PHYSICAL REVIEW B VOLUME 42, NUMBER 17 15 DECEMBER 1990-I

Channel plasmons

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(Received 19 April 1990)

We present a derivation, based on Green's theorem, of the exact dispersion relation for the electrostatic modes guided by a channel of finite width cut into the otherwise planar surface of a solid in contact with vacuum. The solid, which can be either a metal or a polar dielectric medium, is characterized by a real, isotropic, frequency-dependent dielectric constant $\epsilon(\omega)$. The results of numerical solutions of this dispersion relation are presented for two forms of the channel cross section.

The guiding of electromagnetic waves by surfaces and interfaces is now a well-established topic in condensed-matter theory.[1,2] Less well studied is their guiding by structures which confine the power carried by the wave to a region of bounded spatial extent transverse to the direction of propagation. Electrostatic[3–10] and electromagnetic[11] waves propagating along the apex of a dielectric wedge provide an example of such confined guided waves. In this paper we obtain the dispersion relation for electromagnetic waves guided by a channel cut into the otherwise planar surface of a metal in contact with vacuum. The solid can be either a metal or a polar dielectric medium. In either case it is characterized by a real, isotropic, frequency-dependent dielectric constant $\epsilon(\omega)$, and we are interested in the frequency range in which $\epsilon(\omega)$ is negative. In the present work we neglect retardation. The resulting electrostatic wave will be called *channel plasmons,* whether the solid is a metal or a polar dielectric medium. The solution with retardation taken into account is planned to be presented elsewhere.

The channel is assumed to run parallel to the x_2 axis. The region $x_3>\zeta(x_1)$ is vacuum, while the region $x_3<\zeta(x_1)$ is the solid. The surface profile function $\zeta(x_1)$ is assumed to be a single-valued function of x_1, and we further assume that it is an even function of x_1, $\zeta(-x_1)=\zeta(x_1)$, although this is an inessential simplification. It is sensibly nonzero only for $|x_1|$ smaller than some characteristic length R.

We seek the electrostatic potential $\phi(\mathbf{x};t)$ in this structure in the form $\phi(\mathbf{x};t)=\phi(\mathbf{x}|\omega)\exp(-i\omega t)$ where, due to the infinitesimal translational invariance of the structure,

$$\phi(\mathbf{x}|\omega)=f^{>}(x_1,x_3|k\omega)e^{ikx_2},\quad x_3>\zeta(x_1) \tag{1a}$$

$$=f^{<}(x_1,x_3|k\omega)e^{ikx_2},\quad x_3<\zeta(x_1)\ . \tag{1b}$$

The equations satisfied by $f^{\gtrless}(x_1,x_3|k\omega)$ are

$$\left[\frac{\partial^2}{\partial x_1^2}+\frac{\partial^2}{\partial x_3^2}-k^2\right]f^{\gtrless}(x_1,x_3|k\omega)=0,\quad x_3\gtrless\zeta(x_1) \tag{2}$$

and we seek solutions in each medium that vanish as $(x_1^2+x_3^2)^{1/2}\to\infty$. In addition, $f^{\gtrless}(x_1,x_3|k\omega)$ satisfy the boundary conditions

$$f^{>}(x_1,x_3|k\omega)|_{x_3=\zeta(x_1)}=f^{<}(x_1,x_3|k\omega)|_{x_3=\zeta(x_1)}\ , \tag{3a}$$

$$\frac{\partial}{\partial n}f^{>}(x_1,x_3|k\omega)|_{x_3=\zeta(x_1)}=\epsilon(\omega)\frac{\partial}{\partial n}f^{<}(x_1,x_3|k\omega)|_{x_3=\zeta(x_1)}\ , \tag{3b}$$

at the solid vacuum interface, where $\partial/\partial n$ is the derivative along the normal to the surface at each point, directed from the vacuum into the solid,

$$\frac{\partial}{\partial n}=\{1+[\zeta'(x_1)]^2\}^{-1/2}\left[-\zeta'(x_1)\frac{\partial}{\partial x_1}+\frac{\partial}{\partial x_3}\right]\ . \tag{4}$$

We next introduce Green's function $G(x_1,x_3|x_1',x_3')$, which is the solution of

$$\left[\frac{\partial^2}{\partial x_1^2}+\frac{\partial^2}{\partial x_3^2}-k^2\right]G(x_1,x_3|x_1',x_3')=-4\pi\delta(x_1-x_1')\delta(x_3-x_3')\ , \tag{5}$$

and vanishes as $[(x_1-x_1')^2+(x_3-x_3')^2]^{1/2}$ tends to infinity. An explicit expression for $G(x_1,x_3|x_1',x_3')$ is

$$G(x_1,x_3|x_1',x_3')=2K_0(k[(x_1-x_1')^2+(x_3-x_3')^2]^{1/2}) \tag{6a}$$

$$=G(x_1',x_3'|x_1,x_3)\ , \tag{6b}$$

where $K_0(z)$ is modified Bessel function.

