Phase-relaxation interpretation of elastic softening induced by sliding charge-density waves

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(Received 27 December 1989; revised manuscript received 16 May 1990)

The softening of elastic moduli associated with sliding of a charge-density wave is analyzed in the weak-pinning framework of Lee and Rice. The difference in stiffness between the stationary and sliding states is attributed to the difference in energy between phase configurations that have had time to adjust to the applied strain and those that have not. Whereas sliding motion facilitates phase reconfiguration, the bias dependence arises naturally. The temperature, pin density, and frequency dependences are discussed, and experimental tests are proposed.

I. INTRODUCTION

The phenomenon of sliding charge-density waves (CDW's) is observed in several quasi-one-dimensional compounds.[1] The sliding of the CDW with respect to the crystal lattice is induced by application of an electric field E in excess of a threshold E_T. This sliding results in many unusual electrical phenomena, most conspicuously a strong violation of Ohm's law. The sliding is also accompanied by a mechanical effect: the lattice stiffness can be reduced substantially.[2,3] A sliding-induced softening has been observed in Young's modulus Y of orthorhombic TaS_3,[2-4] $NbSe_3$,[3,5] and $(TaSe_4)_2I$,[3,6] and in a shear modulus G of TaS_3.[7]

While a number of theoretical explanations have been presented[2,3,8-11] none has been entirely successful.[12] One category of explanation proposes that the stationary state is inherently stiffer than the sliding state, either because the pins couple distortions of the CDW to those of the lattice, or because the pins impede screening by the CDW. In the early approach of Mozurkewich *et al.*,[3(a)] the CDW and lattice are each regarded as an array of balls connected by springs. The pins couple the arrays together; thus a distortion of the lattice feels a restoring force from both sets of springs. The coupling vanishes in the sliding state *if* the pinning potential averages to zero. The more sophisticated, microscopic theory of Maki and Virosztek[9] explicitly considers screening by the CDW, but the bias dependence is introduced by an artificial assumption about the pinning frequency's bias dependence and its possible distribution. While softenings of the right order of magnitude can be obtained in these models, each is vulnerable to a trenchant criticism: the dependence on electric field is artificially introduced, essentially by fiat.

A different category of explanation was suggested by Brill and Roark.[2] They proposed that the softening is due to relaxation of some defect in the CDW. The relaxation time τ is long in the stationary state but is dramatically reduced when the CDW slides. According to the standard theory of relaxation processes, reducing τ reduces the modulus M while driving the internal friction δ through a maximum, as observed.[2] (See Fig. 1.) The major strength of this approach is that, unlike the models described above, the dependence of stiffness on CDW motion receives a natural explanation. So far the model has failed to explain quantitatively the observed shapes of $Y(E)$ and $\delta(E)$ and their dependence on frequency.[12] Furthermore, the nature of the relaxing quantity has not been identified.

This paper explores the possibility that the relaxing quantity is the phase configuration of the CDW. This approach retains the principal advantage of Brill and Roark: the dependence of stiffness on sliding receives a natural explanation. Furthermore, by suggesting correlations with band calculations and with piezoresistance and diffraction measurements, this approach provides a framework for quantitative analysis of the softening in shear as well as Young's moduli.

Within the widely accepted Lee-Rice theory of weak pinning,[13] the equilibrium phase configuration is distorted so as to minimize the sum of pinning and distortion energies. Because of randomness, there exist many local, metastable minima which are nearly degenerate with the global minimum, but which are separated from it and from each other by energy barriers.[14] The energies of

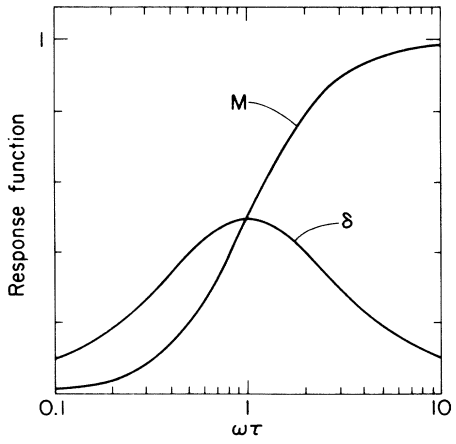

FIG. 1. Dependence of modulus M and internal friction δ on $\omega\tau$ for a simple relaxation process.

