

Thermally activated flux creep in strongly layered high-temperature superconductors

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Thermal activation energies for single vortices and vortex bundles in the presence of a magnetic field parallel to the layers are calculated. The pinning considered is intrinsic and is due to the strongly layered structure of high-temperature superconductors. The magnetic field and the current dependence of the activation energy are studied in detail. The calculation of the activation energy is used to determine the current-voltage characteristic. It may be possible to observe the effects discussed in this paper in a pure enough sample.

I. INTRODUCTION

Thermally activated flux creep plays an important role in the resistive measurements of high-temperature superconductors in a magnetic field.¹⁻⁶ As is well known, in order to maintain a nondissipative transport current it is necessary to pin the vortices. Most commonly, pinning is associated with the impurities and defects present in the sample. However, in strongly layered superconductors such as $\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_8$ an intrinsic pinning mechanism exists for a vortex motion in a direction perpendicular to the layers.⁷ Because the order parameter is strongly modulated in a direction perpendicular to the layers, the free energy of the superconductor is minimized when the vortices are situated in between the layers. In other words, the strong modulation of the order parameters acts as pinning centers. Clearly, it costs superconducting condensation energy for a vortex core to cross a layer. Because most known high-temperature superconductors have a pronounced layered structure, it appears to be both useful and interesting to study the effects of the intrinsic pinning.

In the present paper we shall consider temperatures high enough such that thermal activation of vortices is possible. The central problem is to calculate the nucleation energy of a critical nucleus that separates the stability points of a vortex lattice. This nucleation energy is the activation energy that depends on the transport current and the magnetic field.

For currents not especially small, more specifically larger than j_1 , defined in the following, the dominant activation process consists of the activation of a single vortex. The nucleus consists of a single deformed vortex with a segment situated in a neighboring layer. For smaller currents, the most probable event is the activation of a vortex bundle.⁸ The size of this bundle grows inversely as the transport current j , as $j \rightarrow 0$. Intrinsic pinning, in the context of high-temperature superconductors, has also been discussed in Refs. 9 and 10; see also Ref. 11. The activated creep phenomenon in high-temperature superconductors was also discussed by Tinkham¹² and subsequently by Inui *et al.*¹³

The calculation of the activation energy allows us to determine the exponential factor of the probability of activation, but not the preexponential factor. For a calculation of the preexponential factor it is necessary to know the dynamics of the vortex motion; thermodynamic considerations alone will not suffice. Such an understanding of the dynamics is not currently available.

II. THE MODEL

We consider a model of a layered superconductor^{14,15} which consists of a periodic system of planes with Josephson coupling between the planes. Close to the superconducting transition the Ginzburg-Landau free-energy functional is given by

$$F = d \sum_n \int dx dy \left[-\frac{\xi_{ab}^{-2}}{2} |\psi_n|^2 + \pi \left(\frac{2\kappa\pi}{\phi_0} \right)^2 |\psi_n|^4 + \frac{1}{2} \left| \left(i\nabla + \frac{2e}{c} \mathbf{A} \right) \psi_n \right|^2 + \frac{\xi_c^2}{2\xi_{ab}^2 d^2} \left| \psi_{n+1} \exp \left[-\frac{2ie}{c} \int_{nd}^{(n+1)d} dz A_z \right] - \psi_n \right|^2 \right] + \frac{1}{8\pi} \int d^3r [(\nabla \times \mathbf{A})^2 - 2\mathbf{H} \cdot (\nabla \times \mathbf{A})], \quad (1)$$

where ξ_{ab} and ξ_c are the correlation lengths in the plane and perpendicular to the plane, respectively, and are proportional to $(T_c - T)^{-1/2}$. T_c is the critical temperature, d the interlayer separation, and ϕ_0 the flux quantum,

$\pi\hbar c/e$. The summation over n in Eq. (1) extends over all the layers, and the gradient operator is two-dimensional, acting on a plane. We have also introduced the Ginzburg-Landau (GL) parameter

$$\kappa = \frac{\lambda_{ab}}{\xi_{ab}}, \quad (2)$$

where λ_{ab} is the London penetration depth when the magnetic field is perpendicular to the layers, and the screening current flowing along the layers.

