Consistent derivation of impurity resistivity from the force-balance equation

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The adiabatic impurity resistivity calculation based upon the force-balance equation is studied with use of the method of closed-time-path Green's functions. In this, the effects due to both the noncommutability of the center-of-mass fluctuations at different times and the exact noncanonical commutation relations between the coordinates and momenta of relative electrons are considered. We show that the leading higher-order resistivity terms generated by the noncommutability of center-of-mass fluctuations fully cancel among themselves and make no contribution to the impurity resistivity. As a result the center-of-mass fluctuations can be regarded as classical variables. When canonical commutation relations are approximated for relative electrons, the resulting adiabatic resistivity vanishes. We also demonstrate that the standard adiabatic resistivity formula can be obtained only if the exact noncanonical commutation relations are employed for relative electrons. In the presence of a momentum-conserving inelastic scattering time due to a heat bath and/or direct electron-electron interactions, we confirm that the impurity resistivity (isothermal) is devoid of the divergences of Van Hove's "$\lambda^2 t$"-series expansion. This result is in agreement with a phenomenological study based on the Boltzmann equation.

I. INTRODUCTION: PHENOMENOLOGY AND METHODOLOGY

The theory of electronic transport relating to linear impurity resistivity[1,2] has been the subject of some controversy[3-7] over the past years, principally concentrated on adiabatic models which ignored the role of any form of electron-electron interactions. In a recent paper[4] we advanced cogent arguments that strong photon-mediated electron-electron interactions, or strong direct Coulomb electron-electron interactions, would substantially change the linear impurity resistivity to that of an isothermal conduction process, which can be represented by the lowest-order term of electron-impurity scattering in the Lei-Ting force-balance equation[5] for impurity resistivity, to the exclusion of the divergent terms of Van Hove's "$\lambda^2 t$"-series expansion[8] (λ measures impurity potential strength, t is direct time). In this, we describe the transport process in terms of center of mass (c.m.) and relative electron coordinates, where the c.m. Hamiltonian alone feels the full effect of the uniform constant electric field, and electron-electron interactions are ascribed solely with the relative electrons. Interaction between the c.m. and relative electrons came about through the impurity scattering interaction. We also envisioned a relatively strong bath interaction, needed to quickly remove large amounts of dissipated energy from the electron system at high electric fields, which is requisite for steady-state "isothermal" transport[9] in which energy is removed from the system as quickly as it is pumped in. Even within the framework of linear response, wherein no description of energy transfer to the heat bath is possible, the strong bath interaction can have important effects by producing a strong effective electron-electron interaction mediated by bath quanta (phonons). Thus electrons can exchange energy among themselves and become thermalized, while such processes never occur in adiabatic transport. Although the bath interaction may itself have an associated linear resistivity contributing to momentum relaxation, we focus attention on the impurity resistivity as it is affected by the strong phonon-mediated (or direct Coulomb) electron-electron interaction. In this, we recognize that the unperturbed state to zeroth order in the impurity interaction is one in which the relative electrons, with their full complement of electron-electron interactions, are decoupled from the field as well as from the c.m., so that one may expect the usual relaxation phenomena to unfold following the Boltzmann H theorem in the relative-coordinate system. (On the other hand, there is *a priori* no clear view of how such relaxation phenomena unfold in the laboratory coordinate system, where the electrons do have direct coupling with the electric field). The underlying phenomenology is that strong electron-electron scattering tends to thermalize the relative electrons, with a momentum conserving (total for all electrons) inelastic scattering time τ_{in}: Since the thermalization is taking place in the relative coordinate system, it is clear that a short τ_{in} promotes rapid thermalization about the drifted state. This reasoning is at the heart of the "drifted-temperature model," which has recently been proven to be equivalent to the balance-equation method in the random-phase approximation.[10] Earlier,[11] such an equivalence had been established using the nonequilibrium statistical operator method[12] with a density matrix

closely related to a quasiequilibrium drifted-temperature counterpart. Alternative interpretations have emerged in terms of dielectric energy loss of the c.m. under isothermal conditions,[9] as well as in the very recent work of Cai *et al.*[13] Furthermore, similar results were obtained by Hu and O'Connell from the viewpoint of a generalized quantum Langevin equation.[14] It should also be noted that the isothermal Lei-Ting balance equation closely resembles the well-established transport dynamics of gas plasma physics. In its classical limit, the Lei-Ting resistivity corresponds to the resistivity of gas plasma theory,[15–19] whose well-known difference from the adiabatic noninteracting "Lorentz-gas" resistivity has long been attributed to the important role of strong electron-electron interactions in rapid thermalization about the drifted state.[15–19] It may be interesting to note that the dynamic conductivity discussed recently by Argyres and Resendes[20] in terms of the memory-function approach is in adiabatic limits. For the isothermal case this problem has been studied previously by Götze and Wölfle.[21]

Finally, we also point out another view of this interpretation of the phenomenology of the role of strong electron-electron scattering interactions in an electric field, as it is manifested in the Boltzmann equation embodying a model of a momentum-conserving inelastic scattering time, as studied by Wingreen *et al.*[22] with the collision term promoting thermalization about the drift momentum $\langle k \rangle$:

$$e\mathbf{E}\cdot\frac{\partial}{\partial\mathbf{k}}f_0(\mathbf{k})=\frac{f(\mathbf{k})-f_0(\mathbf{k})}{\tau_k}+\frac{f(\mathbf{k})-f_0(\mathbf{k}-\langle\mathbf{k}\rangle)}{\tau_{\rm in}} .$$

In linear order, where energy dissipation is not involved, the inelastic scattering time $\tau_{\rm in}$ (inelastic in the sense that the energy of a given electron is not conserved, albeit the energy of electron pairs is conserved) tends to relax the distribution about the drift momentum $\langle \mathbf{k} \rangle$, and we have already verified[4] that this physically transparent description yields precisely the same linear impurity resistivity that we obtained previously and reconfirm here under isothermal conditions $\tau_{\rm in} \ll \tau$.

Our earlier work[4] focused on the structure of the Green's functions describing electron dynamics in response to an impressed electric field. In this, we dealt with relative electron Green's functions, and we examined all parts of the two-particle relative electron Green's function known to contribute divergent terms of Van Hove's "$\lambda^2 t$"-series expansion,[8] including the product of two one-particle relative electron Green's functions. In this, inelastic electron-electron scattering is clearly distinguished by its role in the relative-electron Green's function—promoting thermalization about the drifted state in a way that is inaccessible to elastic scattering. Although vertex corrections were neglected, we fully disposed of Van Hove's $\lambda^2 t$-series divergencies, demonstrating that the inelastic scattering interaction "dressed" in the relative electron Green's functions with an imaginary component to the self-energy which removed the divergencies and rendered the terms negligible. While it will surely be of interest to further explore vertex corrections to the two-particle relative-electron Green's function, the removal of the Van Hove divergencies suffices to establish the validity of the Lei-Ting isothermal impurity-resistivity balance equation. (It should be noted that we also assumed the imaginary component of the relative electron self-energy $1/2\tau_{\rm in}$ to be a constant, which is reasonable for a first investigation, but it could be improved upon).

Although our earlier work[4] generated a clear understanding of linear impurity resistivity in the isothermal limit[9] involving strong electron-electron interactions and rapid thermalization about the drift state for $\tau_{\rm in} \ll \tau$, that work was incomplete in that we did not successfully address the determination of the impurity-resistivity in the adiabatic limit $\tau_{\rm in} \gg \tau$ (when electron-electron and bath interactions are negligible) in terms of the balance-equation method, and in this respect we were previously unable to confirm its full sufficiency. Indeed, there has recently been considerable interest[6,7] in determining how the adiabatic resistivity (long known from the standard transport work involving the Boltzmann equation[2] and Kubo formula[3]) can be derived from the force-balance equation. To put this matter to rest, we present the complete analysis of adiabatic resistivity in this paper, using the balance-equation method jointly with the closed-time-path Green's functions.[23]

Two salient features of the problem at hand need to be clarified in the present discussion: one is the quantum-mechanical treatment of the center-of-mass variables, and the other is the deviation of the commutation relations of the electrons' relative coordinates and momenta from the standard canonical ones. In the original formulation of the force-balance equation[5] for an N-electron system, the electron variables were separated into center-of-mass and relative electron variables. In this connection, two assumptions were made in Ref. 5: (a) the center-of-mass variables were regarded as classical variables, and (b) the exact noncanonical commutation relations between the coordinates and the momenta of relative electrons are approximated by canonical ones. Although the resistivity (isothermal) derived in Ref. 5 from the lowest-order impurity scattering term is not sensitive to these assumptions, the contribution from higher-order divergent terms in the determination of the adiabatic impurity resistivity can only be correctly obtained if the exact noncanonical commutation relations between relative variables are taken into account. This will be demonstrated in Secs. III and IV. In a recent paper,[6] Argyres evaluated the higher-order impurity resistivity terms from the force-balance equation in the absence of electron-electron interactions, following assumptions (a) and (b) cited above. Since these assumptions were not designed to calculate higher-order terms, it is not surprising that a zero resistivity was obtained.[6] This unphysical result is entirely due to the inconsistency between imposition of canonical commutation relations on relative-electron variables and the implicit expectation that the total relative-electron momentum is zero. In this paper we show that only when the exact noncanonical commutation relations for relative electrons are employed can the standard adiabatic resistivity formula be recovered.

