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Impurity-level transitions in two-dimensional magnetoplasmas

S.-R. Eric Yang
Division of Physics, National Research Council of Canada, Ottawa K1A OR6, Canada

A. H. MacDonald
Physics Department, Indiana University, Bloomington, Indiana 47405

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Donor and acceptor levels of δ-doped two-dimensional magnetoplasmas are investigated in the strong-magnetic-field limit. Disorder and impurity scatterings are treated self-consistently and the effects of nonlinear screening are included. We find that the frequency-dependent conductivity exhibits satellites near the main cyclotron resonance peak which are associated with impurities. Our results can explain some new features recently observed in far-infrared-magnetospectroscopy studies of shallow impurities in a two-dimensional electron gas.

The confining effect of a magnetic field has many qualitative consequences for a two-dimensional (2D) electron gas (2D EG) in a perpendicular magnetic field. For example, at zero magnetic field there are both bound and free single-electron eigenstates in the ideal 2D EG with a single ionized impurity, while only bound states occur in the presence of a perpendicular magnetic field.[1,2] The study of low-lying impurity levels has played a prominent role in optical studies[3,4] of heterostructures. Recently there has been considerable experimental interest[5-8] in impurity levels in a magnetoplasma. It is clear that a finite density of electrons can alter the impurity levels, because the electron–ionized-impurity interaction may be strongly screened.[9] However, the screening in a 2D EG is itself qualitatively changed by a magnetic field. In the simplest description, the screening wave vector in the 2D EG is proportional to the density of electronic states at the Fermi level. For an ideal 2D EG, the quantization of kinetic energy in units of $\hbar\omega_c$, where ω_c is the cyclotron frequency, produces a singular density of states and causes the screening wave vector to oscillate between infinity, when a Landau level (the set of states with one allowed kinetic-energy value) is partly filled, and zero when the Landau-level filling factor is an integer. This strongly singular behavior is largely mitigated by the disorder broadening of Landau levels and a realistic description of the screening of the electron-impurity interaction is made possible only by accounting for disorder.

In this article we present a study of impurity levels in a strictly 2D EG in a strong magnetic field. The interactions of electrons with an ionized impurity and with a disorder potential are treated self-consistently. Nonlinearities in the screening[10] of the electron-impurity interaction, which are found to be very important,[9] are accounted for by solving a set of self-consistent Hartree equations. We calculate the effect of impurities on the infrared conductivity in the dilute-impurity limit and find that our theory is able to explain recently observed[7] and as yet unexplained features in the cyclotron resonance (CR) spectrum of δ-doped 2D EG systems.

We begin by considering the case of an isolated ionized impurity. It is convenient to take the origin of coordinates to be at the impurity site and to choose the symmetric gauge $[\mathbf{A}=B(-y,x,0)/2]$ for the vector potential. In the absence of disorder and impurity scattering, the eigenfunctions of the Schrödinger equation,[2,11] $\phi_{nm}(\mathbf{r})$, may be labeled by a Landau-level label $[n;\varepsilon_{nm}=\hbar\omega_c(n+\frac{1}{2})]$ and an intra-Landau-level label $(m=0,1,2,\ldots)$. [The angular momentum of the state (n,m) is $\hbar(n-m)$.] In this paper we limit our attention to the case of a strong magnetic field where the mixing of Landau levels by impurity scattering or disorder scattering can be neglected. In the absence of disorder and screening, the effect of an ionized donor is simply to lift the Landau-level degeneracy by shifting the eigenvalue for each angular momentum state. Within each Landau level the shift is largest for the $s(n=m)$ eigenstate whose eigenfunction has a finite amplitude at the impurity site. The shift decreases as $m^{-1/2}$ for large m, since the large m orbitals are localized farther from the impurity site. (Explicit results are given in Ref. 2.)

In the presence of disorder, the angular momentum about the impurity site is no longer a good quantum number. We evaluate the disorder self-energy in the representation of disorder-free eigenstates defining

$$\Sigma_n(m,m';\omega)=\langle nm|\Sigma(\omega)|nm'\rangle =\int d\mathbf{r}d\mathbf{r}'\phi^*_{nm}(\mathbf{r})\Sigma(\mathbf{r},\mathbf{r}';\omega)\phi_{nm'}(\mathbf{r}') . \quad (1)$$

In the self-consistent Born approximation (SCBA), the disorder-averaged self-energy is given by

$$\Sigma_n(\mathbf{r},\mathbf{r}';\omega)=\sum_{m,m'}\phi_{nm}(\mathbf{r}')\phi_{nm'}(\mathbf{r}')G_n(m,m';\omega) \times\langle V_D(\mathbf{r})V_D(\mathbf{r}')\rangle , \quad (2)$$

where $G_n(m,m';\omega)$ is the Green's function.