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these minima depend on the parameters of the Lee-Rice phase energy, which depends in turn on the electronic band structure. By modifying the electronic overlap integrals, an externally imposed strain will change the Lee-Rice parameters, thus shifting the energies and phase configurations corresponding to the local minima. But because of the energy barriers, the actual phase configuration of a pinned CDW cannot smoothly follow the optimum configuration. The optimum could be attained only approximately, and only after a complex sequence of random thermal hops which could not be realized on the time scale of the experiment.[15] Thus the CDW will be trapped in a metastable state near the old configuration, in which its energy will be higher than that in the optimum configuration by an amount which depends on the imposed strain. If the extra energy depends quadratically on the strain, the measured stiffness will be increased.

Despite theoretical arguments that the sliding state is "unique,"[16] experiments suggest that the metastable states somehow persist in the phase configuration of the sliding CDW. Bhattacharya *et al.*[17] showed that the frequency spectrum of broad-band noise fluctuations in the sliding state is determined (through a modified fluctuation-dissipation theorem) by the frequency dependence of the dielectric function in the *stationary* state. This suggests that the energy surface in configuration space has a topology that is unchanged by sliding. Transitions between the various favored configurations would account for the existence of the broad-band noise and probably also for the Gaussian fluctuations in the amplitude of the narrow-band noise.[18]

The central assumption of this paper is that the optimum configuration can be approached much more rapidly in the sliding state. The sliding process is believed to take place not uniformly but by a sequence of local jumps.[16] A cycle of jumps is completed in one period of the narrow-band noise, which for typical currents is less than 1 μsec. Thus on the time scale of the elastic measurement, each region of the CDW jumps many times, and the agitated motion provides an opportunity for the phase to reconfigure incrementally. The net effect is that the complex phase relaxation is facilitated by the sliding motion. In consequence, the configuration will evolve in response to an applied strain so as to reduce the energy, thereby making the measured stiffness in the sliding state smaller than in the stationary one.

The parameters of the Lee-Rice phase energy which are likely to depend on strain are the CDW spring constant K, the amplitude ρ_1, and the wave vector Q. The analysis described in this paper appears to rule out variation of K or ρ_1 because the predicted magnitude of the softening is too small, and because it should vary with pin density, in conflict with experiment.[19] On the other hand, variation of Q yields a prediction of the right order of magnitude and independent of pin density. For this case, the softening has a simple physical interpretation: A fraction g^2 of the CDW stiffness is added to the lattice stiffness for the pinned case, but not for the unpinned case. The value of g should be determinable from band-structure calculations, or it might be deduced from diffraction experiments in statically strained specimens.

The analysis for the case of strain-dependent K or ρ_1 can be described straightforwardly in terms of the Lee-Rice length; this is done in Sec. II A. Section II B deals with the case of strain-dependent wave vector. Section III discusses the dependence of the stiffness change on pin density, temperature, frequency, and mode symmetry.

II. CALCULATIONS

The charge density associated with a CDW may be represented by $\rho(\mathbf{r})=\bar{\rho}+\rho_1\cos[\mathbf{Q}\cdot\mathbf{r}+\phi(\mathbf{r})]$, where $\mathbf{Q}$ is the average CDW wave vector. The phase configuration $\phi(\mathbf{r})$ is determined by minimizing the Lee-Rice phase energy, which is the sum of an elastic distortion energy and a pinning term.[13] In d dimensions,

$$F=\int\frac{K}{2}|\mathbf{Q}+\nabla\phi-2\mathbf{k}_F|^2d^dx-\sum_i\rho_1V\cos[\mathbf{Q}\cdot\mathbf{R}_i+\phi(\mathbf{R}_i)], \tag{1}$$

where $2\mathbf{k}_F$ is the optimum wave vector; it is usually assumed that $\mathbf{Q}=2\mathbf{k}_F$. K (in units of energy per distance) will be called the CDW spring constant, to distinguish it from the CDW stiffness KQ^2 (in the traditional units of energy per volume). The amplitude of the CDW is ρ_1, and the strength of the impurity potential is V. The impurities (number density n) are distributed at random, and the sum runs over all impurity sites $\mathbf{R}_i$.