Superficially, the derivation of the adiabatic resistivity from the force-balance equation seems to have been

achieved in the recent paper of Fishman and Mahan,[7] wherein the effect of the quantum-mechanical character of center-of-mass variables is considered only to lowest order. However, a closer examination of their method indicates[4] that the noncommutability between center-of-mass fluctuations (CMF) at different times also generates another class of divergent terms of equal importance, which were neglected in Ref. 7. When this class of divergent terms is considered and summed to all orders, we show in this paper that the effects due to the noncommutability of CMF on the impurity resistivity fully cancel among themselves, and we find that the proper result based upon the method of Ref. 7 reduces exactly to that of Argyres,[6] who has regarded the c.m. variables as commutative and classical. Moreover, the quantum operators used for relative electrons in Ref. 7 are considered under the assumption that the coordinates and momenta of relative electrons satisfy the standard canonical commutation relations. Therefore the approach of Ref. 7, if treated correctly, faces the same difficulty as that of Ref. 6.

In the present paper we shall study the determination of impurity resistivity from the force-balance equation by using the method of closed-time-path Green's functions.[23] A consistent treatment of CMF and the exact noncanonical commutation relations between the variables of relative electrons will be presented. In Sec. II, we show that the imposition of approximate canonical commutation relations for relative electrons results in the complete cancelation of all the divergent terms generated by the noncommutability of CMF's. As a result, the adiabatic impurity resistivity predicted in this way vanishes. In Sec. III, the macroscopic error of nonvanishing drift of relative electrons brought about by the imposition of approximate canonical commutation relations on the relative-electron variables will be examined. After this relative-electron drift is incorporated into the current, the standard adiabatic resistivity formula is seen to emerge. In Sec. IV, we show that when the exact commutation relations between the relative coordinates and momenta are utilized, our result for adiabatic impurity resistivity again reduces to the standard form. We would like to emphasize here that once a finite inelastic scattering time due to strong electron-electron interactions is introduced, the associated resistivity is devoid of the divergences of Van Hove's "$\lambda^2 t$"-series expansion[8] and has the isothermal expression given in Ref. 4. The final section contains a summary and discussion.

II. QUANTUM-MECHANICAL TREATMENT OF CENTER-OF-MASS VARIABLES

Fishman and Mahan[7] focused attention on the quantum-mechanical aspect of the heavy center-of-mass "particle" (with mass Nm, where m is the electron mass, and N is the number of electrons, N being a large number) for the purpose of determining adiabatic resistivity from the force-balance equation. In this section, we shall show that their considerations were incomplete and that there exists an infinite number of divergent terms which they overlooked. Based on a consistent treatment to be presented below, it will become evident that all the divergent terms related to center-of-mass fluctuations fully cancel among themselves and make no contribution to the resistivity formula.

To begin our discussion, we briefly review the force-balance equation method.[5,7] The Hamiltonian of an electron-impurity system in the presence of a constant electric field E is

$$H = H_e + \frac{\mathbf{P}^2}{2Nm} + Ne\mathbf{E}\cdot\mathbf{R} + H_i \ , \tag{1}$$

$$H_e = \sum_{i=1}^{N} \frac{p_i^2}{2m} + \sum_{1\le i<j\le N} v(r_i - r_j) \ , \tag{2}$$

$$H_i = \lambda \sum_{\mathbf{q}} \rho_{\mathbf{q}} e^{i\mathbf{q}\cdot\mathbf{R}} V(\mathbf{q})$$

$$= \lambda \sum_{\mathbf{q}} \rho_{\mathbf{q}} e^{i\mathbf{q}\cdot\delta\mathbf{R}} V(\mathbf{q}) e^{i\mathbf{q}\cdot\mathbf{R}_c} \ , \tag{3}$$

where $\rho_{\mathbf{q}} = \sum_{\mathbf{k}} c^{\dagger}_{\mathbf{k}+\mathbf{q}} c_{\mathbf{k}}$ is the density operator of relative electrons. H_e is the relative electron Hamiltonian with electron-electron interaction $v(r_i - r_j)$, and p_i and r_i are relative momentum and coordinate of the ith electrons, respectively. $c_{\mathbf{k}}$ ($c^{\dagger}_{\mathbf{k}}$) is the destruction (creation) operator of a relative electron with momentum $\mathbf{k}$. $\mathbf{P}$ and $\mathbf{R}$ describe the momentum and position of the center of mass of N electrons, and $\delta\mathbf{R} = \mathbf{R} - \mathbf{R}_c$ is the fluctuation of the center of mass from its classical trajectory R_c. The electron-impurity potential of strength λ is

$$V(\mathbf{q}) = \sum_{\alpha} \phi(\mathbf{q}) e^{-i\mathbf{q}\cdot\mathbf{R}_\alpha} \ , \tag{4}$$

where $\mathbf{R}_\alpha$ is the αth impurity site. In the following discussion, $\mathbf{R}$ and $\mathbf{P}$ will be treated quantum mechanically exactly. However, approximate canonical commutation relations $[r_i, p_j] = i\delta_{ij}$ are assumed for the electrons' relative coordinates and momenta, and the exact noncanonical correction to these relations will be considered in the latter two sections.

Employing an interaction picture with H_i and $Ne\mathbf{E}\cdot\mathbf{R}$ being regarded as perturbative interactions, and separating the center of mass from the electrons' relative motion, the force-balance equation can be written as[5,7]

$$Ne\mathbf{E} = \mathrm{Tr}\rho(t) F_i(t) \ , \tag{5}$$

where the density matrix $\rho(t)$ satisfies

$$i\partial_t \rho(t) = [H_i, \rho(t)]_- + [Ne\mathbf{E}\cdot\mathbf{R}, \rho(t)] \ , \tag{6}$$

subject to the initial condition

$$\rho(t=0) = \rho_0 = e^{-\beta H_e} |P\rangle\langle P| \ , \tag{7}$$

in which $P|P\rangle = Nmv_d|P\rangle$ and v_d is the drift velocity of electrons. The frictional force operator due to electron-impurity scattering is obtained as

$$\mathbf{F}_i(t) = \lambda \sum_{\mathbf{q}} i\mathbf{q}\rho_{\mathbf{q}} e^{i\mathbf{q}\cdot\delta\mathbf{R}} V(\mathbf{q}) e^{i\mathbf{q}\cdot\mathbf{R}_c} \ . \tag{8}$$

Expanding the right-hand side of the force-balance equation Eq. (5) in powers of λ and to linear order in $\mathbf{E}$, and

then randomly averaging over impurity configurations, the force-balance equation for impurity scattering has been expressed in the form[7]

$$NeE = F(\delta)v_d + \alpha(\delta)NeE \ , \tag{9}$$

where the long-time (steady-state) limit $\lim_{t\to\infty} f(t)$ is realized by the $\delta\to 0$ limit of the corresponding Laplacian component $\lim_{\delta\to 0} f(\delta) = \lim_{\delta\to 0}[\delta\int_0^\infty dt\, f(t)\exp(-\delta t)]$. $F(\delta)$ and $\alpha(\delta)$ are, respectively (N_i is the impurity concentration),

$$\begin{aligned} F(\delta)v_d = & \sum_{n=1}^{\infty}\int_0^\infty dt_1\,\delta e^{-\delta t_1}\int_0^{t_1} dt_2\cdots\int_0^{t_{2n-2}} dt_{2n-1}\int_0^{t_{2n-1}} dt_{2n}\,\lambda^{2n}N_i^n \\ & \times \sum_{\mathbf{q}_1,\ldots,\mathbf{q}_n} iq_{1x}|\phi(\mathbf{q}_1)|^2 e^{i\mathbf{q}_1\cdot v_d(t_1-t_2)}\cdots|\phi(\mathbf{q}_n)|^2 e^{i\mathbf{q}_n\cdot v_d(t_{2n-1}-t_{2n})} \\ & \times (i)^{2n}\mathrm{Tr}(\rho_0[A(-\mathbf{q}_n,t_{2n}),[A(\mathbf{q}_n,t_{2n-1}),\ldots,[A(-\mathbf{q}_1,t_2),A(\mathbf{q}_1,t_1)]\cdots]]) \ , \end{aligned} \tag{10}$$

and

$$\begin{aligned} \alpha(\delta) = & \sum_{n=1}^{\infty}\int_0^\infty dt_1\,\delta e^{-\delta t_1}\int_0^{t_1} dt_2\cdots\int_0^{t_{2n-2}} dt_{2n-1}\int_0^{t_{2n-1}} dt_{2n}\int_0^{t_{2n}} dt_{2n+1} \\ & \times\lambda^{2n}N_i^n \sum_{\mathbf{q}_1,\ldots,\mathbf{q}_n} iq_{1x}|\phi(\mathbf{q}_1)|^2 e^{i\mathbf{q}_1\cdot v_d(t_1-t_2)}\cdots|\phi(\mathbf{q}_n)|^2 e^{i\mathbf{q}_n\cdot v_d(t_{2n-1}-t_{2n})}(i)^{2n+1} \\ & \times\mathrm{Tr}(\rho_0[R(t_{2n+1}),[A(-\mathbf{q}_n,t_{2n}),[A(\mathbf{q}_n,t_{2n-1}),\ldots,[A(-\mathbf{q}_1,t_2),A(\mathbf{q}_1,t_1)]\cdots]]]) \ . \end{aligned} \tag{11}$$