An application of Green's theorem to the region $x_3>\zeta(x_1)$, together with the boundary conditions at infinity, yields

$$\Theta(x_3-\zeta(x_1))f^{>}(x_1,x_3|k\omega)=\frac{1}{4\pi}\int_s ds'\left[\left[\frac{\partial}{\partial n'}G(x_1,x_3|x_1',x_3')\right]f^{>}(x_1',x_3'|k\omega)-G(x_1,x_3|x_1',x_3')\frac{\partial}{\partial n'}f^{>}(x_1',x_3'|k\omega)\right]\ , \tag{7a}$$

where $\Theta(z)$ is the Heaviside unit step function, the curve s is the intersection of the surface $x_3=\zeta(x_1)$ with the x_1x_3 plane, and ds' is the element of arc length along this curve. The application of Green's theorem to the region $x_3<\zeta(x_1)$, together with the boundary conditions at infinity, yields

$$\Theta(\zeta(x_1)-x_3)f^{<}(x_1,x_3|k\omega)=-\frac{1}{4\pi}\int_s ds'\left[\left[\frac{\partial}{\partial n'}G(x_1,x_3|x_1',x_3')\right]f^{<}(x_1',x_3'|k\omega)-G(x_1,x_3|x_1',x_3')\frac{\partial}{\partial n'}f^{<}(x_1',x_3'|k\omega)\right]. \tag{7b}$$

At this point we use the fact that $\zeta(x_1)$ is a single-valued function of x_1 to write

$$ds'=\{1+[\zeta'(x_1')]^2\}^{1/2}dx_1' \tag{8}$$

in Eqs. (7), and use the boundary conditions (3) in Eq. (7b). the result is the pair of equations

$$\Theta(x_3-\zeta(x_1))f^{>}(x_1,x_3|k\omega)=\frac{1}{4\pi}\int_{-\infty}^{\infty}dx_1'\left\{\left[\left[-\zeta'(x_1')\frac{\partial}{\partial x_1'}+\frac{\partial}{\partial x_3'}\right]G(x_1,x_3|x_1',x_3')\right]_{x_3'=\zeta(x_1')}f(x_1'|k\omega)-[G(x_1,x_3|x_1',x_3')]_{x_3'=\zeta(x_1')}g(x_1'|k\omega)\right\}, \tag{9a}$$

$$\Theta(\zeta(x_1)-x_3)f^{<}(x_1,x_3|k\omega)=-\frac{1}{4\pi}\int_{-\infty}^{\infty}dx_1'\left\{\left[\left[-\zeta'(x_1')\frac{\partial}{\partial x_1'}+\frac{\partial}{\partial x_3'}\right]G(x_1,x_3|x_1',x_3')\right]_{x_3'=\zeta(x_1')}f(x_1'|k\omega)-\frac{1}{\epsilon(\omega)}[G(x_1,x_3|x_1',x_3')]_{x_3'=\zeta(x_1')}g(x_1'|k\omega)\right\}, \tag{9b}$$

where

$$f(x_1|k\omega)=f^{>}(x_1,x_3|k\omega)|_{x_3=\zeta(x_1)}, \tag{10a}$$

$$g(x_1|k\omega)=\left[-\zeta'(x_1)\frac{\partial}{\partial x_1}+\frac{\partial}{\partial x_3}\right]\times f^{>}(x_1,x_3|k\omega)|_{x_3=\zeta(x_1)}. \tag{10b}$$

Equations (9a) and (9b) give the scalar potential in the vacuum and in the solid, respectively, in terms of the values of the scalar potential in the vacuum and its normal derivative evaluated at the interface.

We can obtain a pair of coupled, homogeneous, integral equations for $f(x_1|k\omega)$ and $g(x_1|k\omega)$ by setting $x_3=\zeta(x_1)+\epsilon$ in both equations, where ϵ is a positive infinitesimal:

$$f(x_1|k\omega)=\int_{-\infty}^{\infty}dx_1'[H(x_1|x_1')f(x_1'|k\omega)-L(x_1|x_1')g(x_1'|k\omega)], \tag{11a}$$

$$0=\int_{-\infty}^{\infty}dx_1'\left[H(x_1|x_1')f(x_1'|k\omega)-\frac{1}{\epsilon(\omega)}L(x_1|x_1')g(x_1'|k\omega)\right], \tag{11b}$$

with

$$H(x_1|x_1')=\frac{1}{4\pi}\left[\left[-\zeta'(x_1')\frac{\partial}{\partial x_1'}+\frac{\partial}{\partial x_3'}\right]\times G(x_1,x_3|x_1',x_3')\right]_{\substack{x_3'=\zeta(x_1'),\\ x_3=\zeta(x_1)+\epsilon}}, \tag{12a}$$

$$L(x_1|x_1')=\frac{1}{4\pi}[G(x_1,x_3|x_1',x_3')]_{\substack{x_3'=\zeta(x_1'),\\ x_3=\zeta(x_1)+\epsilon}}. \tag{12b}$$

