Conductance fluctuations in large metal-oxide-semiconductor structures in the variable-range hopping regime

Dragana Popović*
Department of Physics, Brown University, Providence, Rhode Island 02912

A. B. Fowler and S. Washburn
IBM Thomas J. Watson Research Center, Yorktown Heights, New York 10598

P. J. Stiles
Department of Physics, Brown University, Providence, Rhode Island 02912
(Received 12 March 1990)

Conductance fluctuations due to variable-range hopping have been studied in 8-mm-wide silicon inversion layers of large area (3.2 mm^2). The temperature dependence of the average logarithm of conductance $\langle \ln G \rangle$ varies with the carrier density N_s from nearly activated to very weak. Fluctuations of $\ln G$ with the chemical potential μ occur on two different scales. The distribution function of the fluctuations in $\ln G$ is also analyzed, and the results are consistent with the model of conduction via exponentially rare, highly conducting, quasi-one-dimensional chains of hops.

The origin of the fluctuations in the variable-range hopping (VRH) conduction of one-dimensional (1D) devices[1] may be understood[2] by replacing the system with a chain of resistors,[3] each resistor representing an individual hop. A change in the chemical potential μ leads to an exponential variation of the values of the resistors in the chain, changing, in this way, their relative contributions to the sample resistance. Since the total resistance is dominated by the largest resistor, the sample resistance fluctuates by up to several orders of magnitude. The temperature dependence of the ensemble averaged logarithm of resistance in a finite sample is expected[2] to obey Mott's law for 1D. In the presence of large fluctuations of the random potential, however, the sample resistance will be determined by exponentially rare regions of space containing no localized states within a few $k_B T$ of μ. Since energetically unfavorable hops cannot be avoided in 1D, the temperature dependence, in sufficiently long samples, will become activated.[4,5] In the opposite limit of very short samples, however, the temperature dependence will be very weak.[5,6] As the sample width increases, a crossover from 1D to 2D behavior may be expected intuitively. A transition from 1D to 2D Mott hopping has been predicted by numerical simulations,[7] where the channel width was increased until a symmetric 2D situation was reached. This effect may have been observed experimentally.[8] On the other hand, it has been proposed[9] that, for wide and sufficiently short samples, the infinite percolating cluster will be shunted by exponentially rare isolated chains of hops with conductivities that are exponentially larger than the conductivity of the infinite cluster. It was originally assumed that these chains are formed by localized states located at anomalously close distances from one another and subsequently shown[10] that they are almost rectilinear and equidistant. In this model, the sample conductance is then determined by the highest resistance link of the best-conducting chain (the "critical" hop). *Therefore, even though the total number of hops in a sample may be fairly large, its conductance is actually determined by very few hops.* This implies[11–13] that conductance fluctuations can be observed in relatively large systems, of size[12] up to 100 mm. Indeed, fluctuations have been observed as early as 1965[14] but have been studied[15,16] for the first time only recently, using a strip geometry sample of length 2 μm and width 200 μm, where the total number of hops was estimated to be of the order of 10^4. The experimental results were interpreted in terms of the above model, which also predicts[10] very weak temperature dependence. Large fluctuations of the random potential, however, were ignored[10,17,18] in this calculation, which may account for the quantitative discrepancies between the theory and the experiment. It has also been suggested[18] that the highly conducting chains ("punctures") actually form not in the regions of anomalously strong potential fluctuations as originally thought, but rather in the regions where fluctuations are considerably smaller than their typical value. In this paper, we report the observation of conductance fluctuations on much larger samples with length $L = 0.4$ mm and width $W = 8$ mm. Several features of our data are consistent with the model of conduction via quasi-1D chains (punctures).

The measurements were carried out on two n-channel circular metal-oxide-semiconductor field-effect transistors (MOSFET's) fabricated on the (100) surface of silicon doped to a level of $\approx 8.3 \times 10^{14}$ acceptors/cm^3. The gate oxide thickness was 435 Å and the amount of oxide charge, estimated from the threshold voltage at 77 K, was about 3×10^{10} cm^{-2}. The sample conductances were measured in a dilution refrigerator with a lock-in at a frequency of ~ 10 Hz and source-drain voltage 10^{-4} V. (Reducing the source-drain voltage had no effect on the

experiment.) Representative conductance versus gate voltage V_g (and hence μ and N_s) are shown in Fig. 1. The structure was fully reproducible over a period of months as long as the sample was kept below 1.2 K.