In the weak-pinning limit, the phase configuration may be characterized by the typical distance over which the phase varies by order π. This typical distance L, called the Lee-Rice length,[13] is estimated by minimizing the sum of the distortion and pinning energies in a volume L^d. One assumes that $\mathbf{Q}=2\mathbf{k}_F$, substitutes π/L for the phase gradient in Eq. (1), and observes that the effect of nL^d impurities is $(nL^d)^{1/2}$ times the maximum effect of one impurity. The energy per unit volume then is

$$W=W_{\text{elastic}}+W_{\text{pin}}=\frac{K}{2}\left[\frac{\pi}{L}\right]^2-\frac{\rho_1V\sqrt{n}}{L^{d/2}}\ . \tag{2}$$

The minimum occurs for

$$L=\left[\frac{2\pi^2}{d}\frac{K}{\rho_1V\sqrt{n}}\right]^{2/(4-d)} \tag{3}$$

with energy density (subscript 0 indicates equilibrium)

$$W_0=-\frac{4-d}{4}\left[\frac{d}{2\pi^2}\right]^{d/(4-d)}\left[\frac{(\rho_1V)^4n^2}{K^d}\right]^{1/(4-d)}\ . \tag{4}$$

The curvature at the minimum is

$$\frac{d^2W}{dL^2}=3\pi^2\frac{K}{L^4}-\frac{d(d+2)}{4}\frac{\rho_1V\sqrt{n}}{L^{d+4/2}}=\frac{\pi^2}{2}\frac{K}{L^4}\ . \tag{5}$$

The threshold field E_T is calculated by equating the energy density gained by advancing the CDW through one wavelength, $\bar{\rho}E_T2\pi/Q$, to Eq. (4):

$$E_T=\frac{\pi Q}{\bar{\rho}}\left[\frac{4-d}{4d}\right]\frac{K}{L^2}\ . \tag{6}$$

Using Eq. (3), $E_T \propto n^2$ for weak pinning in three dimensions.

To determine CDW effects on elastic moduli, it is necessary to calculate the dependence of energy on strain. The possibilities are divided into two cases. Variation of K or ρ_1 with strain affects W in Eq. (2) directly, and also by changing L. On the other hand, variation of Q with strain, which leaves L unchanged, affects W by violating the usual assumption that the average wave vector equals $2k_F$.

A. Case A: Strain-sensitive spring constant or amplitude

The energy of a pinned CDW depends not only on the parameters K and ρ_1 but also on the particular configuration, which may be characterized by L. Figure 2 shows W versus L from Eq. (2) for two difference values of K or ρ_1, assuming that $\mathbf{Q}=2\mathbf{k}_F$ at all times. Each curve represents a locus of local energy minima. Although neighboring points are close in energy, they are distant in configuration space; that is, the existence of energy barriers inhibits evolution along either curve. According to the central assumption of this paper, evolution is negligible in the stationary state on the time scale of the elastic experiment, but in the rapidly sliding state the system is able to evolve towards the minimum.

If K or ρ_1 were strain dependent, then the curves in Fig. 2 would represent the energy loci for the unstrained and strained states. If the strain were applied suddenly, the system would move from its initial state O directly to the unrelaxed configuration U that has the same L. Only with the passage of sufficient time would the system evolve to R. The energy difference between states U and R is approximately

$$\Delta W=\frac{1}{2}\frac{d^2W}{dL^2}(L_R-L_U)^2\ . \tag{7}$$

The modulus difference between the stationary and rapidly sliding states, $M_{\rm CDW}\equiv M(E=O)-M(E\gg E_T)$, is determined by taking two strain derivatives of ΔW.