When we eliminate $g(x_1|k\omega)$ from Eqs. (11) we obtain

$$\frac{1}{1-\epsilon(\omega)}f(x_1|k\omega)=\int_{-\infty}^{\infty}dx_1'H(x_1|x_1')f(x_1'|k\omega). \tag{13}$$

The solvability condition of this homogeneous integral equation yields the dispersion relation for the channel plasmons.

To solve Eq. (13) we replace the infinite range of integration by the finite range $(-(L+\frac{1}{2}\Delta x),(L+\frac{1}{2}\Delta x))$, where $L=N\Delta x$, and N is a large positive integer. We will define L and Δx more precisely below. We then introduce the set of $2N+1$ points $\{x_n\}$, where

$$x_n=n\Delta x,\quad n=-N,-N+1,\ldots,N-1,N \tag{14}$$

and rewrite Eq. (13) as

$$\frac{1}{1-\epsilon(\omega)}f(x_1|k\omega)=\sum_{n=-N}^{N}\int_{x_n-(1/2)\Delta x}^{x_n+(1/2)\Delta x}dx_1'H(x_1|x_1')f(x_1'|k\omega). \tag{15}$$

On the assumption that $f(x_1|k\omega)$ is a slowly varying function of x_1 in each interval $(x_n-\frac{1}{2}\Delta x, x_n+\frac{1}{2}\Delta x)$, we replace Eq. (15) by

$$\frac{1}{1-\epsilon(\omega)}f(x_1|k\omega) = \sum_{n=-N}^{N}\int_{x_n-(1/2)\Delta x}^{x_n+(1/2)\Delta x} dx_1' H(x_1|x_1')f(x_n|k\omega) \ . \quad (16)$$

On replacing x_1 by x_m, we obtain a homogeneous matrix equation for $f(x_m|k\omega)$:

$$\frac{1}{1-\epsilon(\omega)}f(x_m|k\omega) = \sum_{n=-N}^{N} H_{mn}(k)f(x_n|k\omega) \ , \quad (17)$$

where

$$H_{mn}(k) = \int_{(-1/2)\Delta x}^{(1/2)\Delta x} du\, H(x_m|x_n+u) \ . \quad (18)$$

The matrix element $H_{mn}(k)$ can be written in the form

$$H_{mn}(k) = \tfrac{1}{2}\delta_{mn} + \tfrac{1}{2}\mathcal{H}_{mn}(k) \quad (19)$$

to first order in Δx, where

$$\mathcal{H}_{mn}(k) = -\Delta x\frac{k^2}{\pi}\frac{K_1(k\{(x_m-x_n)^2+[\zeta(x_m)-\zeta(x_n)]^2\}^{1/2})}{k\{(x_m-x_n)^2+[\zeta(x_m)-\zeta(x_n)]^2\}^{1/2}}\{(x_m-x_n)\zeta'(x_n)-[\zeta(x_m)-\zeta(x_n)]\}, \quad m\neq n \quad (20a)$$

$$\mathcal{H}_{mn}(k) = \Delta x\frac{\zeta''(x_m)}{2\pi\gamma_m^2} \ , \quad (20b)$$

with

$$\gamma_m = \{1+[\zeta'(x_m)]^2\}^{1/2} \ . \quad (21)$$

The equation for $f(x_m|k\omega)$ finally takes the form

$$\frac{1+\epsilon(\omega)}{1-\epsilon(\omega)}f(x_m|k\omega) = \sum_{n=-N}^{N}\mathcal{H}_{mn}(k)f(x_n|k\omega) \ . \quad (22)$$

Let us now consider the eigenvalue equation

$$\lambda_s(k)f_s(x_m|k) = \sum_{n=-N}^{N}\mathcal{H}_{mn}(k)f_s(x_n|k) \ , \quad (23)$$

where $\lambda_s(k)$ is the sth eigenvalue of the matrix $\overleftrightarrow{\mathcal{H}}(k)$, while $f_s(x_m|k)$ is the corresponding eigenvector. From a comparison of Eqs. (22) and (23) we see that the dispersion relation for the channel plasmons becomes

$$\frac{1+\epsilon(\omega)}{1-\epsilon(\omega)} = \lambda_s(k) \ . \quad (24)$$

We can simplify Eq. (23) somewhat by the use of symmetry considerations. Because we have assumed that $\zeta(x_1)$ is an even function of x_1, we see from Eqs. (20) that the matrix $\mathcal{H}_{mn}(k)$ has the property