In order to study the temperature dependence of the conductance in this regime, one must average $\ln G$ over different impurity realizations or gate voltages (ensemble average). For this purpose, the autocorrelation function

$$C(\Delta V_g)=\langle F(V_g+\Delta V_g)F(V_g)\rangle \ , \tag{1}$$

where $F(V_g)=\ln G(V_g)-\langle \ln G(V_g)\rangle$, was calculated over various ranges of V_g. The correlation voltage V_c increased with V_g but it remained of the same order of magnitude (~ 1 mV). The averaging of $\ln G$ was performed over the ranges of width equal to $(15-20)V_c$. The data were then fitted to

$$\left\langle \ln\frac{G}{G_0}\right\rangle = -\left[\frac{T}{T_0}\right]^n \tag{2}$$

with the least-squares method, and the exponent n was varied to find the fit with the smallest deviation. The "best" values of n are displayed inset in Fig. 1 for several V_g. n increases continuously with N_s, changing the temperature dependence from almost activated to very weak. While Eq. (2) presents an excellent fit to the data for $V_g<0.43$ V, this is no longer true at higher gate voltages, where the data can be fitted successfully by, for example, a second-order polynomial in $\ln T$. Such a weak temperature dependence is consistent with theoretical predictions[10] based on a model of quasi-1D chains. Since $\langle \ln G\rangle$ is expected[10] to be a stronger function of temperature in longer samples, the discrepancies between the theory and the experiment observed at the lowest gate voltages may be understood intuitively in analogy to the 1D case discussed above. In this region, the fluctuations of the random potential may be very strong (compared to μ) making the localization length ξ short and L/ξ long. In other words, the sample will be effectively longer and the temperature dependence will approach activation. Since increasing N_s appears to improve screening,[18] ξ lengthens and reduces the ratio L/ξ. The sample is, therefore, effectively shorter and the temperature dependence weaker.

For $0.40\le V_g\le 0.42$ V, Mott's law for 1D is obeyed very well. Assuming that the width of a typical chain is W_0, then the 1D density of states D_1 will be about W_0D_2, where D_2 is the 2D density of states, and $T_0=4/k_B\xi D_1(E_F)\approx 30-60$ K. Using reasonable values[19] of $\xi\approx 300-500$ Å and $D_2\le 1.6\times 10^{14}$ $\mathrm{eV}^{-1}\,\mathrm{cm}^{-2}$, we find that W_0 is typically 100–300 Å and W_0/R, where R is the most probable hopping distance for the system, is typically 0.1–0.5. These values are consistent with 1D hopping. On the other hand, it does not seem likely that the exponent $-\frac{1}{2}$ in this range of V_g is due to a Coulomb gap[20] in the density of states because of the presence of the metal gate which screens out the Coulomb interactions. Here as in other experiments, the parameters (such as ξ and permittivity) are *not* consistent with observed T_0. It is possible that the mechanism mentioned above might be the source of $n=-\frac{1}{2}$ in other experiments.

The fluctuations are displayed in Fig. 2(a), after the monotonic component $\langle \ln G\rangle$, found by the quadratic least-squares fit, was subtracted. It is immediately ap-

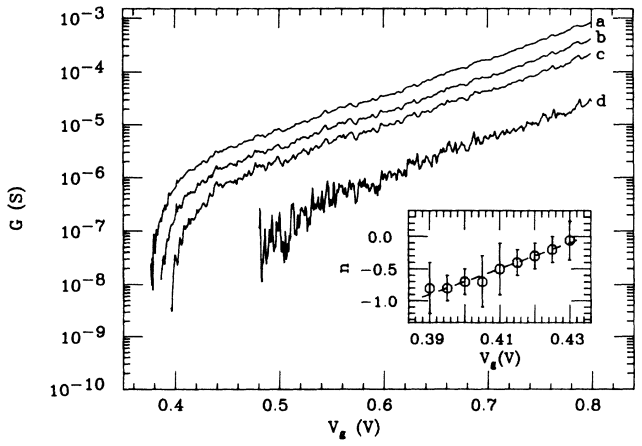

FIG. 1. Conductance vs gate voltage at (a) $T=0.555$ K, (b) $T=0.420$ K, (c) $T=0.330$ K, (d) $T=0.090$ K. Inset: The "best" exponent n for different gate voltages. The dashed line is a guide to the eye.

