RAPID COMMUNICATIONS

PHYSICAL REVIEW B VOLUME 42, NUMBER 2 15 JULY 1990-I

Magneto-optics in parabolic quantum dots

F. M. Peeters*
Bellcore, 331 Newman Springs Road, Red Bank, New Jersey 07701-7020

(Received 3 April 1990)

We show that the position of the resonance lines in the magneto-optical absorption spectrum of a quantum dot with a (asymmetric) parabolic confinement potential is independent of the electron-electron interaction and the number of electrons in the quantum dot.

State-of-the-art lithographic and etching techniques make it possible to confine quasi-two-dimensional (Q2D) electrons at the interface of semiconductors into *dots* of diameter which, at low temperature, is less than the mean free electron path. The electron energy spectrum of such quantum dots is fully quantized. The size and shape of the dot, and the number of electrons in such a dot, can be tuned by technological means. These quantum dots can be considered as the analog of conventional atoms.

Sikorski and Merkt[1] found experimentally the surprising result that the resonance frequency in such quantum dots do not depend on the number of electrons in the quantum dot. The independence of the excitation energy on the number of electrons indicates that the excitation spectrum of a quantum dot is not influenced by the electron-electron interaction. This is in contrast to the situation in real atoms.

Chaplik[2] proved theoretically that for the two-electron system there is an exact compensation of the Coulomb effect. But he found that for larger numbers of electrons, shifts in the energy excitation spectrum due to electron-electron interaction should be present.

Recently Brey, Johnson, and Halperin[3] showed that a parabolic quantum well absorbs far-infrared radiation at the bare harmonic-oscillator frequency independent of the electron-electron interaction and the number of electrons in the well. In this paper we generalize this result to a quasi-zero-dimensional system with parabolic confinement. The observation of Sikorski and Merkt is then a direct consequence of *a generalized Kohn's theorem*[4] which we prove to be valid for a many-electron system confined by a quadratic potential in the presence of a magnetic field. We consider a general asymmetric harmonic potential. Furthermore, we found that the above conclusion is true for any kind of interaction between the electrons as long as the interparticle interaction depends only on the relative distance between the electrons.

In the experimental system of Ref. 1 the electrons are much more strongly confined in one direction (taken as the z direction) than in the other two directions. Therefore, we will confine ourselves to consider only the motion of the electrons in the x-y plane. In the effective-mass approximation the Hamiltonian for N noninteracting electrons in an asymmetric quantum dot is given by

$$H_0=\sum_{j=1}^{N}\frac{\mathbf{p}_j^2}{2m}+\tfrac{1}{2}m\sum_{j=1}^{N}(\omega_x^2x_j^2+\omega_y^2y_j^2)\,, \tag{1}$$

where the confinement frequencies ω_x,ω_y may differ from each other. The interacting system is described by the Hamiltonian $H=H_0+U$, with U the electron-electron interaction

$$U=\sum_{i<j=1}^{N}u(\mathbf{r}_i-\mathbf{r}_j)\,, \tag{2}$$

where for Coulomb interaction the potential is given by $u(\mathbf{r})=e^2/\epsilon|\mathbf{r}|$, with ϵ the dielectric constant of the medium.

The noninteracting system can be diagonalized exactly,

$$H_0=\hbar\omega_x(C_x^+C_x^-+\tfrac{1}{2})+\hbar\omega_y(C_y^+C_y^-+\tfrac{1}{2})\,,$$

with the creation operator $C_x^+=\sum_{j=1}^{N}c_{j,x}^+$ and

$$c_{j,x}^+=\left[\frac{m\omega_x}{2\hbar}\right]^{1/2}\left[x_j-i\frac{p_{j,x}}{m\omega_x}\right]. \tag{3}$$

The annihilation operator $c_{j,x}^-$ is the conjugate of $c_{j,x}^+$. Analogous definitions hold for $c_{j,y}^{\pm}$. The eigenfunctions are denoted by $\Psi_{n_x,n_y}(x,y)$ with corresponding energies

$$E_{n_x,n_y}=\hbar\omega_x(n_x+\tfrac{1}{2})+\hbar\omega_y(n_y+\tfrac{1}{2})\,.$$