Here, $A(\mathbf{q},t)$ is defined in the interaction picture as

$$A(\mathbf{q},t) = \rho_{\mathbf{q}}(t)e^{i\mathbf{q}\cdot\delta\mathbf{R}(t)} \ . \tag{12}$$

It should be noted that the noncommutability between $\mathbf{R}(t_{2n+1})$ and the $2n$ $\delta\mathbf{R}$'s involved through $A(\mathbf{q},t)$ in Eq. (11) yields the existence of the last term in Eq. (9). The authors of Ref. 7 only take account of this part of the center-of-mass fluctuation effect and regard the $2n$ variables $\delta\mathbf{R}(t_i)$ ($i=1,2,\ldots,2n$) in Eqs. (10) and (11) as mutually commutable. In fact, it is easy to verify that the noncommutabilities between each pair of the $2n$ $\delta\mathbf{R}$'s are of equal importance and their contributions must also be considered in a proper treatment.

Before a detailed calculation of the impurity resistivity is carried out, we need to establish a relation between $F(\delta)$ and $\alpha(\delta)$ in Eqs. (10) and (11) by utilizing the following commutation relation:[7]

$$[\mathbf{R}(t_{2n+1}),e^{i\mathbf{q}\cdot\delta\mathbf{R}(t_i)}] = -\frac{q_{ix}}{Nm}(t_i-t_{2n+1})e^{i\mathbf{q}\cdot\delta\mathbf{R}(t_i)} \ . \tag{13}$$

Using the above relation to complete the commutations in Eq. (11) between $\mathbf{R}(t_{2n+1})$ and all the factors $A(q,t_i)$ [see Eq. (12) with $i=1,2,\ldots,2n$], Eq. (11) becomes

$$\begin{aligned} \alpha(\delta) = & \sum_{n=1}^{\infty}\int_0^\infty dt_1\,\delta e^{-\delta t_1}\int_0^{t_1} dt_2\cdots\int_0^{t_{2n-2}} dt_{2n-1}\int_0^{t_{2n-1}} dt_{2n}\int_0^{t_{2n}} dt_{2n+1} \\ & \times\lambda^{2n}N_i^n \sum_{\mathbf{q}_1,\ldots,\mathbf{q}_n} iq_{1x}\frac{-1}{Nm}[q_{1x}(t_1-t_2)+q_{2x}(t_3-t_4)+\cdots+q_{nx}(t_{2n-1}-t_{2n})] \\ & \times|\phi(\mathbf{q}_1)|^2 e^{i\mathbf{q}_1\cdot v_d(t_1-t_2)}\cdots|\phi(\mathbf{q}_n)|^2 e^{i\mathbf{q}_n\cdot v_d(t_{2n-1}-t_{2n})} \\ & \times(i)^{2n}\mathrm{Tr}(\rho_0[A(-\mathbf{q}_n,t_{2n}),[A(\mathbf{q}_n,t_{2n-1}),\ldots,[A(-\mathbf{q}_1,t_2),A(\mathbf{q}_1,t_1)]\cdots]]) \ . \end{aligned} \tag{14}$$

Comparing the above equation with Eq. (10), it is straightforward to see that the structure of the two are almost parallel. Expanding Eqs. (10) and (14), respectively, to the linear (and zeroth) order in v_d (drift velocity), and checking their time integrations, we can relate $\alpha(\delta)$ directly to $F(\delta)$ as follows:

$$\alpha(\delta) = \frac{F(\delta)}{Nm\delta} \ . \tag{15}$$

In order to calculate the frictional force $F(\delta)v_d$, we have to treat the retarded multicorrelation function of $2n$ A operators in Eq. (10). Each A contains one electron density fluctuation operator ρ_q and one center-of-mass fluctuation operator δR. The Feynman diagram technique employing closed-time-path Green's functions[23] provides us with a convenient means for the management of such complicated correlation functions. In the Appendix, we present a brief introduction to the closed-time-path Green's functions and express the $2n$th retarded correlation function in a closed-time-path form. There

we take $n=2$ as an example to demonstrate the detailed evaluation of the frictional force. We shall outline the rules for constructing the requisite Feynman diagrams. In this, we use arrowed lines with r, a, and c [Fig. 1(a)], respectively, to denote the retarded, advanced, and correlation Green's functions:

$$iG_r(k,\, t-t')=\theta(t-t')\mathrm{Tr}\rho_0[c_k(t),\, c_k^\dagger(t')]_+ =\theta(t-t')e^{-i\epsilon_k(t-t')-\eta|t-t'|}, \tag{16}$$

$$iG_a(k,\, t-t')=-\theta(t'-t)\mathrm{Tr}\rho_0[c_k(t),\, c_k^\dagger(t')]_+ =\theta(t'-t)e^{-i\epsilon_k(t-t')-\eta|t-t'|}, \tag{17}$$

$$iG_c(k,\, t-t')=\mathrm{Tr}\rho_0[c_k(t),\, c_k^\dagger(t')]_- =[1-2f(\epsilon_k)]e^{-i\epsilon_k(t-t')-\eta|t-t'|}, \tag{18}$$

where the damping factor η for a single-electron propagator is introduced to represent the effect of electron-electron interactions. $f(\epsilon)$ is the Fermi-Dirac distribution function with $\epsilon_k=k^2/2m$. We use a dashed line with a cross [Fig. 1(b)] to represent the factor arising from averaged impurity scattering

$$\lambda^2 N_i|\phi(q)|^2 e^{iqv_d(t-t')}, \tag{19}$$

where the label q is the momentum transfer of an electron during the scattering process. The wavy lines with r and c symbols [Fig. 1(c)] are employed to represent the retarded and correlation Green's functions for center-of-mass quantum fluctuation:

$$F_r(\mathbf{q}_1,\mathbf{q}_2,\, t_1-t_2)=\theta(t_1-t_2)\mathrm{Tr}\rho_0[e^{i\mathbf{q}_1\cdot\delta\mathbf{R}(t_1)},\, e^{i\mathbf{q}_2\cdot\delta\mathbf{R}(t_2)}]_- =\theta(t_1-t_2)(-i)\frac{i\mathbf{q}_1\cdot i\mathbf{q}_2}{Nm}(t_1-t_2) +O((1/N)^2), \tag{20}$$

$$F_c(\mathbf{q}_1,\mathbf{q}_2,\, t_1-t_2)=\mathrm{Tr}\rho_0[e^{i\mathbf{q}_1\cdot\delta\mathbf{R}(t_1)},\, e^{i\mathbf{q}_2\cdot\delta\mathbf{R}(t_2)}]_+ =2+O(1/N). \tag{21}$$

Here, $[A,B]=AB-BA$ and $[A,B]_+=AB+BA$.

In the present paper, only leading divergent terms such as $\lambda^2(\lambda^2/\delta)^n$ $(n=0,1,2,\ldots)$ arising in conjunction with Van Hove's limiting prescription[8] will be considered. Considering the interaction structure of the three participating elements: electron (fermion) field, impurities, and center-of-mass fluctuation (boson field), and utilizing the Feynman diagrammatic representation, it is straightforward to show that the frictional force $F(\delta)v_d$ in Eq. (10) can be expressed by the graphs in Fig. 2(a), i.e.,

$$F(\delta)v_d=(a1)+(a2)+(a3)+\cdots, \tag{22}$$

where the black dot at time t_1 represents the factor iq_{1x} in Eq. (10). The shaded bubble with an impurity line appearing in each (a)-series graph corresponds to the summation of (b)-series graphs shown in Fig. 2(b). For example, graph $(a1)$ can be expressed as

$$(a1)=(b1)+(b2)+(b3)+\cdots+(\mathrm{TSG}). \tag{23}$$

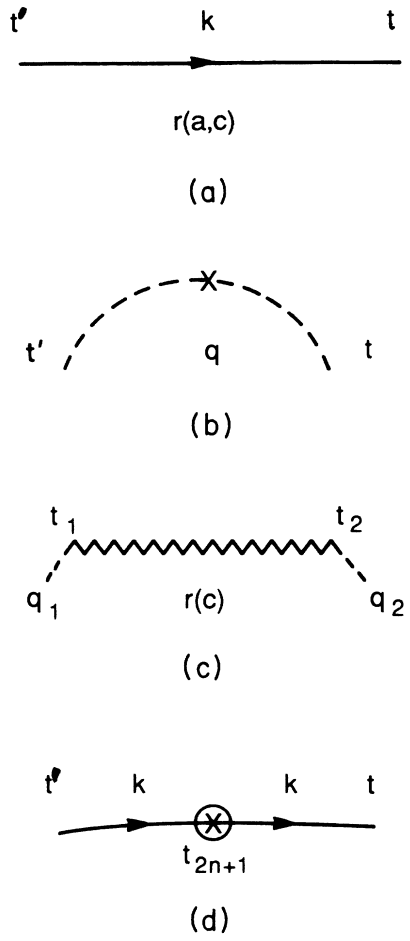

FIG. 1. Graph elements of the Feynman diagrams: (a) retarded (advanced, correlation) Green's function for electrons; (b) the factor from averaged impurity scattering; (c) retarded (correlation) Green's function for the center-of-mass fluctuations; (d) Green's functions influenced by the external electric field.