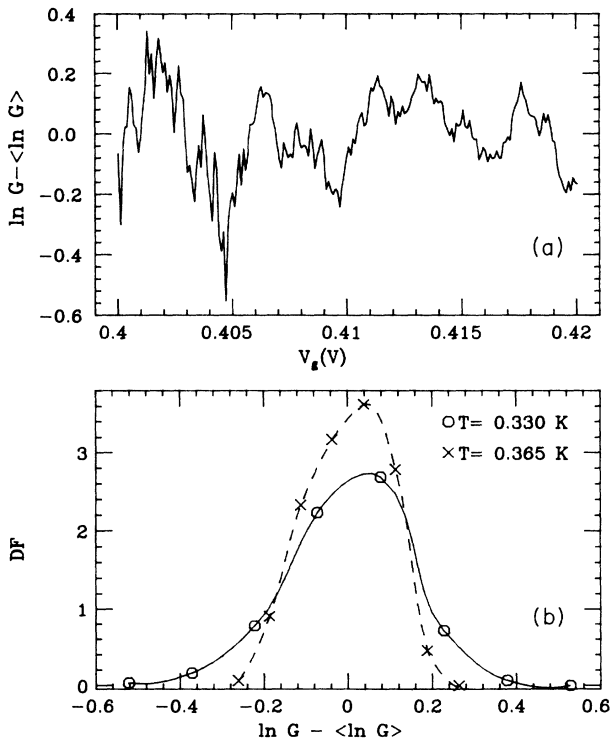

FIG. 2. (a) Fluctuations in the conductance logarithm with the gate voltage at $T=330$ mK. (b) Histograms of the distribution of the conductance logarithm for $0.40\ \mathrm{V}\le V_g\le 0.42$ V at two temperatures.

parent that the conductance fluctuates on two scales in μ: small-scale fluctuations are superimposed on the larger-scale fluctuations, which seem nearly periodic. This fluctuation pattern persists at higher V_g as well, but the "period" varies slightly due to a gate-voltage dependence of V_c. This type of pattern has actually been predicted,[13] with the small-scale fluctuations arising from switching among the "critical" hops within a single chain and the large-scale fluctuations from switching among different chains.

In order to be able to make quantitative comparisons with theory, we need to determine the parameter ν, introduced in Ref. 12 and defined as

$$\nu=\frac{2}{Q_0}\ln\frac{W}{W_0}\ , \tag{3}$$

where $Q_0=-\langle\ln(G/G_0)\rangle$. For $\nu>1$ the fluctuations should be small, because the sample is wide enough to contain many "optimal" chains, for which the product of conductance and probability of formation is maximum. Conversely, $\nu<1$ means that there will be no optimal chains in a typical sample so that the conductance will fluctuate by up to 100% from one sample to another. ν can be accurately determined[12] from $C(\Delta V_g=0)$. Unfortunately, the theoretical expression for the latter exists only for the case when $\nu<1$ and it breaks down when $\nu=1$. For our data, this expression yields[21] the values of ν which are too close to unity [e.g., 0.9972 for the data shown in Fig. 2(a)] to be trusted. Indeed, if we assume that $W_0\approx 100-300$ Å and we use $Q_0\sim 10$ found from the data in Fig. 2(a), Eq. (3) gives $\nu\approx 2.0-2.7$, which corresponds to a wide sample, with many optimal chains. Similarly, theoretical expressions for the fluctuation "period" exist[13] only for the cases of $\nu<1$ and $\nu\sim 1$. Those expressions do not agree with our measurements, which, at least indirectly, is consistent with a large value of ν.

