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Possible vortex-glass transition in a model random superconductor

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We examine, with Monte Carlo simulations, the off-diagonal long-range correlations in a highly simplified model of a three-dimensional disordered superconductor in a magnetic field. Comparisons are made to the three-dimensional Ising and XY Edwards-Anderson spin-glass models. Somewhat surprisingly, the model superconductor behaves very much like the Ising spin glass, suggesting that it too may have a spin-glass-like ordered phase at nonzero temperature, namely the vortex-glass phase.

The order parameter in a superconductor is the complex scalar pair wave function ψ, which may also be viewed as a two-component real vector. One may then make analogies between the various ordered phases of a superconductor and magnetically ordered phases of two-component (XY) classical vector spins. The Meissner phase, which has uniform ψ, has a ferromagnetic phase as its analog. The Abrikosov vortex-lattice phase of a type-II superconductor is analogous to a type of antiferromagnetic phase with a fairly large unit cell, as occurs in *uniformly* frustrated XY models. Quenched random disorder destabilizes the Abrikosov vortex lattice, and the analogous magnetic system would be a *randomly* frustrated XY model and thus a spin glass. Here we study, via Monte Carlo simulations, a highly simplified model for such a disordered superconductor in a magnetic field to see whether it has a spin-glass-like ordered phase at nonzero temperature. Unfortunately, as generally occurs in studies of spin-glass-like models, the evidence we produce is not very strong, but on balance it seems to favor the existence of such a phase.

The analogy between spin-glass models and a random granular superconductor in a magnetic field appears to have been first pointed out by Shih, Ebner, and Stroud.[1] A granular superconductor with weak coupling between grains may be modeled as a Josephson-junction array. The coupling energy between two grains is $-J_{ij}\cos(\phi_i - \phi_j - A_{ij})$, where J_{ij} is the Josephson coupling, ϕ_i and ϕ_j are the phases of the pair condensate wave function [e.g., $\psi_j \propto \exp(i\phi_j)$] at the centers of the two grains, and A_{ij} is the line integral $A_{ij} \propto \int \mathbf{A}\cdot d\mathbf{l}$ of the vector potential $\mathbf{A}$ from one grain center to the other. In a random granular superconductor both J_{ij} and A_{ij} are of random magnitude.

To simplify the model, let us put the grains on a regular simple cubic lattice, assume nearest-neighbor couplings only, which are all of magnitude $J_{ij}=1$, and keep only the random-vector potential A_{ij}. The resulting model has the Hamiltonian

$$H = -\sum_{\langle ij\rangle} \cos(\phi_i - \phi_j - A_{ij}) , \qquad (1)$$

where the sum runs over all nearest-neighbor pairs. This simplification by putting the system on a regular lattice and keeping only the most convenient and essential aspect of the disorder, when applied to spin glasses results in the Edwards-Anderson model.[2] In fact, if the A_{ij} in (1) are all randomly equal to either 0 or π, then the model (1) *is* the Edwards-Anderson $\pm J$ XY spin glass, with randomly ferromagnetic ($A_{ij}=0$) or antiferromagnetic ($A_{ij}=\pi$) coupling between adjacent spins. The angle ϕ_j is that between the x-axis and the fixed-length two-component classical spin at site j, when (1) is viewed as an XY spin-glass model. This XY spin-glass (XYSG) model is one of the three systems we compare in this paper. It has been simulated by Jain and Young,[3] who find no sign of a finite-temperature ordering transition in the temperature range they studied.

The random-superconductor (RSC) model we will study here is (1) with the A_{ij} quenched random numbers uniformly distributed between 0 and 2π. The line integral, modulo 2π, of $\mathbf{A}\cdot d\mathbf{l}$ around any elementary plaquette of the lattice is then also uniformly distributed between 0 and 2π, which amounts to a *net* magnetic flux (modulo the flux quantum) passing through the plaquette that is uniformly distributed between zero and one flux quanta. We examine this as a highly simplified model for a random (but not necessarily granular) superconductor in a penetrating magnetic field. However, there are at least three properties of real random superconductors that are absent in this model: First, a real magnetic field is not static, but has its own fluctuations.[4] We are studying the extreme type-II limit where these fluctuations can be ignored, even though the fluctuations in the ϕ's are important. Second, the disorder in a real superconductor arises from random couplings J_{ij}. Any random component of the magnetic field in the sample just reflects these random couplings because the externally applied field is uniform on microscopic scales. However, once the A_{ij}'s are random there does not appear to be any reason to expect additional disorder in the J_{ij}'s to result in qualitatively different behavior for our model (1). Finally, in a real superconductor the applied field breaks the spatial symmetry of the system, since it is oriented in a particular direction. Our model (1) has on average the full symmetry of the underlying lattice. In high dimensionality ($d \gtrsim 6$) this difference does not appear to matter;[5] whether it matters for $d=3$ is an open question.