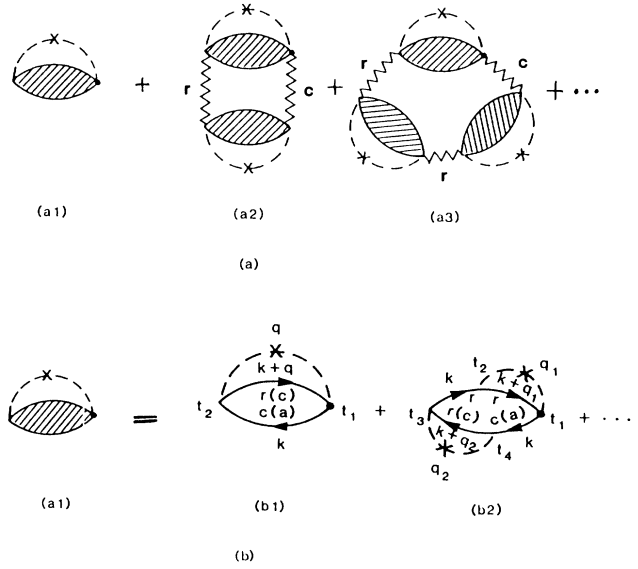

FIG. 2. Feynman diagrams for the frictional force.

Here, (TSG) means time-symmetrized graphs of $(b1)$, $(b2)$, $(b3)$, . . . , e.g., the TSG of (bn) includes all possible nonequivalent graphs obtained from (bn) by exchanging time points $t_2, t_3, \ldots, t_{2n}$ with t_1 being fixed. Using the above rules for constructing Feynman diagrams, the evaluations of these graphs are quite straightforward. Summing up the (b)-series graphs [Fig. 2(b)], we obtain the following result for graph $(a1)$ in Fig. 2(a):

$$(a1)=Nmv_d A(\delta+2\eta) , \tag{24}$$

where

$$\begin{aligned} A(\delta+2\eta)&=\left\langle\frac{\lambda^2}{\tau}\right\rangle-\left\langle\frac{\lambda^2}{\tau}\frac{\lambda^2}{\tau(\delta+2\eta)}\right\rangle \\ &\quad+\left\langle\frac{\lambda^2}{\tau}\left[\frac{\lambda^2}{\tau(\delta+2\eta)}\right]^2\right\rangle-\cdots \\ &=\left\langle\frac{\lambda^2/\tau}{1+\lambda^2/\tau(\delta+2\eta)}\right\rangle . \end{aligned} \tag{25}$$

Here, τ is the relaxation time due to impurity scattering and its average $\langle\cdots\rangle$ has been defined and fully discussed in the literature.[7] Using Eqs. (20) and (21) for wavy lines in other (a)-series graphs of Fig. 2(a), the contributions from each of them can be similarly derived as

$$(a2)=Nmv_d[A(\delta+2\eta)]^2\left[-\frac{1}{\delta}\right] , \tag{26}$$

$$(a3)=Nmv_d[A(\delta+2\eta)]^3\left[-\frac{1}{\delta}\right]^2 , \tag{27}$$

. . . .

It is very important to keep in mind that the higher-order terms generated by the center-of-mass fluctuations are divergent as $(1/\delta)^n$ $(n=1,2,\ldots)$ and these divergencies cannot be removed by the damping factor η. Substituting Eq. (24) and Eqs. (26) and (27) back into Eq. (22), we obtain the final expression for the frictional force:

$$\begin{aligned} F(\delta)v_d&=Nmv_d A(\delta+2\eta)\left[1-\frac{1}{\delta}A(\delta+2\eta)\right. \\ &\quad\left.+\left[\frac{1}{\delta}A(\delta+2\eta)\right]^2-\cdots\right] \\ &=Nmv_d\frac{A(\delta+2\eta)}{1+A(\delta+2\eta)/\delta} . \end{aligned} \tag{28}$$

From the above result and Eq. (15), the expression for the coefficient $\alpha(\delta)$ is seen to be

$$\alpha(\delta)=\frac{A(\delta+2\eta)/\delta}{1+A(\delta+2\eta)/\delta} . \tag{29}$$

With Eqs. (28) and (29), the electric field applied to maintain a steady transport state is related to the center-of-mass velocity v_d through the force-balance equation [Eq. (9)]:

$$NeE=\frac{F(\delta)v_d}{1-\alpha(\delta)}=Nmv_d A(\delta+2\eta) . \tag{30}$$

If the total momentum of the relative electrons is taken to vanish[5-7] and v_d is the conventional drift velocity, then the resistivity ρ_i deduced from Eq. (30) becomes

$$\rho_i=\frac{E}{Nev_d}=\frac{m}{Ne^2}\left\langle\frac{\lambda^2/\tau}{1+\lambda^2/\tau(\delta+2\eta)}\right\rangle . \tag{31}$$

In the above equation, the divergent terms generated from the center-of-mass fluctuations fully cancel among themselves and make no net contribution to ρ_i. In the absence of electron-electron interactions $\eta=0$, the adiabatic resistivity ρ_i vanishes as $\delta\to 0$. This is exactly the result obtained by Argyres.[6] However, when strong electron-electron interactions are presented, we have $2\eta=1/\tau_{\rm in}\gg\lambda^2/\tau$, and in this case the isothermal resistivity of Refs. 4 and 5 is immediately obtained.

III. VANISHING CURRENT AND CORRECT FINITE RESULT FOR ADIABATIC RESISTIVITY

In the last section we demonstrated that the adiabatic resistivity derived from the force-balance equation vanishes even though the center-of-mass fluctuation is treated quantum mechanically in the absence of electron-electron interactions. This unexpected result leads us to reconsider all the approximations made in the force-balance formulation. The first is that the noncanonical commutation relations of the relative electron coordinates and momenta are approximated as canonical ones. The second is the assumption that the total momentum of relative electrons is zero so that electric current J is directly related to the center-of-mass velocity v_d as $J=Nev_d$. Unfortunately, these two assumptions are contradictory to each other. The latter assumption will be true only if the commutation relations of the relative electron variables are treated exactly. However, when one approximates the exact commutation relations, $[r_i,p_j]_-=i(\delta_{ij}-1/N)$, as canonical ones, $[r_i,p_j]_-=i\delta_{ij}$, the space reflection symmetry for relative electrons is artificially broken [see the Hamiltonian in Eq. (1)]. Consequently, the total momentum of relative electrons

$$C=\mathrm{Tr}\rho(t)\sum_p \mathbf{p}c_p^\dagger c_p \tag{32}$$

becomes nonvanishing in this approximation. Since this represents a macroscopic drift of the relative electrons, we incorporate it into the total electric current as

$$J=Nev_d+\frac{e}{m}C \tag{33}$$

in an effort to compensate for the use of incorrect canonical commutation relations for the relative electrons.

The calculation of the total relative momentum in Eq. (32) is parallel to that of the frictional force in Eq. (5). In this section we will treat center-of-mass variables quantum mechanically. Utilizing the perturbative solution of Eq. (6) for the density matrix $\rho(t)$, the leading-order terms for C can be expressed as

$$C(\delta)=C_1(\delta)v_d+C_2(\delta) , \tag{34}$$

in which

$$C_1(\delta)v_d=\sum_{n=1}^{\infty}\int_0^{\infty}dt_1\,\delta e^{-\delta t_1}\int_0^{t_1}dt_2\cdots\int_0^{t_{2n-1}}dt_{2n}\int_0^{t_{2n}}dt_{2n+1}\,\lambda^{2n}N_i^n$$
$$\times\sum_{\mathbf{q}_1,\ldots,\mathbf{q}_n}\sum_{\mathbf{p}}ip_x|\phi(\mathbf{q}_1)|^2e^{i\mathbf{q}_1\cdot v_d(t_1-t_2)}\cdots|\phi(\mathbf{q}_n)|^2e^{i\mathbf{q}_n\cdot v_d(t_{2n-1}-t_{2n})}(i)^{2n}$$
$$\times\mathrm{Tr}(\rho_0[A(-\mathbf{q}_n,t_{2n+1}),[A(\mathbf{q}_n,t_{2n}),\ldots,[A(-\mathbf{q}_1,t_3),[A(\mathbf{q}_1,t_2),c_p^{\dagger}c_p]_-]_-\cdots]_-]_-)\,, \tag{35}$$

$$C_2(\delta)=NeE\sum_{n=1}^{\infty}\int_0^{\infty}dt_1\,\delta e^{-\delta t_1}\int_0^{t_1}dt_2\cdots\int_0^{t_{2n}}dt_{2n+1}\int_0^{t_{2n+1}}dt_{2n+2}\,\lambda^{2n}N_i^n$$
$$\times\sum_{\mathbf{q}_1,\ldots,\mathbf{q}_n}\sum_{\mathbf{p}}ip_x|\phi(\mathbf{q}_1)|^2e^{i\mathbf{q}_1\cdot v_d(t_1-t_2)}\cdots|\phi(\mathbf{q}_n)|^2e^{i\mathbf{q}_n\cdot v_d(t_{2n-1}-t_{2n})}(i)^{2n}$$
$$\times\mathrm{Tr}(\rho_0[R(t_{2n+2}),[A(-\mathbf{q}_n,t_{2n+1}),[A(\mathbf{q}_n,t_{2n}),\ldots,[A(\mathbf{q}_1,t_2),c_p^{\dagger}c_p]_-\cdots]_-]_-]_-)\,. \tag{36}$$