The distribution functions for the conductance logarithm for a population of samples or, equivalently,[12] a series of gate voltages, have been deduced from the data. Figure 2(b) shows some typical results obtained for $0.40\le V_g\le 0.42$ V. The shape of the distribution function[12] (DF) is asymmetric for the cases of $\nu<1$ and $\nu\sim 1$, and it is Gaussian for larger values of the parameter ν. Although the DF's displayed in Fig. 2(b) are asymmetric, we have found that the shape of the DF and its width depend somewhat on the range of V_g so that in some cases the DF is either symmetric or it has a tail towards high conductance. At low V_g, the DF tends to have a tail towards low conductance, which is characteristic of 1D hopping, at intermediate V_g the DF sometimes tails toward higher conductance (characteristic of 2D hopping[22]), and at the highest V_g the DF is symmetric. The observed asymmetry, however, most likely lies within the experimental error and is due to a finite number of data points. This conclusion is supported by the calculations of the third cumulant and its error. The DF is expected[12] to be symmetric for $\nu>2^{1/2}$. In that case, its width is given by[12] $w\sim\exp[-Q_0(\nu-4+2^{3/2})/4]$. For $\nu=2.0$ and $Q_0\sim 10$, $w\sim 0.13$, in a reasonably good agreement with the experimental value of ≈ 0.33 [see Fig. 2(b)], considering that the expression for w is just an order of magnitude estimate. In fact, if the fluctuations of the random potential are taken into account,[18] the formulae for ν and w are somewhat modified. Our results, however, are incapable of distinguishing between those two situations. Finally, Fig. 3 displays $w(V_g)$ and $V_c(V_g)$ at different temperatures. The decrease of w with increasing V_g, i.e., the reduction of the amplitude of the fluctuations, is consistent with ν becoming larger as $\langle\ln G\rangle$ increases [see Eq. (3)].

In summary, we remark that for large nominally 2D samples, the temperature dependence, the pattern of the fluctuations and the width and shape of the distribution function for the conductance logarithm, are in a qualitative agreement with the theoretical predictions[10,12,13] based on the idea that the sample conductance is dominated by the electron transport through exponentially rare, quasi-1D highly conducting regions. Large oscillations have also been observed very recently in the magnetoresistance of large area, InO films with hopping conductivity.[23] In order to be able to make quantitative comparisons between the theory and the experiment, much more realistic models for the punctures are needed, as well as more experiments that provide unambiguous information on where the fluctuations originate. At the

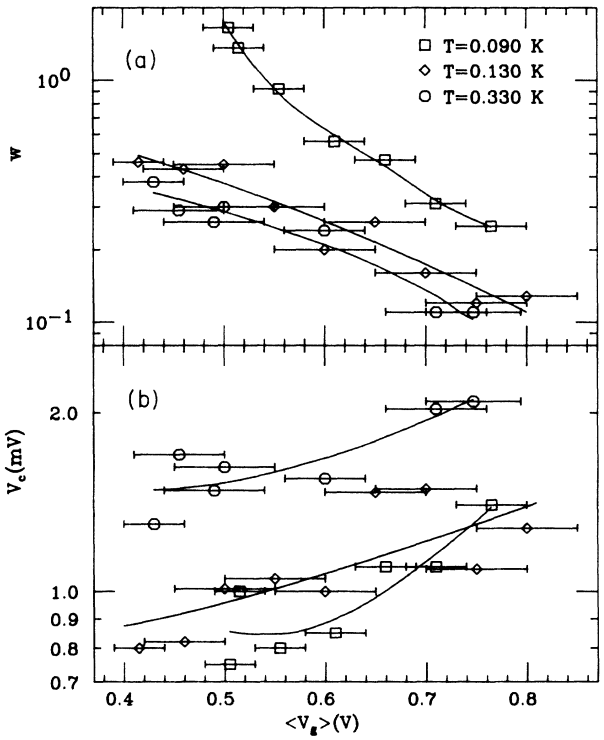

FIG. 3. The dependence on V_g of (a) the width of the distribution function and (b) the correlation voltage at different temperatures. The solid lines guide the eye. The $T=0.130$ K data were taken after the sample was warmed up to 4.2 K.

moment, we do not know whether they arise from the entire sample, or are associated with much smaller regions as a result of inhomogeneities. For example, in spite of considerable sophistication in the manufacturing process (these are self-aligned, implanted contacts), we cannot yet completely rule out the possibility that the contacts are poorly defined.[24] We do believe, however, that this is very unlikely, and therefore that the data represent startling observations of sample specific conductance fluctuations in rather large samples.

