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Destruction of fractional quantum Hall effect in thick systems

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We study systematically the effect of finite thickness of the quasi-two-dimensional layer on the fractional quantum Hall effect using small-system calculations. As the layer thickness increases, there is a crossover from the incompressible Laughlin-type liquid to a compressible state, which has almost vanishing overlap with the Laughlin wave function. We predict that the fractional quantum Hall state will eventually be destroyed with increasing layer thickness as a consequence of the weakening of the short-range component of the interaction. The relevance of our calculation to some recent experiments in thick parabolic wells is discussed.

The fractional quantum Hall effect (FQHE) is observed[1] in high-mobility quasi-two-dimensional (2D) systems in a strong magnetic field. It arises due to the incompressibility of a quantum liquid state best described by the Laughlin wave function.[2] The dynamics of FQHE is governed by the short-range component of the electron-electron repulsive interaction, which is responsible for the incompressibility.[2-4] The FQHE is rigid and stable against small perturbations. From this point of view, the FQHE does not depend crucially on the details of the actual form of the electron-electron interaction. However, if the short-range part of the interaction becomes softened enough, there will be no force to produce the incompressibility and the FQHE will be destroyed.

In this paper, we systematically study the effect of finite layer thickness of the quasi-2D system on the effective electron-electron interaction, and calculate its effect on the FQH state. We show that the FQHE is eventually destroyed with increasing layer thickness, as a consequence of the sufficient softening of the short-range component of the interaction. Our work is motivated by the recent experiment of Shayegan *et al.*,[5] who observed, with increasing layer thickness, a dramatic decrease in the measured energy gap for the FQH state at filling factors $\nu=\frac{1}{3}$ and $\frac{2}{3}$ in a variable-width parabolic quantum well system.

In a real quasi-2D electron system, the electron wave function has a finite extent in the third direction. This modifies the effective electron-electron interaction from the strictly 2D form. Previous work involving the width effect on FQHE is limited to relatively small values of the layer thickness W, as appropriate for a GaAs/Al$_x$Ga$_{1-x}$-As heterojunction. For W/l not so large, where $l=(\hbar e/eB)^{1/2}$ is the magnetic length, it was found that the finite width results in a substantial reduction of the ground-state energy[6] and the excitation gap[7,8] of the FQH state. The qualitative nature of the ground state, however, did not change due to finite layer width—in particular the system remained incompressible. The recent experiment of Shayegan *et al.*[5] in thick parabolic quantum wells has motivated us to revisit this problem. Our main interest here is to investigate the qualitative question of how the fundamental incompressibility of the Laughlin state may be affected by a continuous increase of the layer width. In particular, we study whether the FQHE may be destroyed by a continuous increase of the layer width.

We focus on the $\nu=\frac{1}{3}$ state. Using pseudopotentials suitable to parametrize the effective electron-electron interaction for a quasi-2D system, we show explicitly that the extent of the electron wave function in the third direction substantially weakens the short-range component of the electron-electron interaction relative to the long-range components. Increasing the width eventually changes the qualitative nature of the state, and destroys the FQHE. We carry out exact calculations on small spherical systems to investigate the qualitative change of the ground state due to the finite layer thickness. We find that FQHE is destroyed around $W/l\sim 10$.

We consider a clean quasi-2D system in a strong uniform magnetic field. The effective electron-electron interaction is modeled by

$$V(|\mathbf{r}_1-\mathbf{r}_2|)=\frac{e^2}{\epsilon(r^2+\lambda^2)^{1/2}}, \tag{1}$$

where $\mathbf{r}=\mathbf{r}_1-\mathbf{r}_2$ is the 2D vector separating the two electrons, $\lambda=W/2$ is the effective half-width of the system in the third direction, and ϵ is the background dielectric constant. It has been shown that the model defined by Eq. (1) gives fairly accurate[7] results for the thickness effect on FQHE.