Evidently, the structures of these two equations are parallel to those of Eqs. (10) and (11). In correspondence with the derivation of Eq. (15), we can use Eq. (13) to carry out the commutations between $R(t_{2n+2})$ and $2n$ δR's involved through A's in Eq. (36). We then find that $C_2(\delta)$ is related to $C_1(\delta)$ as

$$C_2(\delta)=\frac{C_1(\delta)}{Nm\delta}NeE\,. \tag{37}$$

The Feynman diagrams for $C_1(\delta)$ in Eq. (35) are demonstrated in Fig. 3, i.e.,

$$C_1(\delta)=(c1)+(c2)+(c3)+\cdots\,. \tag{38}$$

In comparison with the Feynman graphs of Fig. 2 for the frictional force $F(\delta)$ in Eq. (10), here in Fig. 3 we have a small circle at the end of each bubble for the p_x factor in Eq. (35), while in Fig. 2 we have a black dot for the q_{1x} in Eq. (10). Moreover, in Fig. 3 we have one more solid line for the one additional Green's function brought about by $c_p^{\dagger}c_p$ in Eq. (35) [comparing Fig. 3(d) with Fig. 1(b)]. Except for the above-mentioned differences, all other aspects of the graphs in Fig. 3 are the same as those in Fig. 2. Therefore the evaluation of them is fully analogous. Summing up the graph series in Fig. 3(d), we get

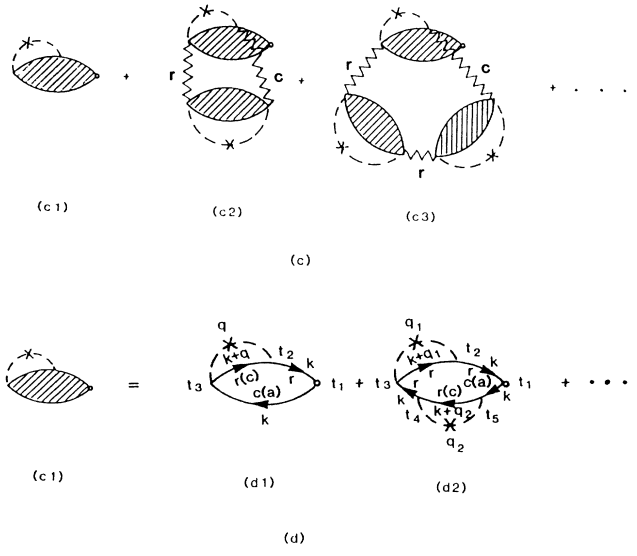

FIG. 3. Feynman diagrams for total momentum of relative electrons.

$$(c1)=(d1)+(d2)+(d3)+\cdots+(\mathrm{TSG})$$
$$=-Nmv_dA(\delta+2\eta)/(\delta+2\eta)\,. \tag{39}$$

Furthermore, the evaluation of the other shaded bubble graphs with the wavy lines here follows exactly the same lines as with the results in the previous section,

$$(c2)=-Nmv_d\frac{A(\delta+2\eta)}{\delta+2\eta}\left[-\frac{A(\delta+2\eta)}{\delta}\right], \tag{40}$$

$$(c3)=-Nmv_d\frac{A(\delta+2\eta)}{\delta+2\eta}\left[-\frac{A(\delta+2\eta)}{\delta}\right]^2, \tag{41}$$

. . . .

Substituting the results of Eqs. (39)–(41) into Eq. (38), we arrive at

$$C_1(\delta)v_d=-Nmv_d\frac{A(\delta+2\eta)/(\delta+2\eta)}{1+A(\delta+2\eta)/\delta}\,. \tag{42}$$

Using this expression for $C_1(\delta)$ jointly with Eqs. (30) and (37), we have

$$C_2(\delta)=-NeE\frac{1}{\delta}\frac{A(\delta+2\eta)/(\delta+2\eta)}{1+A(\delta+2\eta)/\delta}$$
$$=-Nmv_d\frac{A(\delta+2\eta)/(\delta+2\eta)}{\delta}\frac{A(\delta+2\eta)}{1+A(\delta+2\eta)/\delta}\,. \tag{43}$$

Employing Eqs. (33), (34), and (37) the electric current density is given by

$$J=Nev_d\left[1-\frac{A(\delta+2\eta)}{\delta+2\eta}\right]$$
$$=Nev_d\left\langle\frac{1}{1+\lambda^2/\tau(\delta+2\eta)}\right\rangle\,. \tag{44}$$

The impurity resistivity is thus obtained from Eqs. (30) and (44) as

$$\rho_i=\frac{E}{J}=\frac{m}{Ne^2}\left\langle\frac{\lambda^2/\tau}{1+\lambda^2/\tau(\delta+2\eta)}\right\rangle\left\langle\frac{1}{1+\lambda^2/\tau(\delta+2\eta)}\right\rangle^{-1}\,. \tag{45}$$

In the adiabatic limit ($\eta \to 0$ and $\delta \to 0$), $\rho_i \simeq m\lambda^2/Ne^2\langle\tau\rangle$, whereas, in the isothermal limit ($2\eta = 1/\tau_{\rm in} \gg \lambda^2/\tau$ and $\delta \to 0$), $\rho_i \simeq \langle\lambda^2/\tau\rangle m/Ne^2$. We should point out that $2\eta = 1/\tau_{\rm in}$ also renormalizes the value of $\langle 1/\tau\rangle$ in a measure expected to be of secondary importance in comparison with its effect in removing the leading divergences of higher-order resistivity contributions. Such a renormalization of $\langle 1/\tau\rangle$ should not involve any substantial change if the characteristic electron energy E (degenerate case Fermi energy or nondegenerate case thermal energy) is such that $E \gg 1/\tau_{\rm in} \gg 1/\tau$. In this paper the vertex corrections have not been considered, but we do not expect that such corrections would invalidate the our conclusions. As a technical remark, we note that the density-density correlation function $\Pi(\mathbf{q},\omega)$ in this paper needs to be summed up over finite $\mathbf{q}$, and it is well known in many-body theory that the dissipation part of the vertex corrections strongly cancels the dissipation part of the self-energy only at $\mathbf{q} \to \mathbf{0}$. At finite $\mathbf{q}$, the cancellation is incomplete and this implies that the divergencies discussed in the present paper would still be removable by the dissipation of electrons. Our results, although not exact as far as vertex corrections not being included, are consistent with those obtained from the Boltzmann equation.[4] One essential feature of the above results is that the higher-order divergent terms ($\eta \to 0$ and $\delta \to 0$) due to the noncommutability of the center-of-mass fluctuations again cancel among themselves and make no net contribution to the impurity resistivity. This is, of course, in full accord with physical intuition.

It is also important to observe that, not withstanding the emergence of the correct adiabatic resistivity, the compensated current vanishes jointly with the resistive force (in the appropriate ratio). This anomalous feature requires further investigation.

IV. CONSIDERATION OF THE EXACT NONCANONICAL RELATIVE ELECTRON COMMUTATION RELATIONS

The discussion in the previous sections shows that the error induced by using approximate canonical commutation relations for relative electrons must be compensated by accounting for their associated macroscopic drift in the calculation of adiabatic resistivity. In this section, we shall derive the resistivity rigorously using the exact (noncanonical) relative-electron commutation relations

$$[r_i,p_j]_- = i(\delta_{ij} - 1/N) \ . \tag{46}$$

If j ($j = 1,2,\ldots,N$) is summed over in the above equation, one can easily observe that the total relative momentum properly vanishes, $\sum_j p_j \equiv 0$. The $(1/N)$ term in the above equation is the deviation from the standard canonical commutation relations. Its impact, as will be shown below, can be represented by introducing an extra term in the Hamiltonian. In conjunction with this, the force-balance equation is able to yield the correct adiabatic resistivity $\lambda^2/\langle\tau\rangle$ for a system in the absence of electron-electron interactions, with no anomalies.