For quantities that depend on distance scales large compared to the interlayer separation d the layered structure is not important. In this case we shall use the simpler London expression for the free energy, F_L , which is given by

$$F_L = \frac{1}{8\pi} \int d^3r [H^2 + \lambda_{ab}^2 (\nabla \times \mathbf{H})_{ab}^2 + \lambda_c^2 (\nabla \times \mathbf{H})_z^2], \quad (3)$$

where λ_c is the London penetration depth for the magnetic field directed along the plane and the screening currents flowing across the planes. For a strongly layered superconductor which we consider below, $\lambda_c/\lambda_{ab} = \xi_{ab}/\xi_c \gg 1$. We want to emphasize that the London expression can be used for all temperatures provided that λ_{ab} and λ_c are taken from experiments.

III. SINGLE VORTEX ACTIVATION

We now consider the expression for the change in energy, U , due to the activation of a segment of a vortex to the neighboring interlayer spacing. U has the form,

$$U = \delta F_{\text{cond}} + V_{\text{int}}(R) - (j - j_1) \frac{\phi_0 d}{c} R, \quad (4)$$

where δF_{cond} is the loss of the condensation energy due to the destruction of superconductivity at those two points on the layer threaded by the vortex, separated by a distance R ; see Fig. 1. $V_{\text{int}}(R)$ is the interaction energy of the two vortex kinks. The term proportional to the transport current j is due to the Lorentz force. The term proportional to j_1 is the energy that arises due to the deformation of the vortex line and is clearly proportional to the length R . This contribution represents the tension energy. We would like to emphasize that this elastic contribution presupposes only strong short-range order. In particular, this contribution would also be present in melted¹⁶ or glassy¹⁷ phases provided that there is sufficient short-range order. However, the dynamics of such melted phases are expected to be very different.

In order to estimate the current j_1 we use the expression for the elastic energy of a vortex lattice. Thus,

$$j_1 \frac{\phi_0 d}{c} \propto \frac{1}{2} \int dx dz C(x, z) \left[\frac{\partial u_z}{\partial z} \right]^2, \quad (5)$$

where the Fourier transform of the nonlocal compression modulus¹⁸ $C(x, z)$ is given by

$$C(k_x, k_z) = \frac{H^2}{4\pi} \frac{1}{1 + \lambda_{ab}^2 k_z^2 + \lambda_c^2 k_x^2}. \quad (6)$$

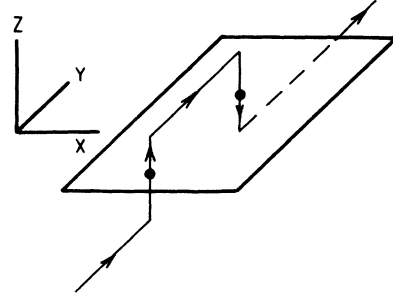


FIG. 1. Single vortex activation.

The integral in Eq. (5) can be easily estimated if we take $dx dz \sim \phi_0/H, u_z \sim d$, and $\partial/\partial z \sim k_z \sim (\lambda_c/\lambda_{ab})k_x$. We get

$$j_1 = b \frac{cHd}{16\pi\lambda_{ab}^2}. \quad (7)$$

Here the numerical coefficient $b \sim 1$. A precise calculation when a single vortex moves a distance d along the z axis and the remaining vortices are considered to be at rest yields b to be 0.96.

For the first two terms in Eq. (4), we shall calculate for the two limiting cases: $R \gg d(\lambda_c/\lambda_{ab})$ and $R \ll d(\lambda_c/\lambda_{ab})$.