Consider the case where the spring constant or the amplitude depends linearly on strain ϵ: $K(\epsilon)=K+K'\epsilon$ or $\rho_1(\epsilon)=\rho_1+\rho_1'\epsilon$. Using Eq. (3), $(L_R-L_U)/L=2/(4-d)(K'/K-\rho_1'/\rho_1)\epsilon$. Substituting into Eq. (7) and taking two strain derivatives gives

$$M_{\rm CDW}=\frac{2\pi^2}{(4-d)^2}\left[\frac{K}{L^2}\right]\left[\frac{K'}{K}-\frac{\rho_1'}{\rho_1}\right]^2\ . \tag{8}$$

At present, L is not directly accessible experimentally. It can be eliminated in favor of E_T:

$$M_{\rm CDW}=\frac{8\pi d}{(4-d)^3}\frac{E_T\bar{\rho}}{Q}\left[\frac{K'}{K}-\frac{\rho_1'}{\rho_1}\right]^2\ . \tag{9}$$

Section III will argue that this result is too small to explain the observed modulus shift and does not have the right temperature and pinning dependences.

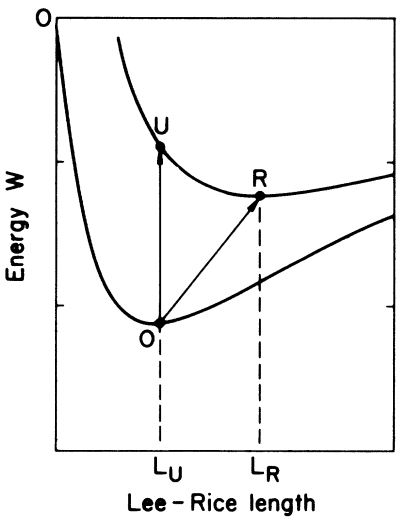

FIG. 2. Energy density vs Lee-Rice length, for analysis of case A. Each curve indicates a locus of states that are near in energy but distant in configuration space. Lower curve, unstrained; upper curve, strained. O, initial state; U, unrelaxed state; R, relaxed state. The curves show the actual shapes for $d=3$, rather than the parabolic approximations used in the text.

B. Case B: Strain-sensitive wave vector

Now consider the alternative possibility, that the strain-dependent parameter is the optimum CDW wave vector $2k_F$. Then straining the lattice according to $R\rightarrow R'=R(1+\epsilon)$ will cause a corresponding wave-vector change that is also proportional to the strain.

First it is instructive to consider a single, purely one-dimensional band, in which case k_F (parallel to z) changes in inverse proportion to R. (This is easily proved for either nearly free electrons or tight-binding bands.) To order ϵ, one has $k_F'=k_F(1-\epsilon)$. Consider the effects of the two terms in Eq. (1). The pinning term forces the phase $\theta(R_i)=QR_i+\phi(R_i)$ to distort along with the lattice in such a way that the phase at the new lattice site retains the value it had at the old lattice site: $\theta'(R_i')=\theta(R_i)$. When the value of the phase simply follows the lattice points, it will be said that the phase configuration has been "transported." Physically, this means that the condensed electrons simply follow the lattice. The average phase-winding rate in the transported state is $Q'=Q(1-\epsilon)$, which is exactly the average rate prescribed to minimize the elastic term, because $Q'-2k_F'=(Q-2k_F)(1-\epsilon)=0$. Therefore, if the initial phase configuration minimized the energy in the unstrained lattice, then the transported phase configuration minimizes the energy in the strained lattice. The trivial nature of such a distortion was emphasized by Xiang and Brill,[7] who concluded for the purely one-dimensional case that "there should be no CDW contribution to the elastic energy needed to uniformly strain the lattice, whether or

not the CDW is pinned. . . ."

Real CDW systems have transverse coupling between chains, imperfect nesting, overlapping bands, etc., so that the inverse proportionality between R and k_F can fail. One may write $k_F'=k_F[1-(1-g)\epsilon]$, $|g|<1$, where g parametrizes the degree of departure from the purely one-dimensional behavior. Initially $Q=2k_F$. Upon straining the lattice, the average phase-winding rate Q' in the transported state no longer minimizes the elastic term, because for this case $Q'-2k_F'=-2k_Fg\epsilon$. Lower energy could be reached by evolution of the new Q' towards $2k_F'$, but this would require relative phase changes of 2π between distant points in the sample. Therefore the configuration of minimum elastic energy can be realized only after a complex phase rearrangement impeded by energy barriers, which is likely to happen on the time scale of the experiment only if the CDW is sliding. Otherwise the phase will be trapped in a metastable configuration near the transported version of the prestrain configuration.