We assume that the disorder is due to a set of identical scatterers with 2D Fourier transform $V_D(q)$, whose positions projected onto the 2D plane are randomly distributed. In this case the impurity-averaged Green's function[12] and self-energy are diagonal. The effect of disorder is to change the spectral distribution of the Green's function (i.e., the density of states) for each angular momentum channel of each Landau level from a δ function to a continuum. The diagonal self-energy matrix elements are given by

$$\Sigma_n(m;\omega)=\sum_{m'}G_n(m';\omega)Q_n(m,m')\,, \qquad (3)$$

where

$$Q_n(m,m')=\int\frac{d^2q}{(2\pi)^2}[L_n(|\mathbf{q}|^2/2)]^2 \times e^{-|\mathbf{q}|^2}|G^{mm'}(\mathbf{q})|^2 n_i|V_D(\mathbf{q})|^2\,, \qquad (4)$$

n_i is the density of disorder scatterers, and

$$G_n(m;\omega)=\frac{1}{\omega-\omega_n-V_n(m)-\Sigma_n(m;\omega)}\,. \qquad (5)$$

In Eq. (5), $\omega_n=\omega_c(n+\frac{1}{2})$ is the Landau-level energy and $V_n(m)$ is the m-dependent energy shift from the screened impurity potential which, as we discuss below, must be determined self-consistently. [$G^{mm'}(\mathbf{q})$ is defined in Ref. 2, $L_n(x)$ is a Laguerre polynomial, and $l=(\hbar c/eB)^{1/2}$ has been adopted as the unit of length.]

In the absence of the impurity potential, both $\Sigma_n(m,\omega)$ and $G_n(m,\omega)$ are independent of m. In this case the sum over m' in Eq. (3) may be performed and the usual SCBA result[13] for the self-energy and Green's function of a disorder-broadened Landau level is recovered. For any fixed set of $V_n(m)$'s Eqs. (3) and (5) must be solved self-consistently. The imaginary part of the self-energy for a given angular momentum channel has a contribution reflecting the rate at which electrons are scattered out of that channel by disorder. We see that electrons can scatter between channels only if they have spectral weight at the same energy. Thus the broadening tends to be smaller for angular momentum channels which are strongly shifted by the impurity potential.

The set of $V_n(m)$'s must be determined by satisfying another level of self-consistency.

$$V_n(m)=V_n^b(m)+2\sum_{n',m'}[\nu_{n'}(m')-\nu_{n'}]\Lambda_{nn'}(m,m')\,, \qquad (6)$$

where ν_n is the filling factor (per spin[14]) far from the impurity. In Eq. (6) $V_n^b(m)$ is the contribution to the energy shift in angular-momentum channel m from the interaction of electrons with the bare ionized impurity potential.[2] The additional contribution to $V_n(m)$ is the contribution to the potential from the screening charges induced by the impurity. The two-particle Coulomb matrix element,

$$\Lambda_{nn'}(m,m')=\langle nm;n'm'|e^2/\epsilon|\mathbf{r}_1-\mathbf{r}_2||nm;n'm'\rangle\,,$$

is given by

$$\Lambda_{n,n'}(m,m')=\int\frac{d^2q}{(2\pi)^2}\frac{2\pi e^2}{\epsilon|\mathbf{q}|}e^{-|\mathbf{q}|^2}L_n(|\mathbf{q}|^2/2)L_{n'}(|\mathbf{q}|^2/2)L_m(|\mathbf{q}|^2/2)L_{m'}(|\mathbf{q}|^2/2)\,, \qquad (7)$$

where ϵ is the background dielectric constant. In our calculations we have determined the self-consistent set of $V_n(m)$'s by iterating starting from the $V_n(m)$'s estimated in a linear screening approximation.[15] At each stage in the iteration procedure Eqs. (3) and (5) are solved as described above. The filling factors in each angular momentum channel can then be determined by integrating the corresponding spectral weight up to the Fermi level. A new estimate of the self-consistently screened impurity potential is then obtained from Eq. (6) and the process is repeated to convergence.