For $\xi_{ab} \ll R \ll d(\lambda_c/\lambda_{ab})$, one can set $\lambda_c = \infty$. In this regime one must use the expression for the free energy given in Eq. (1). Omitting the magnetic energy we get

$$\begin{aligned} \delta F_{\text{cond}} + V_{\text{int}} &= \frac{\phi_0^2 d}{32\pi^3 \kappa^2 \xi_{ab}^4} \int dx dy \left(\frac{1}{2} - f^2 + \frac{1}{2} f^4 + \xi_{ab}^2 |\nabla f e^{i\chi}|^2 \right). \end{aligned} \quad (8)$$

Here f is the dimensionless superconducting order parameter. The equations for f and χ follow from the minimization of Eq. (8), and are

$$\nabla^2 f - f(\nabla\chi)^2 + \frac{1}{\xi_{ab}^2} (f - f^3) = 0 \quad (9)$$

and

$$\nabla^2 \chi = 0. \quad (10)$$

Equations (9) and (10) should be solved with the topological boundary condition that the phase changes at the two points separated by a distance R are 2π and -2π . After some calculations we find that

$$\delta F_{\text{cond}} + V_{\text{int}} = \frac{\phi_0^2 d}{8\pi^2 \lambda_{ab}^2} \ln \left[\frac{R}{\xi_{ab} \sqrt{2}} \right]. \quad (11)$$

However, when $R \gg d(\lambda_c/\lambda_{ab})$, one can use the London expression for the free energy. Minimizing the expression for the free energy given in Eq. (3) we obtain

$$H_y + \lambda_{ab}^2 \frac{\partial}{\partial z} (\nabla \times \mathbf{H})_x - \lambda_c^2 \frac{\partial}{\partial x} (\nabla \times \mathbf{H})_z = \phi_0 \delta(x) \left[\delta \left[z + \frac{d}{2} \right] \Theta \left[y^2 - \frac{R^2}{4} \right] + \delta \left[z - \frac{d}{2} \right] \Theta \left[\frac{R^2}{4} - y^2 \right] \right], \quad (12)$$

$$H_z + \lambda_{ab}^2 \frac{\partial}{\partial x} (\nabla \times \mathbf{H})_y - \lambda_{ab}^2 \frac{\partial}{\partial y} (\nabla \times \mathbf{H})_x = \phi_0 \delta(x) \left[\delta \left[y + \frac{R}{2} \right] - \delta \left[y - \frac{R}{2} \right] \right] \Theta \left[\frac{d^2}{4} - z^2 \right], \quad (13)$$

$$H_x + \lambda_c^2 \frac{\partial}{\partial y} (\nabla \times \mathbf{H})_z - \lambda_{ab}^2 \frac{\partial}{\partial z} (\nabla \times \mathbf{H})_y = 0. \quad (14)$$

The right-hand sides of Eqs. (12)–(14) represent the vortex cores (see Fig. 1): $\{x=0, z=-d/2, |y| > R/2\}$, $\{x=0, z=d/2, |y| < R/2\}$, and $\{x=0, y=\pm R/2, |z| < d/2\}$. To find the interaction between the kinks it is sufficient to consider a single vortex with two kinks as in Eqs. (12)–(14). The solutions of Eqs. (12)–(14) must then be substituted in the London free-energy functional. The calculations simplify in the Fourier space and we get

$$\delta F_{\text{cond}} + V_{\text{int}} = \frac{\phi_0^2}{4\pi^4} \int dk_x dk_y dk_z \frac{\sin^2(k_y R/2) \sin^2(k_z d/2)}{D} \times \left[\frac{1 + \lambda_{ab}^2(k_x^2 + k_z^2) + \lambda_c^2 k_y^2}{k_y^2} + \frac{1 + \lambda_c^2(k_x^2 + k_y^2) + \lambda_{ab}^2 k_z^2}{k_z^2} \right], \quad (15)$$

where

$$D = [1 + \lambda_c^2(k_x^2 + k_y^2) + \lambda_{ab}^2 k_z^2][1 + \lambda_{ab}^2(k_x^2 + k_y^2 + k_z^2)]. \quad (16)$$