Generalizing to the fully three-dimensional case, a lattice distortion $R_i'=R_i+\epsilon_{ij}R_j$ requires $Q_i'=Q_i-\epsilon_{ij}Q_j$ in the transported state, to first order in ϵ_{ij}. What about k_i', the ith component of $\mathbf{k}_F$? With the effects of transverse coupling, etc., the general, strain-dependent variation may be written

$$k_i'=k_i-(\delta_{ik}\delta_{jl}-g_{ijkl})k_j\epsilon_{kl} \ . \tag{10}$$

The compounds that support sliding CDW's all have wave vectors that are commensurate in the directions transverse to the chain axis, requiring many elements of g_{ijkl} to vanish or to equal unity. However, this form allows the incommensurate component of $\mathbf{k}_F$ along the chain axis to depend nontrivially on transverse as well as longitudinal lattice distortions. The rest of the preceding paragraph goes through unchanged, with g and ϵ interpreted as the appropriate tensor components.

The task of this section is to calculate the energy difference between the unrelaxed, transported configuration and the relaxed one of minimum energy.

The contribution of the pinning term in Eq. (1) does not change between the unrelaxed and relaxed states. The pinning energy in the unrelaxed configuration is the same as that in the initial, unstrained system, because the phase at each impurity is transported along with the impurity. The relaxed state has an entirely new configuration, but it also has the same pinning energy (within statistical fluctuations) as the initial, unstrained state, because, by the usual Lee-Rice arguments, the pinning energy depends only on the length L, which is unchanged as long as K and ρ_1 are unchanged. This conclusion is true only for case B, in which K and ρ_1 are assumed to be constant. Any variation of K or ρ_1 with strain, which necessarily changes L, has already been considered in case A.

The energy difference is contained entirely in the elastic term. For the relaxed configuration

$$W^R_{\text{elastic}}=(K/2)\langle|\nabla\phi''|^2\rangle \ , \tag{11}$$

where $\langle\ldots\rangle$ represents a volume average, and the double prime is a reminder that the relaxed state has an entirely different phase configuration from the unrelaxed (primed) one. For the unrelaxed configuration

$$\begin{aligned} W^U_{\text{elastic}}&=(K/2)\langle|\mathbf{Q}'-2\mathbf{k}_F'+\nabla\phi'|^2\rangle \\ &=(K/2)[(-2k_Fg\epsilon)^2+2(-2\mathbf{k}_Fg\epsilon)\cdot\langle\nabla\phi'\rangle \\ &\quad+\langle|\nabla\phi'|^2\rangle] \ . \end{aligned} \tag{12}$$

The second term averages to zero, because $\nabla\phi'$ is defined as the departure from the average winding rate. The third term is the usual Lee-Rice elastic energy, which also depends only on L and therefore is the same (within statistical fluctuations) as in the unrelaxed state, Eq. (11). So the entire energy difference between the unrelaxed and relaxed states is

$$\Delta W=(K/2)(2k_Fg\epsilon)^2 \ , \tag{13}$$

whence

$$M_{\text{CDW}}=g^2K(2k_F)^2 \ . \tag{14}$$

(Actually there are additional energy changes introduced through the dependences of $\nabla\phi$ and of volume on ϵ, but these are equivalent to changes of K and of ρ_1 linear in ϵ, which have already been considered under case A.)

Physically this result for M_{CDW} says that a fraction g^2 of the CDW's stiffness $K(2k_F)^2$ adds to the lattice stiffness, if the CDW is unable to relax. When the lattice is distorted by ϵ, the unrelaxed wave vector changes by ϵ, but the optimum wave vector changes by $(1-g)\epsilon$. Hence the effective strain in the CDW is only $g\epsilon$. In certain special cases, such as a single, purely one-dimensional band, $g=0$, and there would be no stiffness change upon sliding, but for real CDW compounds, the determination of g becomes a challenge for band theory.