Combining Eqs. (2) and (3), it is simple to prove the important property $[U,C_{x,y}^{\pm}]=0$, which is valid for any U that depends only on the relative distance between any two electrons. As a consequence the total Hamiltonian sastisfies the commutation relation $[H,C_{x,y}^{\pm}]=\pm\hbar\omega_{x,y}\times C_{x,y}^{\pm}$, which implies that if Φ_{n_x,n_y} is an eigenstate with energy $E(n_x,n_y)$ so are $C_x^{\pm}\Phi_{n_x,n_y}$ with energy $E(n_x,n_y)\pm\hbar\omega_x$ and $C_y^{\pm}\Phi_{n_x,n_y}$ with energy $E(n_x,n_y)\pm\hbar\omega_y$. From this observation we conclude that the excitation spectrum of the interacting system consists of two sets of equidistant levels with separation equal to the bare harmonic potential frequencies ω_x,ω_y. The electron-electron interaction will shift the total-energy spectrum and will modify the eigenstates. But the electron-electron interaction does not alter the separation between the energy levels and will thus not influence the excitation that is observed in an optical excitation experiment.

When the system is placed in an electric field with frequency ω: $\mathbf{E}(t)=e^{-i\omega t}E_0(\cos\theta,\sin\theta)$, this leads to the following interaction term in the Hamiltonian $H'=-e\sum_j\mathbf{r}_j\cdot\mathbf{E}(t)$ which is responsible for dipole transitions. Because $\sum_j x_j=(\hbar/m\omega_x)^{1/2}(C_x^-+C_x^+)$ and $\sum_j y_j=(\hbar/m\omega_y)^{1/2}(C_y^-+C_y^+)$ such an electric field will only induce transitions from an eigenstate Φ_{n_x,n_y} to the eigenstates $C_{x,y}^{\pm}\Phi_{n_x,n_y}$. We find that the optical absorption spectrum is proportional to

$$\cos^2\theta[D(\omega-\omega_x)+D(\omega+\omega_x)] \\ +\sin^2\theta[D(\omega-\omega_y)+D(\omega+\omega_y)] , \quad (4)$$

with $D(x)=\Gamma/(x^2+\Gamma^2)$, where Γ is the inverse of a characteristic relaxation time. The absorption spectrum (4) consists of two peaks at frequency ω_x and ω_y with relative oscillator strength $\cos^2\theta$ and $\sin^2\theta$, respectively.

Next we will generalize the above results to the case of a magnetic field applied in the z direction. The only change in the Hamiltonian (1) is that we have to replace the kinetic-energy term $\mathbf{p}_j^2$ by $[\mathbf{p}_j-(e/c)\mathbf{A}_j]^2$ with $\mathbf{A}_j$ the vector potential which is chosen in the symmetrical gauge $\mathbf{A}_j=B(-y_j,x_j,0)/2$. Because H_0 is a quadratic function of the electron coordinates the Hamiltonian can still be diagonalized exactly. This leads to a similar diagonalized form for H_0 as before but now with the eigenfrequencies

$$\omega_{1,2}^2=\tfrac{1}{2}\{(\omega_x^2+\omega_y^2+\omega_c^2) \\ \pm[(\omega_x^2+\omega_y^2+\omega_c^2)^2-4\omega_x^2\omega_y^2]^{1/2}\} , \quad (5)$$

where $\omega_c=eB/mc$ is the cyclotron resonance frequency. For $\omega_c\rightarrow 0$ one finds $\omega_{1,2}\rightarrow\omega_{x,y}$, and for $\omega_c\rightarrow\infty$ one has $\omega_1\rightarrow\omega_c$ and $\omega_2\rightarrow 0$. The two creation and annihilation operators are $A_{1,2}^{\pm}=\sum_{j=1}^{N}a_{1,2}^{\pm}(j)$, where

$$a_{1,2}^{\pm}(j)=u_{1,2}\left[x_j(-\omega_{1,2}^2+\omega_y^2+\tfrac{1}{2}\omega_c^2)\mp i\frac{p_{j,x}}{m\omega_{1,2}}(-\omega_{1,2}^2+\omega_y^2)\mp iy_j\frac{\omega_c}{2\omega_{1,2}}(\omega_{1,2}^2+\omega_y^2)-\frac{p_{j,y}}{m}\omega_c\right] , \quad (6)$$

with

$$u_{12}=(m\omega_{1,2}/2\hbar)^{1/2}[(\omega_{1,2}^2-\omega_y^2)^2+\omega_c^2\omega_y^2]^{-1/2} .$$