$$\mathcal{H}_{-m,-n}(k) = \mathcal{H}_{mn}(k) \ . \quad (25)$$

If we then introduce the functions $f^{(e)}(x_m|k)$ and $f^{(o)}(x_m|k)$ by

$$f^{(e)}(x_m|k) = \tfrac{1}{2}[f(x_m|k)+f(x_{-m}|k)] \ , \quad (26a)$$

$$f^{(o)}(x_m|k) = \tfrac{1}{2}[f(x_m|k)-f(x_{-m}|k)] \ , \quad (26b)$$

which are odd and even functions of x_m, respectively, the equations for these functions decouple, and we obtain

$$\lambda_s^{(e)}(k)f^{(e)}(x_m|k) = \sum_{n=0}^{N}\mathcal{H}_{mn}^{(e)}(k)f^{(e)}(x_n|k) \ , \quad m=0,1,\ldots,N \quad (27a)$$

and

$$\lambda_s^{(o)}(k)f^{(o)}(x_m|k) = \sum_{n=1}^{N}\mathcal{H}_{mn}^{(o)}(k)f^{(o)}(x_n|k) \ , \quad m=1,2,\ldots,N \quad (27b)$$

where $\{\lambda_s^{(e,o)}(k)\}$ are the eigenvalues, respectively, of the matrices $\overleftrightarrow{\mathcal{H}}^{(e,o)}(k)$ defined by

$$\mathcal{H}_{m0}^{(e)}(k) = \mathcal{H}_{m0}(k) \ , \quad (28a)$$

$$\mathcal{H}_{mn}^{(e)}(k) = \mathcal{H}_{mn}(k)+\mathcal{H}_{m,-n}(k), \quad 1\leq n\leq N \quad (28b)$$

and

$$\mathcal{H}_{mn}^{(o)}(k) = \mathcal{H}_{mn}(k)-\mathcal{H}_{m,-n}(k) \ . \quad (29)$$

The dimensionalities of the matrices that have to be diagonalized are now essentially half the dimensionality of the matrix in Eq. (23).

The dispersion relations for channel plasmons that correspond to electrostatic potentials that are even and odd functions of x_1 are

$$\frac{1+\epsilon(\omega)}{1-\epsilon(\omega)} = \lambda_s^{(e,o)}(k) \ , \quad (30)$$

respectively.

We also note from Eqs. (20) that when $\zeta(x_1)$ is replaced by $-\zeta(x_1)$, i.e., when a channel is replaced by the corresponding ridge, the elements of the matrix $\mathcal{H}_{mn}(k)$ change their signs. Hence the elements of the matrices $\mathcal{H}_{m,n}^{(e,o)}(k)$ also change their signs. This is equivalent to replacing $\lambda_s^{(e)}(k)$ and $\lambda_s^{(o)}(k)$ by their negatives in Eqs. (27a) and (27b), respectively. The consequence of the preceding results is that the dispersion relations for what might be called *ridge plasmons* that correspond to electrostatic potentials that are even and odd functions of x_1 are

$$\frac{1+\epsilon(\omega)}{1-\epsilon(\omega)} = -\lambda_s^{(e,o)}(k) \ , \quad (31)$$

respectively, where $\lambda_s^{(e,o)}(k)$ are still the eigenvalues of the matrices $\overleftrightarrow{\mathcal{H}}^{(e,o)}(k)$, respectively.

We now apply the preceding results to the determination of the dispersion curves of channel plasmons when the substrate is a metal and a polar dielectric medium.

Metallic substrate. In the case of a metallic substrate we take for $\epsilon(\omega)$ the form appropriate to a simple, free-electron metal,

$$\epsilon(\omega)=1-\frac{\omega_p^2}{\omega^2} , \tag{32}$$

where ω_p is the plasma frequency of electrons in the bulk of the metal. When the expression is used in Eqs. (30) and (31), we find that the frequency of the sth branch of the channel plasmon dispersion curve is given by

$$\omega_s^{(e,o)}(k)_{\rm CP}=\frac{\omega_p}{\sqrt{2}}[1+\lambda_s^{(e,o)}(k)]^{1/2} , \tag{33}$$

while the frequency of the sth branch of the corresponding ridge plasmon dispersion curve is

$$\omega_s^{(e,o)}(k)_{\rm RP}=\frac{\omega_p}{\sqrt{2}}[1-\lambda_s^{(e,o)}(k)]^{1/2} . \tag{34}$$

In these expressions $\omega_p/\sqrt{2}$ is recognized to be the frequency of the surface plasmon at a planar metal-vacuum interface.[12]

If we combine Eqs. (33) and (34), we obtain the relation (sum rule)

$$\omega_s^{(e,o)}(k)_{\rm CP}^2+\omega_s^{(e,o)}(k)_{\rm RP}^2=\omega_p^2 , \tag{35}$$

which can be used to obtain $\omega_s^{(e,o)}(k)_{\rm RP}$ from a knowledge of $\omega_s^{(e,o)}(k)_{\rm CP}$.