III. DISCUSSION

The size of M_{CDW} will now be estimated. According to Lee and Rice,[13] the CDW spring constant along the chain direction is $K=2f_0\xi_z^2=2\epsilon_Fa_z/a_xa_y$, where a_i are the lattice constants along the chain (z) and perpendicular directions. Thus

$$KQ_0^2=\frac{2\epsilon_F}{a_xa_ya_z}(Q_0a_z)^2 \ , \tag{15}$$

where $Q_0=2k_F$. Using typical numbers $\varepsilon_F=3$ eV, $a_xa_ya_z=10^{-21}$ cm^3, and $Q_0=2\pi/4a_z$ gives $KQ_0^2\approx2\times10^{10}$ erg/cm^3. Thus if $g\approx1$, the case B result, Eq. (14), has the right order of magnitude, roughly 1% of a typical lattice stiffness.

Case A requires an estimate for K' or ρ_1'. While the first seems to be unavailable, the second may be extracted from piezoresistance measurements. For orthorhombic TaS_3, for strains along the chain axis, Lear *et al.*[20] quote an activation energy $E_a=72$ meV, its stress sensitivity $dE_a/d\sigma_3=12$ meV/GPa, and the associated Young's modulus $Y_{33}=350$ GPa. Hence $\rho_1'/\rho_1\approx(1/E_a)dE_a/d\epsilon_3=(Y_{33}/E_a)dE_a/d\sigma_3\approx60$. Writing $K/L^2=KQ_0^2/(LQ_0)^2$ and guessing $L\approx10^{-4}$ to 10^{-5} cm,

Eq. (8) gives $M_{CDW} \approx 10^8 - 10^9$ erg/cm^3, which is probably too small.

The pin-density dependences are different for the two cases. For case A, $M_{CDW} \propto n^{2/(4-d)}$ according to Eq. (8) or (9), so that the stiffness increases with pin density in any dimension $d < 4$. For case B, Eq. (14), M_{CDW} is independent of pin density. A recent experiment[19] showed that M_{CDW} is independent of pins introduced by electron irradiation, again indicating case B.[21]

M_{CDW} contains an implicit temperature dependence through the parameters of the Lee-Rice phase energy. For the apparently relevant case B, the dependence is simply that of the spring constant K, because Q_0 is essentially independent of temperature and g is presumably a constant fixed by the band structure. K varies like the square of the order parameter,[13] as should M_{CDW}. Such variation of M_{CDW} was also derived by Maki and Virosztek.[9] It seems to be consistent with experiment.[22] As for case A, Eq. (9) implies an additional strong temperature dependence proportional to E_T. Such a dependence has not been found.[22]

Strictly interpreted, the Lee-Rice theory applies only at $T=0$. Maki and Virosztek have phenomenologically extended the theory to treat the temperature dependence of the threshold field, producing impressive fits to data.[23] They assert that the Lee-Rice length shows a strong temperature dependence due to thermal fluctuations of the CDW phase, reflecting the observed variation of E_T. It is possible that analogous modifications to the present analysis could introduce additional temperature dependences into M_{CDW} beyond those described in the preceding paragraph, but to date experiments show no evidence for such effects.[22]

To summarize, the size of the softening, its insensitivity to pin density, and its temperature dependence all indicate an origin in a strain-dependent wave vector.

The frequency dependence measured by Xiang and Brill[12] presents a serious challenge. The present interpretation predicts that the modulus change between $\omega\tau \gg 1$ and $\omega\tau \ll 1$ is independent of frequency. However, the measurements of Ref. 12 were restricted to electrical bias not greater than $4E_T$, insufficient to realize the full softening. Therefore the decrease of the measured softening with increasing frequency can be interpreted qualitatively using Fig. 1 as a systematic shift towards the right of the points corresponding to both stationary and sliding states. Nevertheless, the predicted shape of the softening with bias is wrong in detail: the observed shape is independent of frequency. Such a frequency-independent shape could imply a power-law dependence of modulus on $\omega\tau$, which might result from a power-law distribution of some microscopic variable, such as relaxation rate.[24]

The present interpretation is also challenged by the behavior of the internal friction. Its magnitude is generally too small to be attributed to a simple relaxation process.[2] Furthermore, while the friction is often observed to peak, as implied by Fig. 1, it has never been reported to return at large bias to its value in the pinned state. This behavior of the internal friction might also result from a distribution of relaxation processes. It is well known that distributions reduce the size of the friction relative to the modulus change, because the friction sees only processes whose τ is near ω^{-1}, while the effect on the modulus is cumulative over smaller τ as well. A wide distribution might also account for the slowness of the decrease of the friction at large bias.