The above operators satisfy the following commutation relations: $[a_1^{\pm}(j),a_2^{\pm}(j)]=0$ which implies that $a_1^{\pm}(j)$ and $a_2^{\pm}(j)$ are independent. Furthermore, we have $[a_s^{-}(j),a_s^{+}(j)]=1$ for $s=1,2$. This leads to $[H_0,A_{1,2}^{\pm}]=\pm\hbar\omega_{1,2}A_{1,2}^{\pm}$ and consequently the operators $A_{1,2}^{\pm}$ induce transitions between neighboring eigenstates. The electron-electron interaction is such that $[U,A_{1,2}^{\pm}]=0$ which leads to $[H,A_{1,2}^{\pm}]=\pm\hbar\omega_{1,2}A_{1,2}^{\pm}$ and tells us that also in the presence of a magnetic field the separation of the energy levels in the interacting system is identical to that in the absence of Coulomb interaction.

Interaction of the system with long-wavelength light will result in resonant absorption at the two frequencies ω_1,ω_2 which are independent of the electron-electron interaction. Thus we can state the following generalization of Kohn's theorem: *The resonance frequencies in the magneto-optical absorption spectrum of a quantum dot with parabolic confinement are independent of the electron-electron interaction and are given by the single-electron transition frequencies.* Although we have considered a quantum dot in two dimensions the same conclusion holds also for a quantum dot in three dimensions with an arbitrary magnetic field as long as the confining potentials in the different spatial directions can be represented by a harmonic one. Kohn's original theorem[4] was proven for a translational invariant three-dimensional electron gas. This generalizes the parabolic quantum-well result of Brey *et al.* to a quasi-zero-dimensional electron.

Any dependence of the position of the resonance frequencies in the magneto-optical absorption spectrum on the number of electrons in a quantum dot is a consequence of the nonharmonicity of the confining potential. This dependence will not necessarily be a consequence of the electron-electron interaction. Recently[5] it was shown that a quantum dot with hard walls exhibits a rich spectrum with more than two resonance frequencies due to the fact that (1) the energy levels are not equally spaced in a hard-wall quantum dot, and (2) the absence of certain selection rules. This already implies that filling up the different energy levels by increasing the number of electrons will result in different possible transitions with different energies. Also, impurity scattering may indirectly lead to a weak electron density dependence of the resonance frequency as was found for the two-dimensional electron gas.[6]

I am indebted to S. J. Allen, Jr. for pointing out the problem and for a fruitful discussion. The author is supported by the Belgian National Science Foundation.

---

*Permanent address: University of Antwerp, Department of Physics, B-2610 Antwerp, Belgium.

[1] Ch. Sikorski and U. Merkt, Surf. Sci. **229**, 282 (1990); Ch. Sikorski and U. Merkt, Phys. Rev. Lett. **62**, 2164 (1989).

[2] A. V. Chaplik, Surf. Sci. (to be published); Pis'ma Zh. Eksp. Teor. Fiz. **50**, 38 (1989) [JETP Lett. **50**, 44 (1989)].

[3] L. Brey, N. F. Johnson, and B. I. Halperin, Phys. Rev. B **40**, 10647 (1989).

[4] W. Kohn, Phys. Rev. **123**, 1242 (1961).

[5] F. Geerinckx, F. M. Peeters, and J. T. Devreese J. Appl. Phys. (to be published).

[6] Z. Schlesinger, S. J. Allen, J. C. Hwang, P. M. Platzman, and N. Tzoar, Phys. Rev. B **30**, 435 (1984).