In high magnetic fields, the cyclotron kinetic-energy dominates, and we need only consider the states within the lowest Landau level. The electron pair interaction can be described[3] by a set of pseudopotentials, V_m, which is given by the potential energy of two electrons with relative angular momentum m

$$V_m = \int d^2\mathbf{r} |\phi_m(\mathbf{r})|^2 V(\mathbf{r}) . \tag{2}$$

In Eq. (2), $\phi_m(\mathbf{r})$ is the normalized wave function of a pair of electrons with relative angular momentum m. Because of the Fermi statistics, only those V_m with odd m are relevant for the spin-polarized system being studied in this paper. The FQH state at $\nu=\frac{1}{3}$ is described by Laughlin wave function

$$\psi = \prod_{i<j} (z_i - z_j)^3 \exp\left[-\sum_i \frac{1}{4} \frac{|z_i|^2}{l^2} \right] , \tag{3}$$

where z_i is the complex representation of the 2D vector for the ith electron. Laughlin's state becomes the exact ground state for a hard-core potential with $V_1 > 0$, and $V_m = 0$ for all other m's. For a purely 2D system with Coulomb interaction, the deviation from the hard-core model is small, and the FQHE survives.

From Eq. (2), we note that if $V(\mathbf{r}) \rightarrow V(\mathbf{r}) + C$, with C a constant, then $V_m \rightarrow V_m + C$, for all m's. The net effect of such a transformation is to shift the energy spectrum of the system by an overall constant. Therefore, only the relative differences of the pseudopotentials $\{V_m\}$ are relevant to the nature of the ground state. To quantitatively describe the deviation of a quasi-2D system from the hard-core model, we introduce a set of dimensionless parameters:

$$f_m = (V_3 - V_m)/(V_1 - V_3) . \tag{4}$$

$\{f_m\}$ are invariant when $\{V_m\} \rightarrow \{V_m + \text{const.}\}$. From Eq. (4), we always have $f_1 = -1$, $f_3 = 0$ for any pair potential $V(\mathbf{r})$. So we only consider f_m with $m \geq 5$. For the hard-core model, $f_m = 0$, for all m. The deviation from the hard-core model is then described by nonzero values of f_m. If many f_m's are large, the system is not well described by the hard-core model, and Laughlin's state, Eq. (3), will no longer be a good description for the true ground state of the system, and, consequently, we do not expect FQHE to occur.

From Eq. (4), we note that if we reduce all V_m by a same constant factor, f_m are unchanged. The only net effect of such a reduction in V_m is to rescale the energy units. The effect of the layer thickness is, however, more complicated. When the width increases, all V_m are reduced, *but not by the same constant.* The reduction depends on the value of m, so that the values of f_m are relatively shifted. The finite width softens the short-range ($m=1$) interaction relative to the long-range component (larger m). In Fig. 1, we plot f_m as functions of the half-width λ for several values of m. The case $\lambda/l = 0$ corresponds to the strict 2D case. As λ/l increases, f_m with $m \geq 5$ increases monotonically, thus driving the system farther and farther away from the ideal hard-core model. One may thus argue that beyond a point in λ/l, the effective interaction loses its short-range nature, and the Laughlin state is not favored. It is, therefore, inevitable

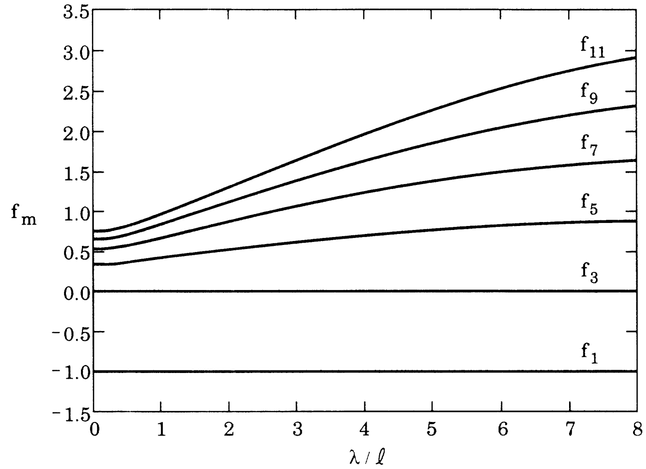

FIG. 1. Dimensionless pseudopotentials f_m [cf. Eq. (4)] of the effective electron-electron interaction as functions of λ/l.

that the FQHE will eventually be destroyed as λ/l increases.

We are primarily interested in the possible destruction of the FQHE at $\nu=\frac{1}{3}$ due to the finite layer width. Since Laughlin's wave function best describes the incompressible FQH state, its overlap with the *exact* ground state in small systems provides a good measure of the qualitative nature of the ground state. In the spherical geometry, Laughlin wave function of Eq. (3) has the following form:

$$\psi = \prod_{i<j} (u_i v_j - v_i u_j)^3 , \tag{5}$$

where u,v are spinor coordinates in a sphere and $u = \cos(\theta/2)\exp(\frac{1}{2} i\phi)$, $v = \sin(\frac{1}{2}\theta)\exp(-\frac{1}{2} i\phi)$ where θ, ϕ are the usual polar angles.