In Fig. 1 we present the branches of the dispersion curve for channel plasmons guided by a channel defined by a deep Gaussian surface profile function $\zeta(x_1)=-A\exp(-x_1^2/R^2)$, with $A/R=8$. In Fig. 1(a) we have drawn the branches of the dispersion curve which correspond to electrostatic potentials that are even functions of x_1, corresponding to the ten largest positive and four largest (in magnitude) negative eigenvalues of the matrix $\overleftrightarrow{\mathcal{H}}^{(e)}(k)$. In Fig. 1(b) the branches of the dispersion curve are drawn, which correspond to electrostatic potentials that are odd functions of x_1, corresponding to the ten largest (in magnitude) negative and four largest positive eigenvalues of the matrix $\overleftrightarrow{\mathcal{H}}^{(o)}(k)$. In both cases the curves corresponding to the remaining eigenvalues of $\overleftrightarrow{\mathcal{H}}^{(e,o)}(k)$ lie too close to the frequency $\omega=\omega_p/\sqrt{2}$ to be resolved on the scale of these plots. All modes are seen to be dispersive, due to the presence of characteristic lengths (A and R) in the structure supporting these waves. The frequencies of the branches all lie below ω_p, i.e., they lie in the stop band for bulk electromagnetic waves in the metallic portion of the structure. They all tend to $\omega_p/\sqrt{2}$ as $k\to\infty$. This is because as k increases the wavelength of the waves decreases as does the spatial extent of the wave transverse to the direction of propagation. Consequently, the wave sees a locally flatter and flatter surface, and its frequency approaches that of a surface plasmon at a planar metal-vacuum interface. The effect of increasing the ratio A/R is to increase the separation of the branches of the dispersion curve in frequency without, however, leading to branches outside the range $0<\omega<\omega_p$.

In Fig. 2 we present the branches of the dispersion curve for channel plasmons guided by a channel defined by the surface profile function

$$\zeta(x_1)=-d\frac{2\cosh^2(\beta w/4)}{\cosh(\beta w/2)+\cosh\beta x_1} , \tag{36}$$

corresponding to the six largest positive and six largest (in magnitude) negative eigenvalues of the corresponding matrices $\mathcal{H}_{mn}^{(e,o)}(k)$. In the limit as $\beta\to\infty$ this profile function defines the rectangular channel of width w and depth d whose surface profile function is given by

$$\zeta(x_1)=-d,\quad |x_1|<\frac{w}{2}$$
$$=0,\quad |x_1|>\frac{w}{2} . \tag{37}$$

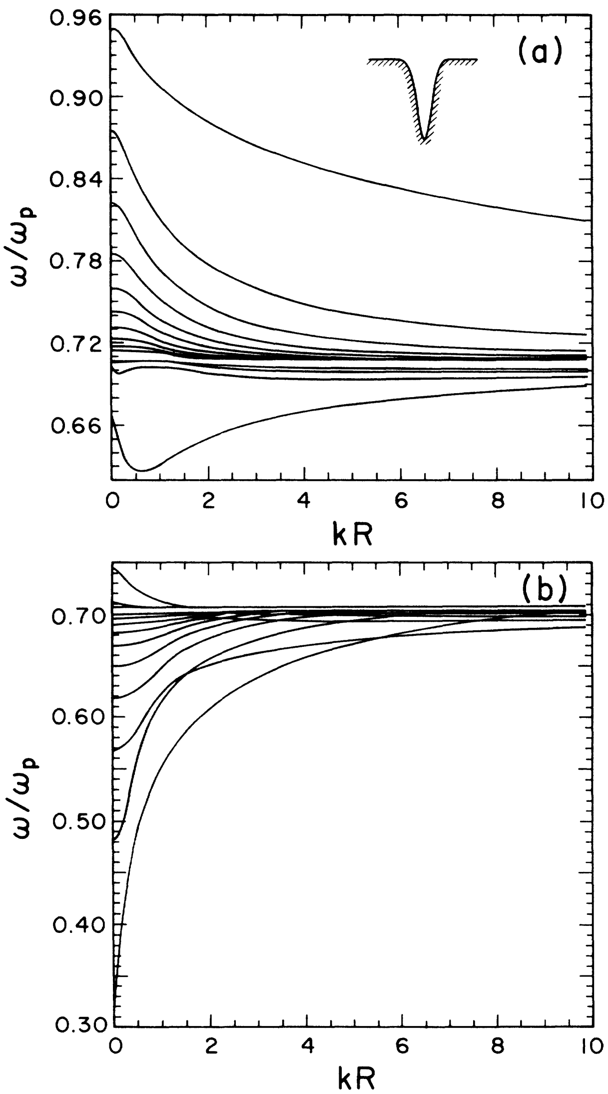

FIG. 1. Branches of the dispersion curve for channel plasmons guided by a channel on a metallic substrate defined by the surface profile function $\zeta(x_1)=-A\exp(-x_1^2/R^2)$. $A/R=8$, $L/R=22$, $N=450$. (a) Even modes; (b) odd modes.

