $x_m(\theta)$ at given anisotropy γ and in Fig. 1(b) for $x_m(\gamma)$ at given vortex orientation θ.

Results of numerical evaluation of $h_z(x,y)=h_1-h_2$ given in Eqs. (3) and (5) are shown in Fig. 2. A shallow minimum in the domain of negative $h_z(x,y)$ is clearly seen both in the perspective plot of Fig. 2(a) and in Fig. 2(b), which shows contours $h_z(x,y)=$const. To emphasize peculiarities of the interaction potential, we have chosen $\gamma=55$ (corresponding to $Bi_2Sr_2CaCu_2O_8$, Ref. 13) and $\theta=60°$.

In some applications another representation of the field h_z may be more convenient:

$$h_z(\mathbf{r})=\sqrt{m_c}\cos^2\theta K_0(\rho_1)+\sqrt{m_{zz}}\sin^2\theta K_0(\rho_0)-\frac{m_c-m_a}{8}\sin^2 2\theta\int_0^1\frac{du}{a^3b}\left(\frac{x^2}{a^2\rho}K_1(\rho)-K_0(\rho)\right). \tag{10}$$

Here $\rho(u)$ is defined in Eq. (5); $\rho_0=\rho(0)$ and $\rho_1=\rho(1)$ [ρ_0 is given in Eq. (3), $\rho_1^2=(x^2+y^2)/m_a$]. This representation is more symmetric: For the limiting values of $\theta=0$ and $\pi/2$, the integral term of (10) vanishes [unlike that of Eq. (5)], leaving either the first or the second term to survive. Equation (10) shows that $h_z(0,y)$ is always positive.

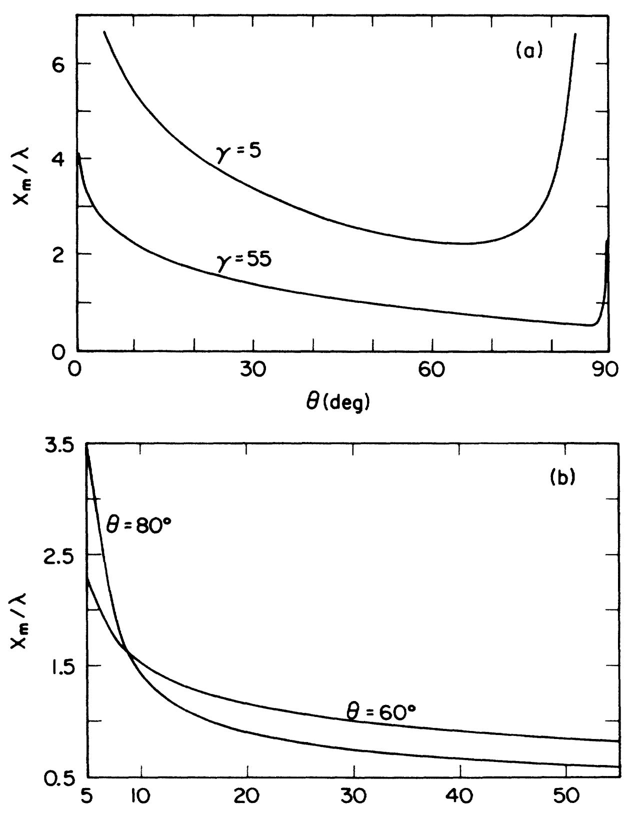

FIG. 1. The position x_m (in units of λ) of the minimum interaction energy: (a) vs orientation θ of vortices with respect to the crystal axis $\mathbf{c}$ for two values of $\gamma=(m_c/m_a)^{1/2}$ and (b) vs γ for two fixed angles θ.