A nonrandom type-II superconductor in a magnetic field between H_{c1} and H_{c2} has an Abrikosov vortex-lattice phase at low temperatures.[6] Quenched disorder destroys the vortex-lattice correlations[7] beyond a finite lattice correlation length. The usual gauge-invariant off-diagonal correlation function is

$$G_0(\mathbf{r},\mathbf{r}') = \left[\left\langle \exp i\left\{\phi(\mathbf{r})-\phi(\mathbf{r}')-\int_{\mathbf{r}}^{\mathbf{r}'}\mathbf{A}\cdot d\mathbf{l}\right\}\right\rangle\right], \quad (2)$$

where $\phi(\mathbf{r})$ is the phase of the condensate wave function at $\mathbf{r}$ and the line integral is along the straight line from $\mathbf{r}$ to $\mathbf{r}'$. Note again that we are using units where the quantum of magnetic flux is 2π. The angular brackets in (2) represent a thermal average, while the square brackets are an average over realizations of the disorder. In the presence of disorder $G_0(\mathbf{r},\mathbf{r}')$ will reflect the short-range vortex lattice order for small $(\mathbf{r}-\mathbf{r}')$, but decay at long distances with some correlation length, ξ_0. Here we want to focus on the correlations for lengths longer than ξ_0. Thus our main justification for choosing a completely random A_{ij} is to reduce ξ_0 as much as possible, making the long-distance regime beyond ξ_0 as accessible as possible. More realistic models with more short-range order presumably have larger ξ_0, making the behavior for distances longer than ξ_0 less accessible.

If a random superconductor in a field has a vortex-glass phase,[8] the simplest fully averaged correlation function to show the off-diagonal long-range order (again neglecting fluctuations in $\mathbf{A}$) should be[9]

$$G(\mathbf{r},\mathbf{r}') = [\,|\langle \exp i\{\phi(\mathbf{r})-\phi(\mathbf{r}')\}\rangle|^2]\,. \quad (3)$$

For our lattice model this correlation function is

$$G_{ij} = [\,|\langle \exp i(\phi_i-\phi_j)\rangle|^2]\,. \quad (4)$$

The usual "spin-glass susceptibility,"[2] that diverges at the phase transition, is

$$\chi_{SG} = \sum_j G_{ij}\,. \quad (5)$$

The third model we will consider here is the Ising spin glass (ISG). This may be viewed (somewhat perversely) as again Hamiltonian (1) but now with the $\{A_{ij}\}$ *and* the $\{\phi_j\}$ *all* restricted to take on only the values 0 or π. More conventionally, the Hamiltonian for the Ising spin glass is[2]

$$H = \sum_{\langle ij\rangle} J_{ij}S_iS_j\,, \quad (6)$$

where $S_i = \pm 1$. Here we will take $J_{ij} = \pm\frac{1}{2}$ so that (1) and (6) have the same mean-field transition temperature and the same first terms in their high-temperature expansions for χ_{SG}.

In Fig. 1 we show a log-log plot of $(\chi_{SG}-1)$ vs T for the three models, all on simple cubic lattices. The Ising data are from Ogielski,[10] the XYSG data are from Jain and Young,[3] while our data for the random-superconductor model (1) were obtained from new Monte Carlo simulations. In the limit of high temperature, the three models have identical $(\chi_{SG}-1)\sim 1/T^2$. Below the mean-field transition temperature $T_c^{MF} = \sqrt{1.5} \simeq 1.22$, the two spin-glass models show significantly different temperature dependences of χ_{SG}. For the XY spin glass the behavior is quite consistent with a zero-temperature transition[3] with $\chi_{SG} \sim T^{-\gamma}$, $\gamma \simeq 5$. The Ising spin glass, on the other hand, shows a much stronger growth of χ_{SG} as the temperature

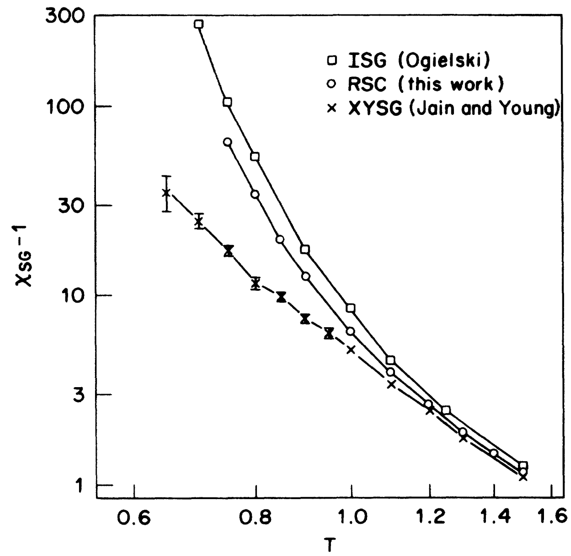

FIG. 1. Spin-glass susceptibility χ_{SG} vs temperature T for the three models compared here. For the RSC (this work) and the XYSG (Ref. 3), the error bars where not shown are smaller than the symbols. For the ISG, Ogielski (Ref. 10) did not estimate the statistical errors. The lines are guides to the eye. In the limit of high temperature the three models have identical χ_{SG} and, in this limit, the slope on this log-log plot of $(\chi_{SG}-1)$ vs T approaches -2.

is reduced, and is believed to have an ordering transition at[10,11] $T_c \simeq 0.6$ (remember we have changed the temperature scale by a factor of 2 from Refs. 10 and 11). Our random-superconductor model[1] shows a χ_{SG} between the two spin-glass models, but at low temperatures its temperature dependence is much closer to that of the Ising spin glass, indicating that it too may have an ordering transition with $0 < T_c \lesssim 0.6$.