Another lesson we learned above is that the effect of the noncommutability of the CMF is not important, and it makes no net contribution to the final result for impurity resistivity. This conclusion is consistent with the conventional wisdom that the center of mass, with heavy mass Nm, can be treated in a semiclassical manner. In our derivation to follow, the center-of-mass coordinates shall be regarded as classical C numbers accordingly.

Using the exact (noncanonical) commutation relations for the relative coordinates r_i and momentum p_j in Eq. (46), the following steady-state kinetic equations for the center of mass and for the relative electrons may be obtained:

$$\dot{R} = P/Nm \ , \tag{47}$$

$$0 = \dot{P} = -NeE - \sum_{i,\alpha} \nabla_i \phi(R_i + R - R_\alpha) \ , \tag{48}$$

$$\dot{r}_i = p_i/m \ , \tag{49}$$

$$\dot{p}_i = -\nabla_i \sum_\alpha \phi(r_i + R - R_\alpha) - eE \ . \tag{50}$$

The derivation of Eq. (50) is partly based upon the expression given by Eq. (48). Considering Eqs. (47)–(50) one can easily observe that the total relative momentum is now assured to vanish when we keep the $(1/N)$ term in the commutation relations [Eq. (46)]. On the other hand, it is rather inconvenient to study the many-particle properties of relative electrons by employing the exact noncanonical commutation relations. Our scheme for incorporating the effects of the $(1/N)$ term in Eq. (46) is to replace the original Hamiltonian [Eq. (1)] with the following effective Hamiltonian:

$$H_{\rm eff} = H + eE \cdot \sum_i r_i = H + ieE \cdot \sum_k \frac{\partial c_k^\dagger}{\partial k} c_k \ . \tag{51}$$

This effective Hamiltonian describes two independent systems which consist of a heavy c.m. particle and N electrons in an applied electric field. These two subsystems couple with each other through the electron-impurity interaction [Eq. (2)]. Using the canonical commutation relations for both the heavy particle variables R,P and the electron variables r_i and p_i ($i = 1,2,\ldots,N$),

$$[R,P]_- = i \quad \text{and} \quad [r_i,p_j]_- = i\delta_{ij} \ , \tag{52}$$

we find that the same set of equations of motion [Eqs. (47)–(50)] is obeyed. Basing the study of impurity resistivity upon $H_{\rm eff}$ and Eq. (52), the total force acting on the many electrons $\sum_i \dot{p}_i$, determined from Eq. (50), always vanishes and thus the total "relative" momentum $\langle \sum_i p_i \rangle = c_0$ is an $\mathbf{E}$- and t-independent constant. Moreover, c_0 has to be equal to zero, since at $\mathbf{E} = \mathbf{0}$ the total momentum must vanish. This conclusion implies that the total current density $J = Nev_d$ comes entirely from the heavy-particle motion, as we should expect in a fully proper theory. Although the net force acting on the total electrons is zero, each electron still experiences the force ($\dot{p}_i \neq 0$) due to $\mathbf{E}$ and the impurity scattering, even in the steady state. Having pointed out in previous sections that the noncommutability of CMF (the heavy-particle fluctuations) does not contribute to the resistivity, we neglect its effects in the following discussion.

Employing the effective Hamiltonian in Eq. (51), the force-balance equation may be written as

$$NeE = F(\delta)v_d + \beta(\delta)NeE \ , \tag{53}$$

where the frictional force $F(\delta)v_d$ takes the same form as in Eq. (10) except that the heavy-particle coordinate R will be treated classically. The coefficient $\beta(\delta)$ is brought about by the last term in H_{eff}. Employing the method described in Sec. II, it is straightforward to show that

$$\begin{aligned} F(\delta)v_d = & \sum_{n=1}^{\infty} \int_0^{\infty} dt_1\, \delta e^{-\delta t_1} \int_0^{t_1} dt_2 \cdots \int_0^{t_{2n-2}} dt_{2n-1} \int_0^{t_{2n-1}} dt_{2n}\, \lambda^{2n} N_i^n \\ & \times \sum_{\mathbf{q}_1,\ldots,\mathbf{q}_n} iq_{1x} |\phi(\mathbf{q}_1)|^2 e^{i\mathbf{q}_1\cdot v_d(t_1-t_2)} \cdots |\phi(\mathbf{q}_n)|^2 e^{i\mathbf{q}_n\cdot v_d(t_{2n-1}-t_{2n})} \\ & \times (i)^{2n} \mathrm{Tr}(\rho_0[\rho_{-\mathbf{q}_n}(t_{2n}),[\rho_{\mathbf{q}_n}(t_{2n-1}),\ldots,[\rho_{-\mathbf{q}_1}(t_2),\rho_{\mathbf{q}_1}(t_1)]_-\cdots]_-]_-) \ , \end{aligned} \tag{54}$$

and

$$\begin{aligned} \alpha(\delta) = & \sum_{n=1}^{\infty} \int_0^{\infty} dt_1\, \delta e^{-\delta t_1} \int_0^{t_1} dt_2 \cdots \int_0^{t_{2n-2}} dt_{2n-1} \int_0^{t_{2n-1}} dt_{2n} \int_0^{t_{2n}} dt_{2n+1}\, \lambda^{2n} N_i^n \\ & \times \sum_{\mathbf{q}_1,\ldots,\mathbf{q}_n} iq_{1x} |\phi(\mathbf{q}_1)|^2 e^{i\mathbf{q}_1\cdot v_d(t_1-t_2)} \cdots |\phi(\mathbf{q}_n)|^2 e^{i\mathbf{q}_n\cdot v_d(t_{2n-1}-t_{2n})} \\ & \times (i)^{2n+1} \mathrm{Tr}\left[\rho_0\left[-ieE\cdot\sum_k \left[\frac{\partial c_k^{\dagger}(t_{2n+1})}{\partial k}\right] \right.\right. \\ & \left.\left. \times c_k(t_{2n+1}),[\rho_{-\mathbf{q}_n}(t_{2n}),[\rho_{\mathbf{q}_n}(t_{2n-1}),\ldots,[\rho_{-\mathbf{q}_1}(t_2),\rho_{\mathbf{q}_1}(t_1)]_-\cdots]_-]_-\right]\right] . \end{aligned} \tag{55}$$

Considering Feynman diagrams, the frictional force is simply represented by the (a)-series graphs of Fig. 2(b) if the contribution from the heavy-particle (center-of-mass) fluctuation is neglected, i.e.,

$$\begin{aligned} F(\delta)v_d &= (a1)+(a2)+(a3)+\cdots+(\mathrm{TSG}) \\ &= Nmv_d\left\langle \frac{\lambda^2/\tau}{1+\lambda^2/\tau(\delta+2\eta)} \right\rangle . \end{aligned} \tag{56}$$

The diagrams for the coefficient $\beta(\delta)$ have one more structural feature as compared to those in Fig. 2(b) for $F(\delta)$. It comes from outermost commutation in Eq. (55) and is represented by an arrowed line with a circled cross at time point t_{2n+1} [Fig. 1(d)] which corresponds to the following factor:

$$(-ieE)\cdot\frac{\partial}{\partial k}(iG(k,t-t_{2n+1}))iG(k,t_{2n+1}-t') \ . \tag{57}$$

Then the leading-order terms of $\beta(\delta)$ in Eq. (55) are represented by the graphs in Fig. 4, i.e.,

$$\beta(\delta)NeE = (e1)+(e2)+(e3)+\cdots+(\mathrm{TSG}) \ . \tag{58}$$

The evaluation of these graphs is completely parallel to that of the (b)-series graphs in Fig. 2(b), and it yields the result

$$\beta(\delta)NeE = NeE\left\langle \frac{\lambda^2/\tau(\delta+2\eta)}{1+\lambda^2/\tau(\delta+2\eta)} \right\rangle . \tag{59}$$

With Eqs. (56) and (59), the impurity resistivity can be shown to have exactly the same form as Eq. (45). In the adiabatic limit ($\eta=0$), the resistivity becomes

$$\rho_i = \left.\frac{F(\delta)}{(Ne)^2[1-\beta(\delta)]}\right|_{\delta\to 0} = \frac{m}{Ne^2}\frac{\lambda^2}{\langle\tau\rangle} \ . \tag{60}$$

Furthermore, we would like to emphasize that the force-balance equation [Eq. (53)] also yields the isothermal resistivity expression[4] when inelastic electron-electron scattering becomes strong: $\eta = 1/\tau_{\text{in}} \gg \lambda^2/\tau$.

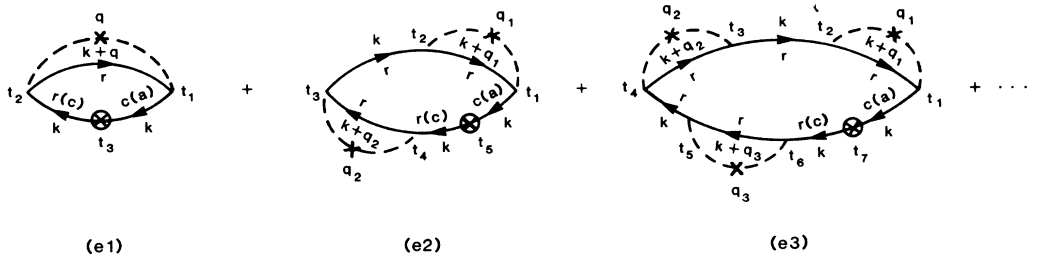

FIG. 4. Feynman diagrams for the electric field coefficient in the force-balance equation.