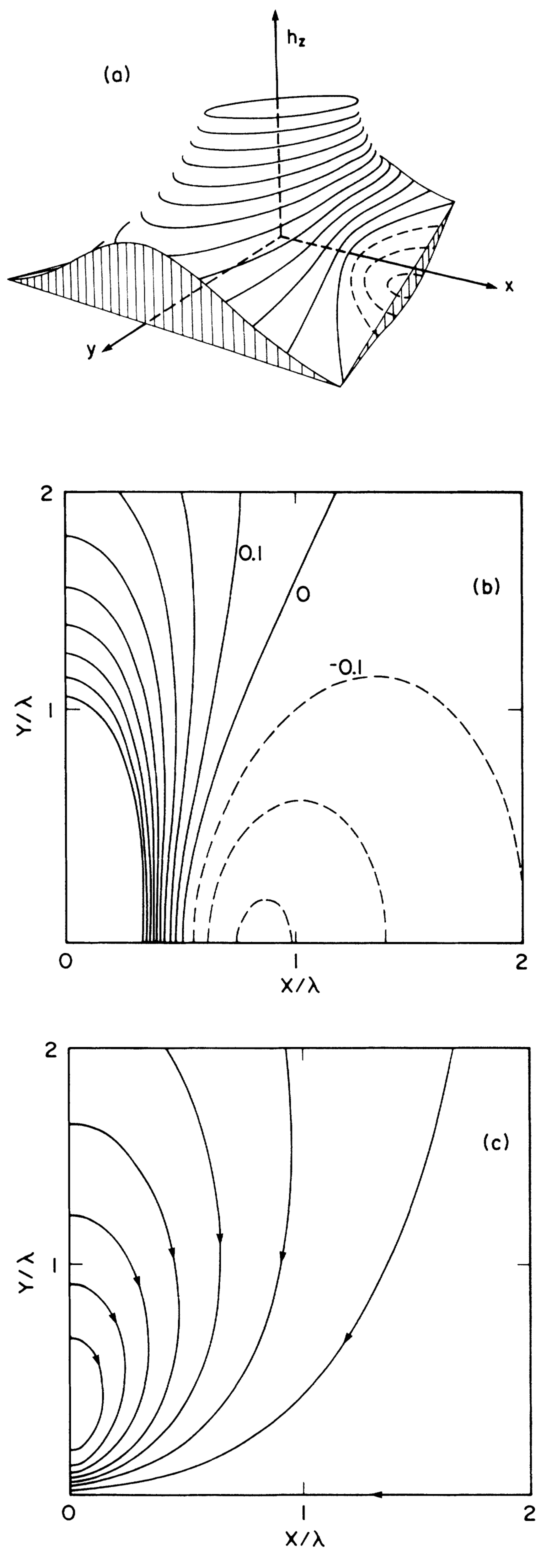

FIG. 2. (a) Three-dimensional perspective plot of the interaction potential for a vortex inclined at 60° to the **c** crystal axis; $\gamma=55$. Dashed lines correspond to negative energies. The contours of constant $h_z(x,y)$ are drawn with an interval $0.1(\phi_0/2\pi\lambda^2)$. (b) Contours of $h_z(x,y)=$const in the first quadrant of the x-y plane (with the same increment); the vortex axis **z** is situated at $x=y=0$. (c) The stream lines of the transverse field $\mathbf{h}_{tr}$ in the first quadrant of the x-y plane.

This feature is seen clearly in Figs. 2(a) and 2(b).

Using the same method, one can evaluate other components of the field, $h_{x,y}(x,y)$. Because there is no z dependence in the problem, these two are derivatives of the vector potential A_z, whose Fourier transform $-im_{xz}k_y/d$ follows from Eq. (1). One obtains

$$A_z(\mathbf{r})=\frac{m_{xz}}{2}y\int_0^1\frac{du}{ab^3}K_0(\rho)\,. \tag{11}$$

The stream lines of the transverse field $\mathbf{h}_{tr}=h_x\mathbf{x}+h_y\mathbf{y}$ are given by the contours $A_z(x,y)=$const. An example of $\mathbf{h}_{tr}(x,y)$ is given in Fig. 2(c).