We model the thickness effect using the pseudopotential $\{V_m\}$ given by Eqs. (2) and (1) *obtained for an infinite system.* We then use these values of V_m to diagonalize the finite spherical system exactly. Because of the finite curvature of the spherical surface, these parameters are the thermodynamic limit of the pseudopotentials for the finite system. It is more appropriate to use the $\{V_m\}$ of the infinite system to study the layer thickness effect, especially when the layer width becomes large. To justify our method, we plot in Fig. 2 the excitation gap for up to eight-electron system at $\nu=\frac{1}{3}$ for $\lambda=0$ using the $\{V_M\}$ of the infinite system. The extrapolation of the gap to the thermodynamic limit is in very good agreement with the results of Fano, Ortolani, and Colombo,[9] who used $\{V_m\}$ of the finite system.

We now discuss our numerical results of the layer thickness effect. In Fig. 3, we plot the wave-function overlap between the exact numerical result and the Laughlin state as a function of the layer width for a six-electron system. There are three distinct regions of λ/l as we can see from the figure. For $\lambda/l \lesssim 3$ the overlap is close to unity. This indicates that the Laughlin state is an excellent description of the true ground state. The overlap is insensitive to the layer width in this region. This may be understood as a consequence of the rigidity of the Laughlin state. The

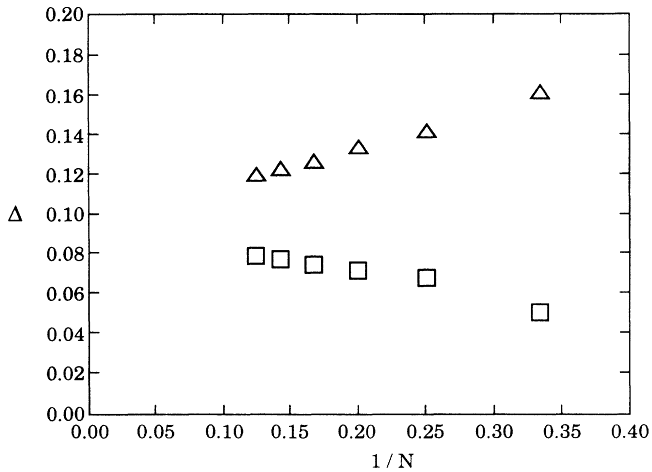

FIG. 2. The excitation gaps of pure 2D systems at $\nu=\frac{1}{3}$ as functions of the number of electrons in the small system calculations. The squares are the results of the present calculation, where the pseudopotential V_m of the infinite system is used. The triangles are the results of Fano *et al.* (Ref. 9), where V_m of the finite systems is used.

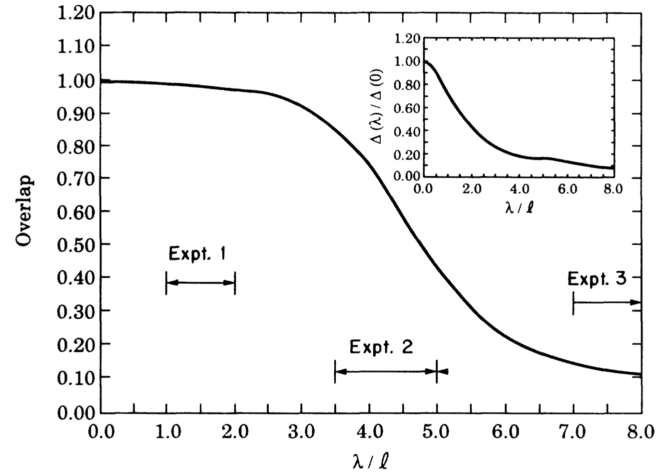

FIG. 3. Wave-function overlap between the exact ground state for a small sphere and the Laughlin state as a function of λ/l. The marked parameter regions: "Expt. 1" corresponds to the experimental situation for the heterostructure samples, "Expt. 2" corresponds to Ref. 5 with the larger layers widths. Region "Expt. 3" has not yet been reached in experiments, where we predict the disappearance of the FQHE. The inset shows the calculated excitation gap (relative to the strict 2D system) as a function of λ/l [one had $\Delta(0)\approx 0.1e^2/l$]. The number of electrons in these calculations are six.