V. SUMMARY AND CONCLUSIONS

The core of the balance-equation method lies in the transformation to center-of-mass and relative-electron coordinates. In this, the exact commutation relations of the relative-electron coordinates and momenta are not canonical, differing by terms of $O(1/N)$ that are required to assure that the total relative electron momentum shall vanish. In neglecting these terms and invoking a canonical approximation for the relative-electron variables, a macroscopic drift of relative electrons is erroneously induced, and when this drift is compensated by incorporating it into the total current, we obtain the correct adiabatic resistivity. However, we have shown that this correction procedure involves the anomalous vanishing of the compensated current along with the resistive friction force, such that their ratio yields the correct adiabatic resistivity. In Sec. IV, we showed that this anomaly may be removed by a fully correct treatment of the exact noncanonical relative-electron commutation relations. Thus, we have conclusively demonstrated the capacity of the balance-equation method to yield the correct adiabatic resistivity. It is noteworthy that the center-of-mass variables are seen to behave in a purely classical manner in this demonstration, with *no* correction from their quantum-mechanical commutation relations, contrary to the earlier claim of Fishman and Mahan.[7]

Our earlier consideration[4] of the impurity resistivity in the isothermal limit devolved upon the "dressing" of the relative-electron Green's functions by the strong electron-electron interaction (phonon mediated or direct) as manifested by an imaginary component of the electron self-energy given by $(2\tau_{in})^{-1}$ in all of the Feynman diagrams involved in the terms of Van Hove's "$\lambda^2 t$" series, which are divergent in the adiabatic limit $\tau_{in} \gg \tau$. Such dressing of the relative-electron Green's function is also represented in the present work through the relative-electron linewidth $\eta = 1/2\tau_{in}$, and in the isothermal limit $\tau_{in} \ll \tau$ both the previous and present analyses demonstrate that the divergencies of Van Hove's "$\lambda^2 t$" series are quenched, leading to the original isothermal balance-equation result[4] for impurity resistivity $\rho_i^I = m\langle \lambda^2/\tau \rangle / Ne^2$ (as opposed to the adiabatic result $\rho_i^A = m\lambda^2/Ne^2\langle \tau \rangle$). While it is true that our analysis neglects vertex corrections, there is no clear reason to expect difficulty from that quarter in the linear limit, and we have proven that the known divergent terms of Van Hove's "$\lambda^2 t$" series are rendered negligible in the isothermal limit.

In the present paper, we have shown that the balance-equation method yields the correct commonly cited "Kubo formula" linear impurity resistivity in the adiabatic limit. While this adiabatic aspect of the Kubo formula is widely known, the Kubo-formula prediction for linear impurity resistivity under isothermal conditions (strong electron-electron scattering) is not so well known. We expect that the balance-equation prediction for linear isothermal impurity resistivity may also be obtained from the Kubo formula[24] by including the self-energy and vertex corrections due to strong electron-electron interactions in the latter. This work is in progress and the results will be presented elsewhere.

Note added in proof. After submitting this manuscript, we were informed that the effective Hamiltonian [Eq. (51)] can also be derived using a Lagrangian multiplier [Robert Sullivan and John Inkson (private communication)]. We thank these authors for helpful discussions.

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APPENDIX: FEYNMAN DIAGRAM REPRESENTATION OF EQ. (10) EMPLOYING CLOSED-TIME-PATH GREEN'S FUNCTIONS

We extend the usual time path (from $-\infty$ to $+\infty$) into a closed one which runs from $-\infty$ to $+\infty$ and returns back to $-\infty$. The former branch (from $-\infty$ to $+\infty$) is regarded as a positive "+" branch, the latter one (from $+\infty$ to $-\infty$) is regarded as a negative "−" branch. Furthermore, the operators $\rho_q(t_+)$ and $\rho_q(t_-)$ are regarded as two different operators before the time ordering is completely. The generalized time-order operator along the closed time path $\mathcal{T}_p$ behaves like the ordinary time ordering $\mathcal{T}$ on the positive branch and it behaves as the antitime ordering $\bar{\mathcal{T}}$ on the negative branch; moreover, it orders all the operators on the negative branch to stand to the left of those on the positive branch. For example, if $A(t)$ and $B(t')$ are two arbitrary operators (bosons), then

$$\mathcal{T}_p A(t_+)B(t'_+) = \mathcal{T} A(t)B(t') , \tag{A1}$$

$$\mathcal{T}_p A(t_-)B(T'_-) = \bar{\mathcal{T}} A(t)B(t') , \tag{A2}$$

$$\mathcal{T}_p A(t_+)B(t'_-) = B(t')A(t) , \tag{A3}$$

$$\mathcal{T}_p A(t_-)B(t'_+) = A(t)B(t') . \tag{A4}$$

Here the subscripts "+" and "−" mean the corresponding operators are on the "+" and "−" branches, respectively. For convenience, we also introduce two symbols ξ_α and η_β with α and β being branch indices, and

$$\xi_+ = 1, \quad \xi_- = 1 , \tag{A5}$$

$$\eta_+ = 1, \quad \eta_- = -1 . \tag{A6}$$

Utilizing the definition of the generalized time-ordering operator along the closed time path and taking notice of the time sequence $t_1 \geq t_2 \geq t_2 \geq \cdots \geq t_{2n}$ required by the time integrations in Eq. (10), the iterated commutation of $2n$ A operators can be expressed as

$$[A(-q_n,t_{2n}),[A(q_n,t_{2n-1}),\ldots,[A(-q_1,t_2),A(q_1,t_1)]_-\cdots]_-]_-$$
$$=-\tfrac{1}{2}\xi_{\alpha_1}\eta_{\alpha_2}\cdots\eta_{\alpha_{2n-1}}\eta_{\alpha_{2n}}\mathcal{T}_p[A_{\alpha_{2n}}(-q_n,t_{2n})A_{\alpha_{2n-1}}(q_n,t_{2n-1})\cdots A_{\alpha_2}(-q_1,t_2)A_{\alpha_1}(q_1,t_1)]\ . \quad \text{(A7)}$$

Here, $\alpha_i=+$ or $-$ $(i=1,2,\ldots,2n)$ means operator $A(t_i)$ is on the α_ith branch and all the repeated subscripts are summed over "+" and "−" branches. Equation (A7) may be verified by direct application of the definition of $\mathcal{T}_p$. To exemplify this procedure, we take $A(t_1)$ and $B(t_2)$ with $t_1>t_2$ as an example. Using Eqs. (A5) and (A6), we have

$$\tfrac{1}{2}\xi_{\alpha_1}\eta_{\alpha_2}\mathcal{T}_p A_{\alpha_1}(t_1)B_{\alpha_2}(t_2)=\tfrac{1}{2}[\mathcal{T}_p A_+(t_1)B_+(t_2)+\mathcal{T}_p A_-(t_1)B_+(t_2)-\mathcal{T}_p A_+(t_1)B_-(t_2)-\mathcal{T}_p A_-(t_1)B_-(t_2)]\ . \quad \text{(A8)}$$

Now, making use of the definition of $\mathcal{T}_p$ in Eqs. (A1)–(A6), it becomes obvious that

$$\tfrac{1}{2}\xi_{\alpha_1}\eta_{\alpha_2}\mathcal{T}_p A_{\alpha_1}(t_1)B_{\alpha_2}(t_2)=-[B(t_2),A(t_1)]\ . \quad \text{(A9)}$$

Repeated application of Eqs. (A1)–(A6) readily confirms Eq. (A7).