Given the field $h_z(\mathbf{r})$, one can evaluate the line energy of a single vortex $\varepsilon_L=\phi_0h_z(r\to0)/8\pi$.[2-4] In doing so we first note that the contribution $h_2(0)$ (in units $\phi_0/2\pi\lambda^2$) is finite:

$$h_2(0)=\frac{m_{zz}-m_a}{2m_a}\int_0^1\frac{du}{a(u)b(u)}$$
$$=\frac{|\cos\theta|}{m_a}\ln\frac{\sqrt{m_a}(1+|\cos\theta|)}{\sqrt{m_{zz}}+\sqrt{m_c}|\cos\theta|}\,. \tag{12}$$

The only divergent part of $h_z(0)$ comes from $h_1(0)$ $\sim K_0(\rho_0)$. Evaluating this contribution one should use a cutoff at some $\rho_0(x,y)=C(\theta)$ with a constant C on the order of the coherence length ξ. Physically, this corresponds to an elliptical contour of the vortex core chosen so as to coincide with the stream lines of the persistent current, i.e., with a cylinder $h_z(x,y)=$const. Any other choice would imply that current lines cross the "core surface" and hence cause dissipation.[14] The constant C cannot be determined within the London approach. One can instead choose it by comparing with the results of the Ginzburg-Landau theory for the line energy:[15] at $\theta=0$, $\varepsilon_L(4\pi\lambda/\phi_0)^2=m_a^{-1}\ln(\kappa m_a)$, while at $\theta=\pi/2$, one has $(m_am_c)^{-1/2}\ln(\kappa\sqrt{m_am_c})$, where κ is the ratio of (geometric) averages λ and ξ, and the core corrections are omitted. We then obtain

$$\varepsilon_L\left(\frac{4\pi\lambda}{\phi_0}\right)^2=\sqrt{m_{zz}}\ln\frac{\kappa}{\sqrt{m_a}}$$
$$+\sqrt{m_c}|\cos\theta|\ln\frac{\sqrt{m_a}(1+|\cos\theta|)}{\sqrt{m_{zz}}+|\cos\theta|\sqrt{m_c}}\,. \tag{13}$$

It is worth noting that the second contribution in (13) has nothing to do with either the artificial cutoff or the core energy; it is a part of the London energy and it affects the angular dependence of ε_L. For this reason it is retained in (13), despite the fact that the correction (of the same order of magnitude) due to the core is omitted. Better treatment of the line energy can be achieved in the frame of Ginzburg-Landu theory; still, Eq. (13) can be used for materials with $\kappa\gg1$. Although $\partial\varepsilon_L/\partial\theta=0$ at $\theta=\pi/2$, for high anisotropies the line energy approaches its minimum at $\pi/2$ very steeply. The energy (13) can be approximated by $\sqrt{m_{zz}}\ln(\kappa\sqrt{m_{zz}})$ with less than 5% error for $\gamma=5$; for $\gamma=55$ the error is under 16%.

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[10]Similar procedure has been used by N. Schopohl (private communication).

[11]There are situations when Eqs. (7) and (8) do not hold. For instance, in the isotropic limit Eq. (7) cannot be satisfied. Also, when $\theta = 0$ Eq. (8) yields zero, and therefore more terms in the asymptotic expansion should be retained. As $\theta \rightarrow \pi/2$ Eq. (8) does not hold because the integration domain in Eq. (6) shrinks to zero. Fortunately, in all of these cases the exact solutions are available.

[12]At first sight, this contradicts the result of Ref. 2 that predicts a positive h_z at $r \gg \lambda$. The asymptotic formulas in Ref. 2 have been obtained by applying the steepest descent method to the integration over k_y. Close examination shows that at $y=0$, the saddle coincides with the branch point. Hence, the asymptotic formulas of Ref. 2 do not hold for $\phi = \tan^{-1}(y/x) = 0$. For a finite azimuth ϕ, however, Ref. 2 gives the correct asymptotics for h_z; one can confirm this, starting with Eqs. (3) and (5). Thus, for a finite ϕ (less than a certain ϕ_{max}), $h_z(r,\phi)$ changes sign twice, when r varies from zero to infinity. However, numerical estimates show that the second change, from negative back to positive, happens at such a large r/λ that it can be disregarded for any practical situation.

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