We can obtain δF_{cond} from Eq. (15) by taking the limit $R \rightarrow \infty$. Thus,

$$\delta F_{\text{cond}} = \epsilon_0 \ln \left[\frac{d}{\xi_c} \right], \quad (17)$$

where ϵ_0 is

$$\epsilon_0 = \frac{\phi_0^2 d}{8\pi^2 \lambda_{ab}^2}. \quad (18)$$

The interaction energy has the form

$$V_{\text{int}}(R) = -\frac{d\epsilon_0}{4\lambda_{ab}} f \left[\frac{R}{\lambda_c} \right], \quad (19)$$

where

$$\begin{aligned} f(z) &= \int_1^\infty dx \left[\frac{x-1}{x} \right]^2 e^{-xz} \\ &= \frac{1}{z}, \quad z \ll 1 \\ &= \frac{2}{z^3} e^{-z}, \quad z \gg 1. \end{aligned} \quad (20)$$

Note that for $R \sim d(\lambda_c/\lambda_{ab})$, $\delta F_{\text{cond}} + V_{\text{int}}$ given by Eqs. (17)–(20) is of the same order of magnitude as the expression given in Eq. (11).

From Eqs. (4) and (7) we can obtain the activation energy for a vortex to penetrate a superconducting layer using Eq. (11) and Eqs. (17)–(20). The critical size of the nucleus is given by the extremum of the free energy given in Eq. (4). We get

$$(j-j_1) \frac{\phi_0 d}{c} = \frac{\partial}{\partial R} (\delta F_{\text{cond}} + V_{\text{int}}) \Big|_{R=R_c}. \quad (21)$$

When $\xi_{ab} \ll R \ll d(\lambda_c/\lambda_{ab})$, we get from Eq. (21) using Eq. (11),

$$R_c = \frac{\xi_{ab}}{I}, \quad (22)$$

where I is defined to be

$$I = \left[\frac{2}{3\sqrt{3}} \right] \left[\frac{j-j_1}{j_{\text{GL}}} \right]. \quad (23)$$

In this case the activation energy, U_0 , is given by

$$U_0 = \epsilon_0 \left[\ln \left[\frac{1}{I\sqrt{2}} \right] - 1 \right]. \quad (24)$$

The Ginzburg-Landau depairing current j_{GL} is defined by

$$j_{\text{GL}} = \frac{\phi_0 c}{12\pi^2 \sqrt{3} \lambda_{ab}^2 \xi_{ab}}. \quad (25)$$

The limiting case discussed above should hold in the regime,

$$\frac{\xi_c}{d} \ll \frac{j-j_1}{j_{\text{GL}}} \ll 1. \quad (26)$$

At smaller currents the size of the critical nucleus, R_c , will be larger than $d\lambda_c/\lambda_{ab}$ and one must use Eqs. (17)–(19). The energy U is given by

$$U = \epsilon_0 \left[\left[\ln \frac{d}{\xi_c} \right] - I \left[\frac{R}{\xi_{ab}} \right] - \left[\frac{d}{4\lambda_{ab}} \right] f \left[\frac{R}{\lambda_c} \right] \right]. \quad (27)$$

The size of the critical nucleus, R_c , can be determined from Eq. (21). When $d(\lambda_c/\lambda_{ab}) \ll R \ll \lambda_c$, corresponding to the regime

$$\frac{\xi_c d}{\lambda_{ab}^2} \ll \frac{j - j_1}{j_{GL}} \ll \frac{\xi_c}{d}, \quad (28)$$

the size of the critical nucleus is given by

$$R_c^2 = \xi_{ab}^2 \left[\frac{d}{4I\xi_c} \right]. \quad (29)$$

The activation energy, U_0 , is therefore

$$U_0 = \epsilon_0 \left[\left[\ln \frac{d}{\xi_c} \right] - \left[\frac{dI}{\xi_c} \right]^{1/2} \right]. \quad (30)$$

The other limiting case $R \gg \lambda_c$, corresponding to the regime,

$$\frac{\xi_c d}{\lambda_{ab}^2} \gg \frac{j - j_1}{j_{GL}}, \quad (31)$$

the size of the critical nucleus is given by

$$R_c = \lambda_c \ln \left[\frac{d\xi_c}{I\lambda_{ab}^2} \right], \quad (32)$$

and the activation energy, U_0 , is

$$U_0 = \epsilon_0 \left[\left[\ln \frac{d}{\xi_c} \right] - \left[\frac{\lambda_{ab} I}{\xi_c} \right] \ln \left[\frac{d\xi_c}{I\lambda_{ab}^2} \right] \right]. \quad (33)$$

The vortex activation energies in the regime $j_1 < j < j_{GL}$ are given by Eqs. (24), (30), and (33). However, as we shall see, for $j < j_1$ the critical nucleus is a bundle of vortices. In the next section we discuss the activation energy for such bundles.