Figure 1 displays the spectral distribution in the presence of an ionized donor for $s(n=m)$ and $p^{\pm}(n-m=\pm1)$ channels and for a higher angular momentum channel for both $n=0$ and $n=1$ Landau levels. In each case the solid line and dashed lines show the result obtained neglecting and including the disorder coupling between angular momentum channels. These results were obtained for the representative case of $B=12.41$ T and $n=n_i=3\times10^{11}$ cm^{-2} with the disorder due entirely[16] to remote ionized donors set back by $d=0.6l$. (At this field and density, $\nu_0=\frac{1}{2}$ and the Fermi energy equals $\hbar\omega_c/2$.) As m increases, the effect of the impurity potential diminishes and the spectral distribution converges toward the SCBA result for a uniform system. The results for $m=9$ shown in Fig. 1 are barely distinguishable from those in the absence of the impurity. The spectral distribution is most strongly altered for the s and p channels. The s channel is shifted well down so that the main peak does not overlap with that from other angular momenta. As a result the main peak is not broadened by interchannel coupling and is narrower than that of other channels. The center of the p channel is shifted *upward* since the impurity is locally overscreened at the position where the p-channel wave functions are localized. This feature occurs even in a linear screening approximation and is a consequence of two-dimensional electrostatics.[9] We find that the charge on the ionized donor is, as expected, completely screened.[17]

Figure 2 shows the spectral distribution in the presence of an ionized acceptor. Because of the repulsive interaction between the electrons and ionized acceptors the s levels in this case are shifted upward. Again the p levels are overscreened and they are shifted *downward.* We emphasize that it is because of the kinetic-energy quantization and magnetic confinement effects in a strong magnetic field that acceptors, at least in the absence of disorder, produce bound states[18] in the conduction band.

In the dilute impurity limit we may evaluate the infrared conductivity by calculating its change for an isolated impurity and multiplying by the number of impurities. For an isolated impurity transitions with $\Delta n=1$ and $\Delta m=0$ are optically active. It is easy to show that the infrared conductivity is the sum over all m channels of the joint density of states between occupied states in the $n=0$ Landau level and the density of states in the $n=1$ Landau level. For a typical situation, such as that discussed above, only $m=0$ and $m=1$ channels are strongly altered by the presence of an impurity. Figure 3 shows the absorption spectrum for an impurity density of 3×10^{10} cm^{-2} for both donor and acceptor cases. In each case the main peak comes from transitions far away from the impurity sites. This peak is centered near the CR peak in the absence of impurities. For the case of donors the

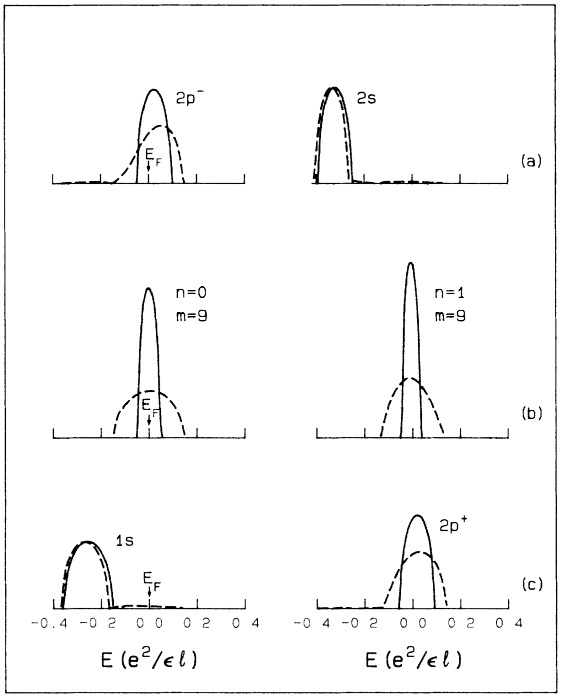

FIG. 1. Spectral distribution of the s and p channels and of the $m=9$ channel in the $n=0$ and $n=1$ Landau levels with an isolated ionized donor. The solid lines show results obtained by neglecting coupling between different angular momentum channels while the dashed lines include this coupling. Energies are in $e^2/\epsilon l$ units and are expressed, for both $n=0$ and $n=1$, relative to the Landau-level eigenenergy in the absence of disorder and the impurity. For these calculations $B=12.4$ T and $e^2/\epsilon l=21$ meV.