As in the case of the Gaussian surface profile function the modes supported by this channel are all dispersive, and their frequencies tend to $\omega_p/\sqrt{2}$ as $k \to \infty$. For both modes of even and odd symmetry we see that for large k there is one pair of branches with frequencies above and below $\omega_p/\sqrt{2}$ that are well separated from this frequency of a surface plasmon at a planar metal-vacuum interface. The remaining branches have frequencies within about 1% of the latter frequency. Increasing the ratio d/w increases the separation of these two branches from the frequency $\omega_p/\sqrt{2}$.

Polar dielectric substrate. Let us now assume that the substrate is a polar dielectric medium of cubic symmetry that contains two ions in a primitive unit cell. It could be a semiconductor with the zinc-blende structure, or an ionic crystal with the rocksalt or cesium chloride structure. The dielectric constant $\epsilon(\omega)$ in this case is given by

$$\epsilon(\omega)=\epsilon_\infty \frac{\omega_L^2-\omega^2}{\omega_T^2-\omega^2} , \tag{38}$$

where ϵ_∞ is the optical frequency dielectric constant, while ω_L and ω_T are the frequencies of the longitudinal and transverse optical phonons, respectively. When Eq. (38) is used in Eqs. (30) and (31), we obtain for the frequency of the sth branch of the channel plasmon dispersion curve

$$\omega_s^{(e,o)}(k)_{\rm CP}=\omega_T \left[\frac{\epsilon_0+1+(\epsilon_0-1)\lambda_s^{(e,o)}(k)}{\epsilon_\infty+1+(\epsilon_\infty-1)\lambda_s^{(e,o)}(k)} \right]^{1/2} , \tag{39}$$

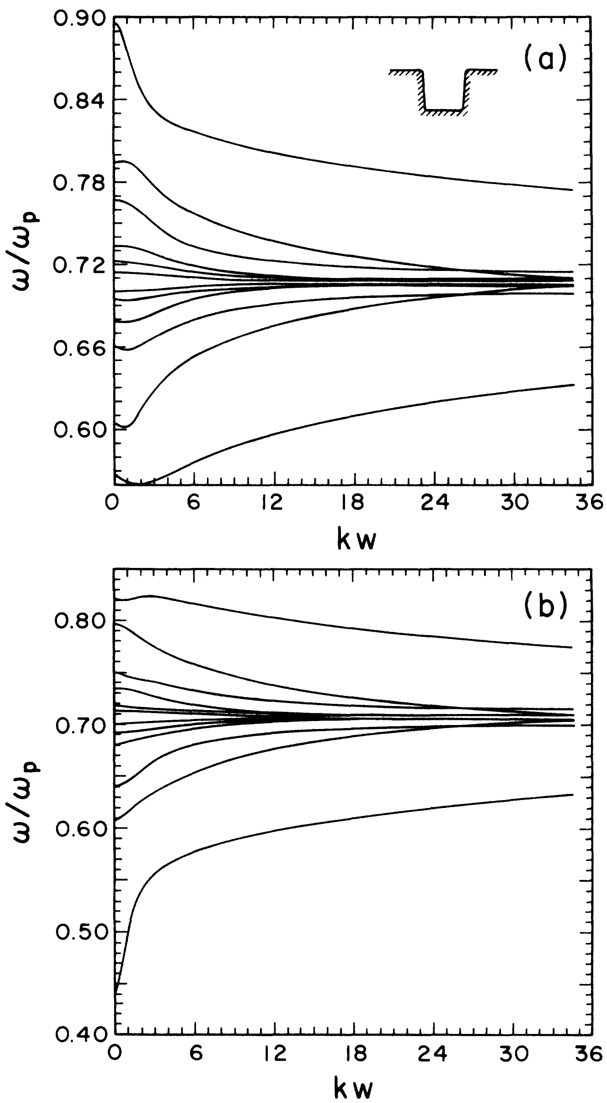

FIG. 2. Branches of the dispersion curve for channel plasmons guided by a channel on a metallic substrate defined by the surface profile function $\zeta(x_1)=-2d\cosh^2(\beta w/4)/[\cosh(\beta w/2)+\cosh\beta x_1]$. $d/w=1$, $\beta w=100$, $L/w=6$, $N=445$. (a) Even modes; (b) odd modes.