Employing the relations in Eq. (A7), we are able to express the retarded correlation function of Eq. (10) in terms of the generalized "contour" time-order correlation functions as follows:

$$F(\delta)v_d=\sum_{n=1}^{\infty}F^{(n)}(\delta)v_d\ , \quad \text{(A10)}$$

$$F^{(n)}(\delta)v_d=\int_0^{\infty}dt_1\,\delta e^{-\delta t_1}\int_0^{t_1}dt_2\cdots\int_0^{t_{2n-2}}dt_{2n-1}\int_0^{t_{2n-1}}dt_{2n}\,\lambda^{2n}N_i^n$$
$$\times\sum_{\mathbf{q}_1,\ldots,\mathbf{q}_n}iq_{1x}|\phi(\mathbf{q}_1)|^2e^{i\mathbf{q}_1\cdot v_d(t_1-t_2)}\cdots|\phi(\mathbf{q}_n)|^2e^{i\mathbf{q}_n\cdot v_d(t_{2n}-t_{2n})}G^{(2n)}(q_1,q_2,\ldots,q_{2n},t_1,t_2,\ldots,t_{2n})\ , \quad \text{(A11)}$$

$$G^{(2n)}(q_1,q_2,\ldots,q_{2n},t_1,t_2,\ldots,t_{2n})=-\tfrac{1}{2}\xi_{\alpha_1}\eta_{\alpha_2}\cdots\eta_{\alpha_{2n-1}}\eta_{\alpha_{2n}}$$
$$\times\mathrm{Tr}[\rho_0\mathcal{T}_p(A_{\alpha_{2n}}(-q_n,t_{2n})A_{\alpha_{2n-1}}(q_n,t_{2n-1}),\ldots A_{\alpha_2}(-q_1,t_2)A_{\alpha_1}(q_1,t_1))]\ . \quad \text{(A12)}$$

For this $2n$-point closed-time-path Green's function in Eq. (A12), we have the generalized Wick's theorem[8] to decouple it into a series of two-point closed-time-path Green's functions. Noticing that each A [Eq. (12)] contains two single-electron (fermion) operators (c_k and $c_{k+q}^{\dagger}$) and one center-of-mass (boson) operator ($e^{iq\cdot\delta R}$), each term of $G^{(2n)}$ contains $2n$ single-electron Green's functions, $G_{\alpha_i,\alpha_j}(t_i,t_j)$, and n center-of-mass fluctuation Green's functions, $F_{\alpha_i,\alpha_j}(t_i,t_j)$. After the above-mentioned decoupling process, we replace the two-point closed-time-path Green's functions F_{α_i,α_j} and G_{α_i,α_j} with the familiar retarded, advanced, and correlation Green's functions [Eqs. (16)–(21)] through the following relations:

$$G_{\alpha\beta}=\tfrac{1}{2}(\xi_\alpha\eta_\beta G_r+\eta_\alpha\xi_\beta G_a+\xi_\alpha\xi_\beta G_c)\ , \quad \text{(A13)}$$

$$F_{\alpha\beta}=\tfrac{1}{2}(\xi_\alpha\eta_\beta F_r+\eta_\alpha\xi_\beta F_a+\xi_\alpha\xi_\beta F_c)\ . \quad \text{(A14)}$$

With this, the representation of the frictional force $F(\delta)$ indicated in Fig. 2 follows directly. In evaluating the graphs of Fig. 2(a), the terms of higher order in $(1/N)$ are dropped. Moreover, the time sequence order $t_1\geq t_2\geq\cdots\geq t_{2n}$ causes many terms, such as those containing $G_a(t_2-t_3)$ to vanish automatically. The final expression for $F(\delta)$ emerges without complication.

In the following discussion, we take $n=2$ as an example to demonstrate the above-stated procedure and provide a detailed evaluation of the corresponding Feynman graphs for $n=2$:

$$G^{(4)}(q_1,q_2,t_1,\ldots,t_4)=-\tfrac{1}{2}\xi_{\alpha_1}\eta_{\alpha_2}\eta_{\alpha_3}\eta_{\alpha_4}\mathrm{Tr}\rho_0\mathcal{T}_p\rho_{q_1\alpha_1}(t_1)e^{iq_1\cdot\delta R_{\alpha_1}(t_1)}$$
$$\times\rho_{-q_1\alpha_2}(t_2)e^{-iq_1\cdot\delta R_{\alpha_2}(t_2)}\rho_{q_2\alpha_3}(t_3)e^{iq_2\cdot\delta R_{\alpha_3}(t_3)}\rho_{-q_2\alpha_4}(t_4)e^{-iq_2\cdot\delta R_{\alpha_4}(t_4)}\ , \quad \text{(A15)}$$

Decoupling the above four-point correlation function into a series of two-point functions:

$$iG_{\alpha\beta}(k,t-t')=\mathrm{Tr}\rho_0\mathcal{T}_p c_{k\alpha}(t)c_{k\beta}^{\dagger}(t')\ , \quad \text{(A16)}$$

$$F_{\alpha\beta}(q,q',t,t')=\mathrm{Tr}\rho_0\mathcal{T}_p e^{iq\cdot\delta R_\alpha(t)}e^{iq'\cdot\delta R_\beta(t')}\ , \quad \text{(A17)}$$

we may express the traced part on the right-hand side of Eq. (A15) as follows:

$$\sum_k [iG_{\alpha_1\alpha_2}(k+q_1, t_1-t_2)iG_{\alpha_2\alpha_3}(k, t_2-t_3)iG_{\alpha_3\alpha_4}(k+q_2, t_3-t_4)iG_{\alpha_4\alpha_1}(t_4-t_1)+(\text{TSG})]$$
$$\times[F_{\alpha_1\alpha_2}(q_1, -q_1, t_1-t_2)F_{\alpha_3\alpha_4}(q_2, -q_2, t_3-t_4)+(\text{TSG})]$$
$$+\sum_{k_1}\sum_{k_2}[iG_{\alpha_1\alpha_2}(k_1+q_1, t_1-t_2)iG_{\alpha_2\alpha_1}(k_1, t_2-t_1)F_{\alpha_2\alpha_3}(-q_1, q_2, t_2-t_3)$$
$$\times iG_{\alpha_3\alpha_4}(k_2+q_2, t_3-t_4)iG_{\alpha_4\alpha_3}(k_2, t_4-t_3)F_{\alpha_4\alpha_1}(-q_2, q_1, t_4-t_1)+(\text{TSG})] . \quad \text{(A18)}$$

In Eq. (A18), TSG (time symmetrized graphs) include all possible nonequivalent terms obtained by exchanging the time points t_2, t_3, t_4 while fixing t_1. If we replace the closed-time-path Green's functions in Eq. (A18) with the usual retarded, advanced, and correlation functions through Eqs. (A13) and (A14), and then put all those terms back into Eq. (A15), we arrive at a quite simple expression:

$$G^{(4)}(q_1,q_2,t_1,\ldots,t_4)=\sum_k iG_r(k+q_1, t_1-t_2)iG_r(k, t_2-t_3)$$
$$\times\tfrac{1}{2}[iG_r(k+q_2, t_3-t_4)iG_c(k, t_4-t_1)+iG_c(k+q_2, t_3-t_4)iG_a(k, t_4-t_1)]$$
$$+\sum_k iG_r(k+q_1, t_1-t_2)\tfrac{1}{2}[iG_r(k, t_2-t_4)iG_c(k+q_2, t_4-t_3)$$
$$\times iG_c(k, t_2-t_4)iG_a(k+q_2, t_4-t_3)]iG_a(k, t_3-t_1)$$
$$+\sum_k iG_r(k, t_1-t_3)\tfrac{1}{2}[iG_r(k+q_2, t_3-t_4)iG_c(k, t_4-t_2)$$
$$\times iG_c(k+q_2, t_3-t_4)iG_a(k, t_4-t_2)]iG_a(k+q_1, t_2-t_1)$$
$$+\sum_k \tfrac{1}{2}[iG_r(k, t_1-t_4)iG_c(k+q_2, t_4-t_3)+iG_c(k, t_1-t_4)iG_a(k+q_2, t_4-t_3)]$$
$$\times iG_a(k, t_3-t_2)iG_a(k+q_1, t_2-t_1)$$
$$+\sum_k \tfrac{1}{2}[iG_r(k_1+q_1, t_1-t_2)iG_c(k_1, t_2-t_1)$$
$$+iG_c(k_1+q_1, t_1-t_2)iG_a(k_1, t_2-t_1)]F_r(-q_1, q_2, t_2-t_3)$$
$$\times\sum_k \tfrac{1}{2}[iG_r(k_2+q_2, t_3-t_4)iG_c(k_2, t_4-t_3)+iG_c(k_2+q_2, t_3-t_4)iG_a(k_2, t_4-t_3)] . \quad \text{(A19)}$$

In the process of deriving the above result, many terms such as those containing $G_a(t_3-t_4)$ vanish automatically because of the time sequence $t_1>t_2>t_3>t_4$. Moreover, Eqs. (20) and (21) ($F_c=2$) are applied here and all the terms which are of higher order in $(1/N)$ are dropped.

With the use of Eq. (A19) and remembering the Feynman diagram rules stated in Sec. II, we observe that the $n=2$ term of the frictional force [Eq. (A11)] corresponds to graph $(b2)$ and its three TSG's and graph $(a2)$ with the shaded bubble being replaced with $(a1)$, i.e.,

$$F^{(2)}(\delta)v_d=[(b2)+(\text{TSG})]$$
$$+\text{lowest-order graph of } (a2) . \quad \text{(A20)}$$

Using the Green's functions defined in Eqs. (16)–(18) and (20) and in Eq. (A19) for $G^{(4)}$ and carrying out the time and the momentum integrations, we obtain the following results for the Feynman graphs for $F^{(2)}(\delta)$:

$$[(b2)+(\text{TSG})]=-\left\langle\frac{\lambda^2}{\tau}\frac{\lambda^2}{\tau(\delta+2\eta)}\right\rangle , \quad \text{(A21)}$$

$$\text{lowest-order graph of } (a2)=-\left\langle\frac{\lambda^2}{\tau}\right\rangle\frac{1}{\delta}\left\langle\frac{\lambda^2}{\tau}\right\rangle . \quad \text{(A22)}$$

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