IV. ACTIVATION OF A VORTEX BUNDLE

For a current j smaller than j_1 the approximation of a single vortex activation is not valid; in this case the critical nucleus consists of a bundle of vortices. It can be seen from Fig. 2 that the bundle consists of region of length R . The ratio of its size in the z direction to its size in the x direction is of the order of ξ_c / ξ_{ab} .

We can write down the macroscopic energy of such a bundle as a sum of a volume (Lorentz) energy and a surface energy. Thus the total energy U is

$$U = -\frac{jHd}{c}RS + \delta F_{\text{cond}} \frac{H}{\phi_0}S + \frac{j_1 d}{c}(SH\phi_0)^{1/2}R. \quad (34)$$

The second term in this equation are the energies of the right and the left surfaces as shown in Fig. 2. It is $2\delta F_{\text{cond}}$ from Eq. (17) multiplied by the number of vortices threading the surfaces which is SH/ϕ_0 . The third term in Eq. (34) represents the energy of the middle surface of the bundle as shown in Fig. 2. This energy is equal to the elastic energy, $(j_1 d \phi_0 R)/c$, of a single vortex multiplied by the number of shifted vortices along the middle surface of the bundle, i.e., $(SH/\phi_0)^{1/2}$.

The energy (34) should be minimized with respect to S and R . We get

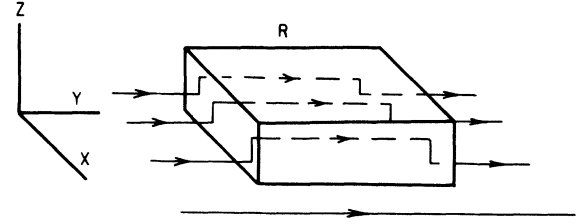


FIG. 2. Activation of a vortex bundle.

$$S_c = \frac{\phi_0}{H} \left[\frac{j_1}{j} \right]^2, \quad (35)$$

and

$$R_c = \frac{c\phi_0}{4\pi^2\lambda_{ab}^2 j}. \quad (36)$$

The corresponding activation energy is

$$U_0 = \delta F_{\text{cond}} \left[\frac{j_1}{j} \right]^2, \quad (37)$$

where δF_{cond} is given by Eq. (17). This macroscopic approximation is valid if the bundle consists of many vortices, i.e., if $S_c \gg \phi_0/H$ or if $j \ll j_1$. One should note that, in order of magnitude, the expressions given in Eqs. (33) and (37) match at $j \sim j_1$.

V. THE CURRENT-VOLTAGE CHARACTERISTIC

The results already obtained allow us to calculate the current-voltage characteristic. The resistive mechanism is related to the activated hopping of segments of vortices to the neighboring layers and to the subsequent motion of the vortex kinks along the layers. We shall suppose that the kinks can reach the sample boundaries before new bundles are created. In this case the mean value of the electric field is

$$E = \frac{H}{c} W d L S, \quad (38)$$

where L is the length of the sample along the direction of the magnetic field, S is the cross section of the bundle determined by Eq. (35). For the case of a single vortex activation S is simply ϕ_0/H . W is the activation probability per unit volume, per unit time, and is

$$W = \mathcal{B} \exp(-U_0/T). \quad (39)$$

We have determined U_0 for various regimes. However, the preexponential factor \mathcal{B} cannot be determined without the knowledge of the vortex dynamics; thermodynamic considerations alone do not suffice.