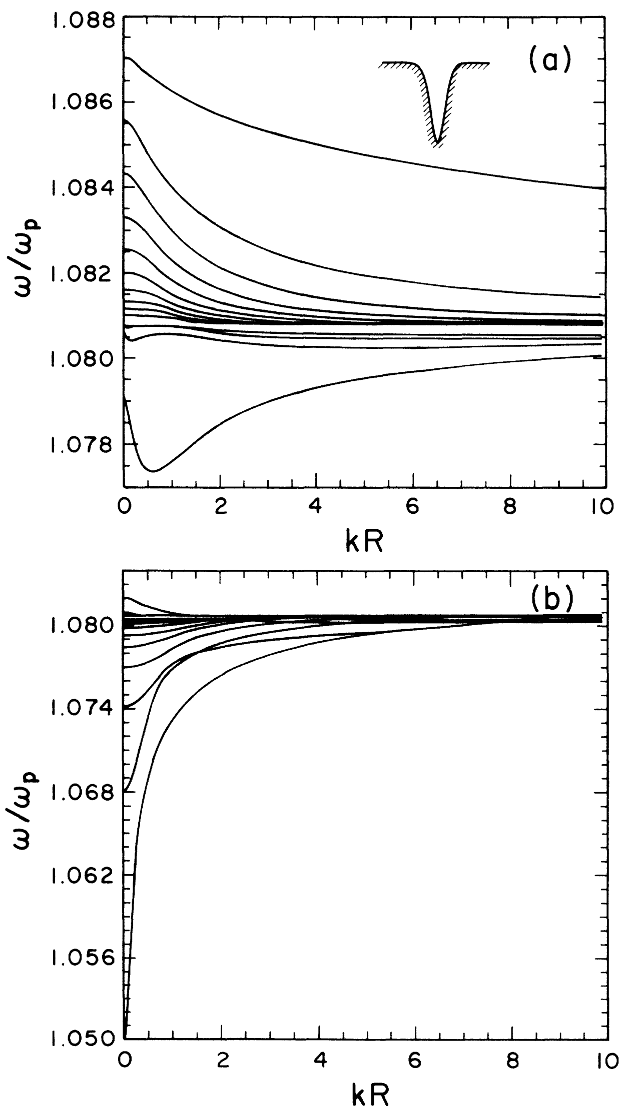

FIG. 3. Branches of the dispersion curve for channel plasmons guided by a channel on a GaAs substrate defined by the surface profile function $\zeta(x_1)=-A\exp(-x_1^2/R^2)$, with $A/R=8$, $\omega_L=297$ cm^{-1}, $\omega_T=273$ cm^{-1}, $\epsilon_0=12.9$, $\epsilon_\infty=10.9$, $L/R=22$, $N=450$. (a) Even modes; (b) odd modes.

while for the frequency of the sth branch of the corresponding ridge plasmon dispersion curve we obtain

$$\omega_s^{(e,o)}(k)_{\rm RP}=\omega_T\left[\frac{\epsilon_0+1-(\epsilon_0-1)\lambda_s^{(e,o)}(k)}{\epsilon_\infty+1-(\epsilon_\infty-1)\lambda_s^{(e,o)}(k)}\right]^{1/2} . \tag{40}$$

In Eqs. (39) and (40) ϵ_0 is the static dielectric constant, which is given by the Lyddane-Sachs-Teller relation $\epsilon_0=\epsilon_\infty(\omega_L^2/\omega_T^2)$. The frequency

$$\omega_{\rm SP}=\omega_T\left[\frac{\epsilon_0+1}{\epsilon_\infty-1}\right]^{1/2} \tag{41}$$

is the frequency of the surface plasmon at the planar interface between vacuum and a polar dielectric medium characterized by the dielectric constant (38).[13]

In Fig. 3 we present branches of the dispersion curve for channel plasmons guided by a channel defined by a deep Gaussian surface profile $\zeta(x_1)=-A\exp(-x_1^2/R^2)$, with $A/R=8$, cut into a GaAs surface. In the case of channel plasmons of even symmetry [Fig. 3(a)] we have plotted the curves corresponding to the ten largest positive and four largest (in magnitude) negative eigenvalues of the matrix $\overleftrightarrow{\mathcal{H}}^{(e)}(k)$. In the case of channel plasmons of odd symmetry [Fig. 3(b)] we have plotted the curves corresponding to the ten largest (in magnitude) negative and four largest positive eigenvalues of $\overleftrightarrow{\mathcal{H}}^{(o)}(k)$. The curves depicted in Figs. 3(a) and 3(b) are seen to be qualitatively very similar to their counterparts for channel plasmons guided by a channel of the same form cut into a metal substrate, depicted in Figs. 1(a) and 1(b), respectively.