The shape of the bias dependence and its surprising frequency dependence might productively be compared to the ac dielectric function, which also measures distortions of the CDW phase configuration. When measured in the kHz range, the dielectric function increases strongly immediately above threshold,[25] which might reflect an increased polarizability due to the agitated motion described in the Introduction. Like the softening, most of this dielectric enhancement occurs within a few times threshold, and it decreases with increasing frequency. Also, the dielectric constant shows strange frequency dependences below threshold.[26-28] Numerical analysis of the Lee-Rice model might illuminate the effects of bias and frequency on both dielectric function and stiffness.

Perhaps the most intriguing aspect of the elastic softening with CDW sliding is the large effect in a shear mode.[7] The present analysis provides a framework for understanding the role of mode symmetry. In an anisotropic system, K, ρ_1, and Q_0 are equally likely to depend on longitudinal or shear strains. Band calculations should be capable of predicting their sensitivities to strains of different symmetries.

IV. CONCLUDING REMARKS

This new interpretation of the elastic softening that is induced by sliding CDW's involves three key assumptions. (1) By modifying the band structure, an imposed strain changes the optimal, lowest-energy phase configuration. (2) Energy barriers between metastable states impede relaxation of the actual phase configuration towards the optimal one, even in the sliding state. (3) With respect to the time scale of the elastic experiment, the relaxation is slow (and the modulus is large) in the stationary state, but the relaxation becomes faster (decreasing the modulus) as the CDW velocity increases.

This phase-relaxation interpretation is reasonably successful if one assumes that the principal effect of the strain is to modify the CDW wavelength with respect to the underlying lattice. The predicted size of the softening has the right order of magnitude. Furthermore, its size is independent of pin density and varies with temperature as the square of the order parameter, in agreement with experiments.

This interpretation might be distinguished from the screening theory of Maki and Virosztek[9] by an elastic measurement at very low frequency. If the softening is relaxational, an experiment at sufficiently low frequency would measure no modulus change between the stationary and sliding states, because the phase configuration would have time to relax fully even in the stationary state. Thus the modulus change must pass through a maximum at some frequency below those accessible to the vibrating reed technique. (Unfortunately it is not presently possible to say how low this frequency will be.) According to the screening theory, in contrast, the sta-

tionary state is stiffer because, as long as the pinning frequency is nonzero, the requisite phase distortion uses too much phason energy. Thus, according to that theory, there would be a modulus change between stationary and sliding states however low the measuring frequency.

This theory could also be tested by exploring the postulated strain dependences of the parameters. A strain-dependent wave vector might be detected by x-ray diffraction, a strain-dependent CDW amplitude by diffraction or by piezoresistance. (Unfortunately, the spring constant appears to be experimentally inaccessible at present.) A major goal of such experiments should be to measure the sensitivity factors K', ρ_1', and g for strains of different symmetries, and thereby to interpret the relative sizes of the softenings in Young's and shear moduli.

ACKNOWLEDGMENTS

I am deeply indebted to Sue Coppersmith for many fruitful discussions. I also thank Joe Brill, Paul Goldbart, and Ron Jacobsen. This work was supported by the National Science Foundation under Grant No. DMR 84-51935.

[1]For reviews, see P. Monceau, *Electronic Properties of Inorganic Quasi-One-Dimensional Compounds* (Reidel, Boston, 1985); G. Grüner, Rev. Mod. Phys. **60**, 1129 (1988).