This work was supported in part by the National Science Foundation (NSF) under Grant No. DMR-8717817.

---

*Present address: IBM Thomas J. Watson Research Center, York Town Heights, New York 10598.

[1]See, for example, A. B. Fowler, J. J. Wainer, and R. A. Webb, IBM J. Res. Dev. **32**, 372 (1988), and references therein.

[2]P. A. Lee, Phys. Rev. Lett. **53**, 2042 (1984).

[3]A. Miller and E. Abrahams, Phys. Rev. **120**, 745 (1960).

[4]J. Kurkijärvi, Phys. Rev. B **8**, 922 (1973).

[5]M. É. Raĭkh and I. M. Ruzin, Zh. Exp. Teor. Fiz. **95**, 1113 (1989) [Sov. Phys.—JETP **68**, 642 (1989)].

[6]R. A. Serota, R. K. Kalia, and P. A. Lee, Phys. Rev. B **33**, 8441 (1986).

[7]X. C. Xie and S. Das Sarma, Phys. Rev. B **36**, 4566 (1987).

[8]A. B. Fowler, A. Hartstein, and R. A. Webb, Phys. Rev. Lett. **48**, 196 (1982).

[9]M. Pollak and J. J. Hauser, Phys. Rev. Lett. **31**, 1304 (1973).

[10]A. V. Tartakovskiĭ, M. V. Fistul', M. É. Raĭkh, and I. M. Ruzin, Fiz. Tekh. Poluprovodn. **21**, 603 (1987) [Sov. Phys.—Semicond. **21**, 370 (1987)].

[11]M. É. Raĭkh and I. M. Ruzin, Pis'ma Zh. Eksp. Teor. Fiz. **43**, 437 (1986) [JETP Lett. **43**, 562 (1986)].

[12]M. É. Raĭkh and I. M. Ruzin, Zh. Eksp. Teor. Fiz. **92**, 2257 (1987) [Sov. Phys.—JETP **65**, 1273 (1987)].

[13]M. É. Raĭkh and I. M. Ruzin, Fiz. Tekh. Poluprovodn. **22**, 1262 (1988) [Sov. Phys.—Semicond. **22**, 799 (1988)].

[14]W. E. Howard and F. F. Fang, Solid State Electron. **8**, 82 (1965).

[15]A. O. Orlov, A. K. Savchenko, E. V. Chenskiĭ, É. A. Il'chev, and É. A. Poltoratskiĭ, Pis'ma Zh. Eksp. Teor. Fiz. **43**, 421 (1986) [JETP Lett. **43**, 540 (1986)].

[16]E. I. Laĭko, A. O. Orlov, A. K. Savchenko, É. A. Il'ichev, and É. A. Poltoratskiĭ, Zh. Eksp. Teor. Fiz. **93**, 2204 (1987) [Sov. Phys.—JETP **66**, 1258 (1987)].

[17]M. É. Raĭkh (private communication).

[18]A. O. Orlov, M. É. Raĭkh, I. M. Ruzin, and A. K. Savchenko, Zh. Eksp. Teor. Fiz. **96**, 2172 (1989).

[19]C.J. Adkins, S. Pollitt, and M. Pepper, J. Phys. (Paris) Colloq. **37**, C4-343 (1976).

[20]A. F. Efros and B. I. Shkovskii, J. Phys. C **8**, L49 (1975).

[21]Dragana Popović, in *Hopping and Related Phenomena,* edited by H. Fritzsche and M. Pollak (World Scientific, Singapore, 1990), p. 243.

[22]A. O. Orlov, M. É. Raĭkh, I. M. Ruzin, and A. K. Savchenko, Solid State Commun. **72**, 169 (1989).

[23]F. P. Milliken and Z. Ovadyahu (unpublished).

[24]M. Pepper, M. J. Uren, and R. E. Oakley, J. Phys. C **12**, L897 (1979).