Finally, in Fig. 4 we present branches of the dispersion curve for channel plasmons guided by a channel defined by the surface profile function (36) cut into a GaAs surface. The curves corresponding to the six largest positive and six largest (in magnitude) negative eigenvalues of the matrices $\overleftrightarrow{\mathcal{H}}^{(e,o)}(k)$ have been drawn. Again, the curves presented in Figs. 4(a) and 4(b) are qualitatively very similar to their counterparts for channel plasmons on a metal substrate depicted in Figs. 2(a) and 2(b), respectively.

Thus, in this paper we have defined a new kind of guided surface electromagnetic wave, a channel plasmon, and have obtained its dispersion relation. The channels supporting such a guided surface wave are multimode structures, but the frequencies of the modes they support are dispersive, and many are sufficiently separated from the frequency of a plasmon on a planar metal-vacuum surface that they should be distinguishable from the latter. The forms of the dispersion curves of these waves are largely insensitive to the nature of the substrate into which the guiding channel is cut, i.e., whether it is a metal or a dielectric medium. We have also examined the dispersion curves for the ridge plasmons associated with surface profile functions that are the negatives of those that give rise to channel plasmons. In the case of channels on metallic substrates, we have related the frequencies of ridge plasmons to those of channel plasmons. An interesting question, whose answer must be left to another investigation, is how the results of the present work are changed when, e.g., a channel cut into a metallic substrate is filled with a dielectric material. We hope that the results of this investigations will be useful in applications where the guiding of surface electromagnetic waves is required.

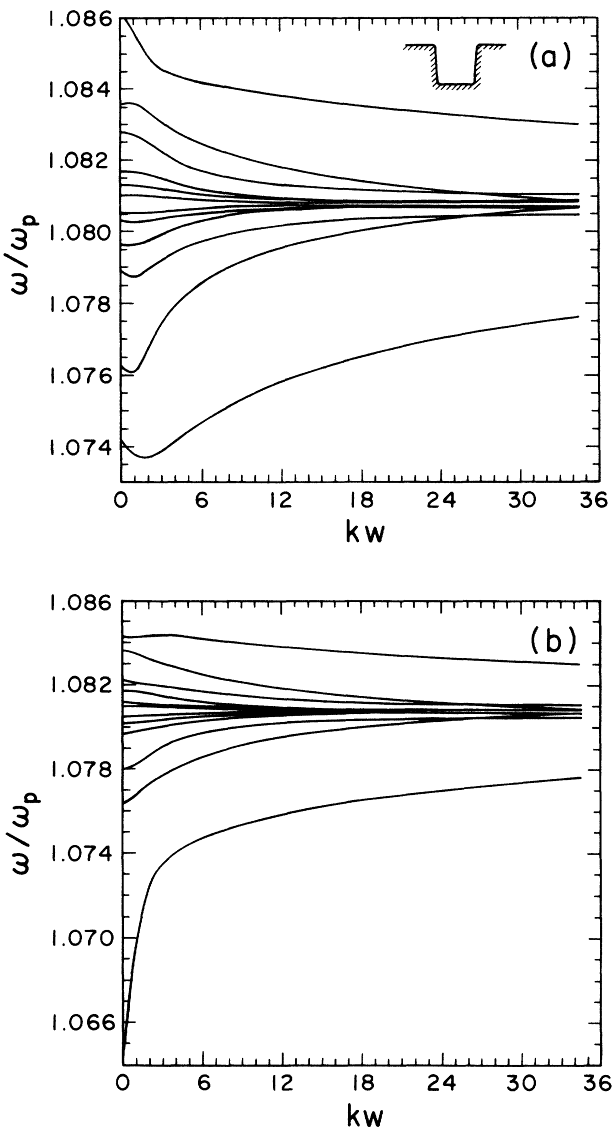

FIG. 4. Branches of the dispersion curve for channel plasmons guided by a channel on a GaAs substrate defined by the surface profile function $\zeta(x_1)=-2d\cosh^2(\beta w/4)/[\cosh(\beta w/2)+\cosh\beta x_1]$, with $d/w=1$, $\beta w=100$, $\omega_L=297$ cm^{-1}, $\omega_T=273$ cm^{-1}, $\epsilon_0=12.9$, $\epsilon_\infty=10.9$. $L/w=6$, $N=445$. (a) Even modes; (b) odd modes.

The authors are grateful to Professor A. Apkarian for discussions which stimulated the work reported here. This work was supported in part by Army Research Office Grant No. DAAL-88-K-0067. This research was also supported in part by the University of California, Irvine, through an allocation of computer time.

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