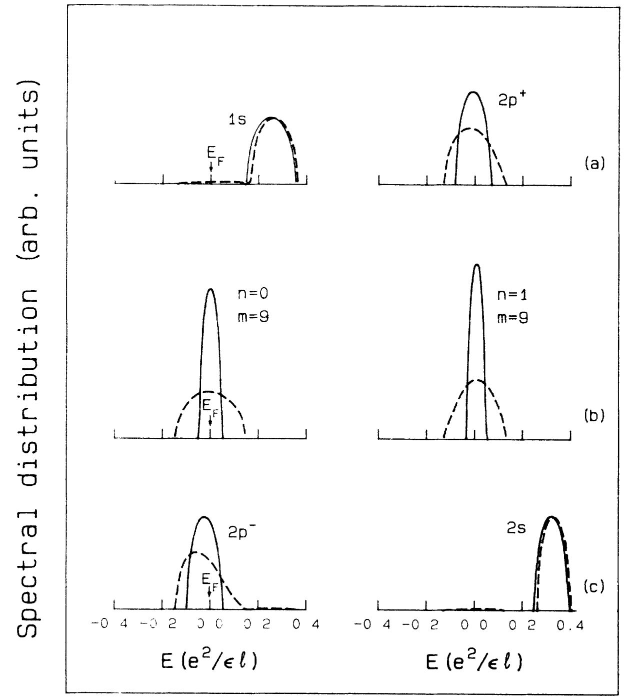

FIG. 2. As in Fig. 1 but for an isolated acceptor.

shoulder on the high energy side of the main CR peak comes from transitions between the downward shifted $1s$ channel to the upward shifted $2p^+$ channel, while the peak on the low-energy side of the main CR peak comes from transitions between the upward shifted $2p^-$ channel to the downward shifted $2s$ channel. (In undoped quantum wells this transition is not seen since $2p^-$ is not occupied.) In the case of acceptors, the situation is qualitatively different. The feature on the high-energy side of the CR peak comes from transitions between the *downward* shifted $2p^-$ channel to the *upward* shifted $2s$ channel. In this case the *upward* shift of the $1s$ channel is sufficiently large that it is nearly unoccupied and there is no feature on the low-energy side of the CR peak associated with $1s$-to-$2p^-$ transitions.

We believe that our theory provides an understanding of several qualitative features seen in magneto-optical studies of impurity levels at strong fields in magnetoplasmas. For donor impurities, features have been seen both on the high-frequency and on the low-frequency side of the main cyclotron resonance peak. Features on the high-energy side have been interpreted as $1s$-to-$2p^+$ transitions,[5] in agreement with our calculation. The origin of the features on the low-energy side[5,7] which we attribute here to $2p^-$ to $2s$ transitions, has not been understood. For acceptors no feature has been seen on the low-energy side, in agreement with our calculation. The feature on the high-energy side of the cyclotron-resonance transition in the acceptor case is associated, in our calculations, with $2p^-$ to $2s$ transitions.[19] Our calculations could be generalized to the case where $\nu > 2$ and improved by accounting for the finite thickness of the 2D EG layer and by including the coupling of different Landau levels by electron-impurity, electron-disorder, and electron-electron interactions. Even with these improvements, however, we believe that a quantitative understanding of the magneto-optical spectrum in a particular sample will be difficult to achieve since the details of the spectrum depend sensitively on the disorder scattering in the system which has uncontrolled contributions from interface roughness and from accidental doping.

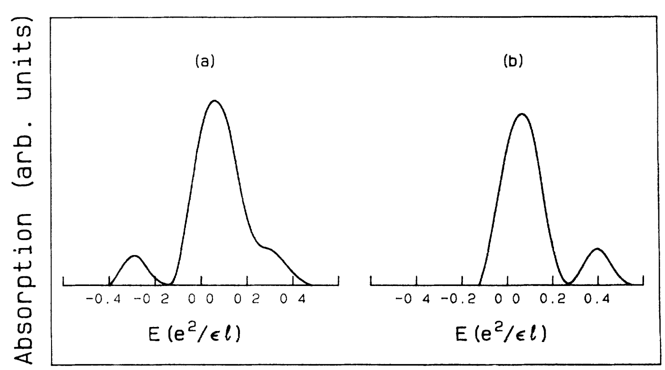

FIG. 3. Optical absorption spectrum in the strong magnetic-field limit for a dilute density of (a) ionized donors and (b) acceptors. The energies are in $e^2/\epsilon l$ units and are measured with respect to $\hbar\omega_c$.

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