[2]J. W. Brill and W. Roark, Phys. Rev. Lett. **53**, 846 (1984); J. W. Brill, W. Roark, and G. Minton, Phys. Rev. B **33**, 6831 (1986).

[3](a) G. Mozurkewich, P. M. Chaikin, W. G. Clark, and G. Grüner, in *Charge Density Waves in Solids,* Vol. 217 of *Springer Lecture Notes in Physics,* edited by Gy. Hutiray and J. Sólyom (Springer, New York, 1985), p. 353; (b) G. Mozurkewich, P. M. Chaikin, W. G. Clark, and G. Grüner, Solid State Commun. **56**, 421 (1985).

[4]M. H. Jericho and A. M. Simpson, Phys. Rev. B **34**, 1116 (1986).

[5]X. -D. Xiang and J. W. Brill, Phys. Rev. B **39**, 1290 (1989).

[6]A. Suzuki, H. Mizubayashi, and S. Okuda, J. Phys. Soc. Jpn. **57**, 4322 (1988).

[7]X. -D. Xiang and J. W. Brill, Phys. Rev. B **36**, 2969 (1987).

[8]L. Sneddon, Phys. Rev. Lett. **56**, 1194 (1986).

[9]K. Maki and A. Virosztek, Phys. Rev. B **36**, 2910 (1987).

[10]R. Zeyher, Phys. Rev. Lett. **61**, 1009 (1988).

[11]A. S. Rozhavsky, Yu. S. Kivshar, and A. V. Nedzvetsky, Phys. Rev. B **40**, 4168 (1989).

[12]X. -D. Xiang and J. W. Brill, Phys. Rev. Lett. **63**, 1853 (1989).

[13]P. A. Lee and T. M. Rice, Phys. Rev. B **19**, 3970 (1979).

[14](a) P. B. Littlewood and T. M. Rice, Phys. Rev. Lett. **48**, 44 (1982); (b) P. B. Littlewood and R. Rammal, Phys. Rev. B **38**, 2675 (1988).

[15]The time scale of the elastic experiments is 10^{-2} sec or less, while rearrangements of the phase within the stationary state continue to occur beyond 10^3 sec. See G. Mihály and L. Mihály, Phys. Rev. Lett. **52**, 149 (1984).

[16]D. S. Fisher, Phys. Rev. B **31**, 1396 (1985); P. B. Littlewood, *ibid.* **33**, 6694 (1986).

[17]S. Bhattacharya, J. P. Stokes, M. J. Higgins, and M. O. Robbins, Phys. Rev. B **40**, 5826 (1989).

[18]G. L. Link and G. Mozurkewich, Solid State Commun. **65**, 15 (1988); T. Miyashita and H. Takayama, J. Phys. Soc. Jpn. **57**, 2889 (1988).

[19]R. L. Jacobsen and G. Mozurkewich, Phys. Rev. B **39**, 10 903 (1989).

[20]R. S. Lear, M. J. Skove, E. P. Stillwell, and J. W. Brill, Phys. Rev. B **29**, 5656 (1985); R. S. Lear, Ph.D. thesis, Clemson University, 1983 (unpublished).

[21]Note, however, that irradiation is generally believed to form *strong* pins, for which the present analysis does not directly apply.

[22]R. L. Jacobsen and G. Mozurkewich, Phys. Rev. B **42**, 2778 (1990).

[23]K. Maki, Phys. Rev. B **33**, 2852 (1986); K. Maki and A. Virosztek, *ibid.* **39**, 9640 (1989).

[24]This sort of idea has already been applied to the dielectric function, whose frequency dependence was interpreted in Ref. 14(b) in terms of a power-law density of relaxational modes.

[25]W. Wu, L. Mihály, and G. Grüner, Solid State Commun. **55**, 663 (1985).

[26]W. Wu, L. Mihály, G. Mozurkewich, and G. Grüner, Phys. Rev. B **33**, 2444 (1986).

[27]R. J. Cava, R. M. Fleming, R. G. Dunn, and E. A. Rietman, Phys. Rev. B **31**, 8325 (1985).

[28]J. P. Stokes, M. O. Robbins, and S. Bhattacharya, Phys. Rev. B **32**, 6939 (1985).