We now give the expressions for the current-voltage characteristic for various regimes. Introducing the quantity α , defined by

$$\alpha = \frac{\epsilon_0}{T}, \quad (40)$$

we get for $j \ll j_1$,

$$E = \frac{\phi_0 d}{c} L \mathcal{B} \left[\frac{j_1}{j} \right]^2 \exp \left[-\alpha \left[\frac{j_1}{j} \right]^2 \left[\ln \frac{d}{\xi_c} \right] \right], \quad (41)$$

for $\xi_c d / \lambda_{ab}^2 \gg j - j_1 / j_{GL} > 0$,

$$E = \frac{\phi_0 d}{c} L \mathcal{B} \exp \left\{ -\alpha \left[\left[\ln \frac{d}{\xi_c} \right] - \left[\frac{2}{3\sqrt{3}} \frac{\lambda_{ab}}{\xi_c} \frac{j - j_1}{j_{GL}} \right] \ln \left[\frac{j_{GL}}{j - j_1} \frac{d \xi_c}{\lambda_{ab}^2} \right] \right] \right\}, \quad (42)$$

for $\xi_c d / \lambda_{ab}^2 \ll j - j_1 / j_{GL} \ll \xi_c / d$,

$$E = \frac{\phi_0 d}{c} L \mathcal{B} \exp \left\{ -\alpha \left[\left[\ln \frac{d}{\xi_c} \right] - \left[\frac{2}{3\sqrt{3}} \frac{d}{\xi_c} \frac{j - j_1}{j_{GL}} \right]^{1/2} \right] \right\}, \quad (43)$$

and for $\xi_c / d \ll j - j_1 / j_{GL} \ll 1$,

$$E = \frac{\phi_0 d}{c} L \mathcal{B} \left[e \left[\frac{2}{3} \right]^{3/2} \left[\frac{j - j_1}{j_{GL}} \right] \right]^\alpha. \quad (44)$$

As can be seen, the current j_1 , plays an important role. From Eqs. (7) and (25), we obtain, with H in T,

$$\frac{j_1}{j_{GL}} \sim \frac{H d \xi_{ab}}{\phi_0} \sim 10^{-3} H \left[\frac{T_c}{T_c - T} \right]^{1/2}. \quad (45)$$

We assumed that $d \sim 10 \text{ \AA}$ and that $\xi_{ab} \sim 30 \text{ \AA}$ $[T_c / (T_c - T)]^{1/2}$. Because the depairing current has the order of magnitude of $10^9 [(T_c - T) / T_c]^{3/2} \text{ A/cm}^2$, the current j_1 is (in A/cm^2)

$$j_1 \sim 10^6 H [(T_c - T) / T_c] \quad (46)$$

If we assume that the magnetic field is larger than $H_{c1} \sim 100 \text{ G}$, and that $\lambda_{ab}^2 \sim 10^3 \xi_{ab}^2$, we get for currents $j < j_1$, in K ,

$$U_0 \cong 2.5 \times 10^3 \left[\frac{j_1}{j} \right]^2 \left[\frac{T_c - T}{T_c} \right]. \quad (47)$$

By contrast, in the regime in which $j > j_1$ the dependence of the activation energy on the transport current is weak. Equations (46) and (47) give the dependence of activation energy on the transport current, magnetic field, and temperature.

In case 4, Eq. (44), the current density is so large that it

can contribute to the magnetic field H if the sample size is not too small. In this case j in Eq. (44) should be interpreted to be the local current density.

VI. CONCLUSION

In conclusion, we have discussed the activation of vortices in the presence of a magnetic field in a strongly layered superconductor. The pinning considered is intrinsic to the layered structure. One of the important results is the magnetic field and temperature dependence of the activation energy $U_0 \sim H^2 j^{-2}$ at small currents. This is due to the property of a vortex bundle when the magnetic field is parallel to the layers. In order to observe this dependence it is necessary to have a pure enough sample. We would like to emphasize that the power law dependence on the current [cf. Eq. (44)] is valid only if the current j is sufficiently large, i.e., $j > j_{GL} (\xi_c / d)$. This property is determined by the intrinsic interlayer pinning when the magnetic field is parallel to the layers.

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