effect of finite width is to reduce the excitation gap (shown as an inset in Fig. 3). For $\lambda/l>7$, we are in region 3 where the overlap becomes very small, and, the Laughlin state is no longer a good description. We interpret the ground state of the system in this region to be a compressible state. Such an assignment is based on our understanding of the incompressible FQH state being the Laughlin state. We conclude that the FQHE is destroyed in this parameter range (region 3) due to the softening of the short-range interaction. From our small system calculations, we see no evidence to indicate the ground state to be a crystal-like state. The total orbital angular momentum of the electron system is zero, suggesting the state to be a liquid. This, however, may be due to the finite size of the sphere, which seems to favor liquid states. Between the above two regions is a crossover (region 2), where the overlap drops from almost unity to almost zero. Intuitively, one may argue that the excitation gap reduces quickly in the crossover region, because the ground state becomes less incompressible. One can, however, see from the inset of Fig. 3 that actually the excitation gap drops rapidly in regions 1 and 2, becoming very small for $\lambda/l>5$ and remaining almost a constant in region 3—we attribute this to finite-size effects. In summary, the layer width in quasi-2D electron systems has two distinct effects. When the width is small, it reduces the excitation gap while still preserving the Laughlin state, when the width becomes large, it destroys the FQHE.

We now discuss our numerical results in comparison with experiments. The FQHE at $\nu=\frac{1}{3}$ or $\frac{2}{3}$ have been observed at magnetic field $B\cong 6$–28 T, or the magnetic length $l\cong 50$–100 Å. For the GaAs-$Ga_{1-x}Al_xAs$ heterostructure,[10] the half-width of the layer thickness $\lambda\simeq 100$ Å. So $\lambda/l=1$–2, and the system is well within the small width region. The finite width only reduces the gap, while the ground state is still a Laughlin state. In the selectively doped parabolic $Al_xGa_{1-x}As$ quantum well,[5] the electron layer thickness increases with increasing areal density in the well, and λ can be varied between 400 and 1000 Å. In the work[5] of Shayegan *et al.*, it was reported that the excitation gap decreases dramatically between $\lambda/l\simeq 3.5$ and 5. This is in the crossover region 2 of Fig. 3. The dramatic decrease in the gap is in accord with the apparent decrease of the wave function overlap between the ground state and the Laughlin state. We note that the directly calculated energy-gap saturates in this region due to the discrete spectra of the small system. Therefore, a more sophisticated estimate of the energy gap is needed to give a quantitative description in this region. In addition, the experimental measurements of the gap, particularly for the smaller values of the gap in Ref. 5, must be strongly affected by disorder. Naively, one would not trust the transport measurement of the excitation gap Δ to be very accurate when $\Delta\sim\Gamma$, where Γ is the disorder-induced broadening. For the samples of Ref. 5, the condition of $\Delta\sim\Gamma$ is certainly satisfied for $\Delta\sim 0.5$ K (and, probably is satisfied for $\Delta\sim 1$ K). Thus, the actual numbers for the excitation gap extracted in Ref. 5 in our regions 2 and 3 of Fig. 3 are not very reliable. It should be interesting to test experimentally whether the FQHE is destroyed as λ/l is increased above $\lambda/l\sim 7$ as our calculation predicts. For a given sample with a fixed electron density and width, $\lambda/l_{1/3}=\sqrt{2}\lambda/l_{2/3}$; with $l_{1/3}$ and $l_{2/3}$ being, respectively, the magnetic length at $\nu=\frac{1}{3}$ and $\frac{2}{3}$. Therefore, we predict that it is possible, for a fixed λ, to observe the $\frac{2}{3}$ FQHE while the $\frac{1}{3}$ state has already been destroyed. This may be realized in variable-width systems.

In our theoretical study, we have not included disorder, which always exists in real samples. As the layer width increases, the electric subband mixing also becomes more important. While a complete theory should include these factors, we want to emphasize that the increasing layer width alone will destroy the FQHE if the width is large enough so that the short-range component of the electron-electron interaction is sufficiently weakened with respect to the long-range component. We believe that this situation has already been achieved in wide parabolic wells of Ref. 5, but clearly more detailed experimental work is needed for a quantitative verification of our theory.

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