The random superconductor in a magnetic field has symmetry that is intermediate between the two spin-glass models. The XY spin-glass Hamiltonian is invariant under global proper and improper rotations in spin space. For the random superconductor the only symmetries are global proper phase "rotations" $\phi_i \to \phi_i + \delta$. The global improper "rotation" $\phi_i \to -\phi_i$ is time reversal, which is not a symmetry due to the presence of the magnetic field. Thus the symmetry of the random superconductor in a field is a subgroup of that of the XY spin glass. The Ising spin glass has as its only symmetry global inversion in spin space which is the rotation $\phi_i \to \phi_i + \pi$. Thus the symmetry of the Ising spin glass is a subgroup of that of the random superconductor in a field. One might have guessed that the presence or absence of the continuous rotational symmetry is the primary source of the differences between the Ising and XY spin glasses. However, that would suggest that the random superconductor in a field, with its continuous rotational phase symmetry, would behave more like the XY spin glass. Figure 1 suggests otherwise.

A few words about our simulations: They were performed on various different computers, with the longest runs being done on the Connection Machine Model CM-2. Our program ran at a rate of approximately 100 Monte Carlo spin flips per processor per second on the

Connection Machine, and we used up to $2^{14} \simeq 1.6 \times 10^4$ processors in parallel. In all cases we ran multiple realizations of the disorder $\{A_{ij}\}$ in order to obtain objective estimates of the statistical error. Measurements were made over differing ranges of time in order to determine when equilibrium results were obtained. Samples of different sizes were run in order to study the finite-size effects (see below). The longest runs are those represented by the $T=0.75$ point in Fig. 1 where 2 replicas each of 23 realizations of the disorder for samples of size 32^3 with periodic boundary conditions were each run for $2^{18} \simeq 2.6 \times 10^5$ Monte Carlo steps per site. Samples up to size 16^3 were run for $0.85 \leq T \leq 1.1$, while 8^3 sufficed at higher temperatures to obtain χ_{SG}.

Some finite-size results comparable to those obtained by Bhatt and Young[11] for the Ising spin glass are shown in Fig. 2. For each realization of the disorder for samples of size L^3 and periodic boundary conditions we have simulated two replicas with the same $\{A_{ij}\}$ but different initial conditions and updated with different random numbers, monitoring the (complex) overlap

$$q = \sum_j \exp\{i(\phi_j^1 - \phi_j^2)\} , \tag{7}$$

where ϕ_j^α is the phase at site j in replica α. We then obtain $[\langle |q|^2 \rangle]$ and $[\langle |q|^4 \rangle]$, combining them into the "renormalized coupling constant"

$$g = 2 - \frac{[\langle |q|^4 \rangle]}{[\langle |q|^2 \rangle]^2} . \tag{8}$$

In the disordered phase for a large system, q will have a two-component (Req and Imq) Gaussian distribution with $g=0$. At zero temperature $|q| = L^3$ does not fluctuate so $g=1$. Bhatt and Young[11] calculated the analogous quantity for the Ising spin glass and found that g is essentially size independent for $T \lesssim T_c$, while g drops to zero with increasing L for $T > T_c$, as it must. Our results in Fig. 2 are quantitatively quite similar to theirs, again showing that, in the temperature and size range studied, our random superconductor model (1) behaves very much like the Ising spin glass, and may order at $T_c \lesssim 0.6$. Finite-size data for the XY spin glass are not presently available for comparison.

It has recently been suggested[8,9] that random type-II superconductors in a field may quite generally exhibit a vortex-glass ordered phase. Preliminary experimental evidence for a vortex-glass ordering transition has been obtained[12] for films of the cuprate superconductor $YBa_2Cu_3O_7$. Here we have examined what appears to be in some sense the simplest three-dimensional model that could exhibit such a vortex-glass ordered phase. Just as for three-dimensional Ising spin-glass models, it is not possible to obtain very solid evidence of the presence of the ordered phase (if it indeed exists), due to our inability to equilibrate the systems at low temperatures. However, the behavior of the random-superconductor model is quantitatively quite similar to that of the Ising spin glass, and for the latter system there is fairly good *experimental* evidence for the transition[13] that probes time scales far beyond those accessed in simulations. Thus we have obtained preliminary evidence that the three-dimensional random-superconductor model (1) may have a vortex-glass ordered phase at positive temperature.

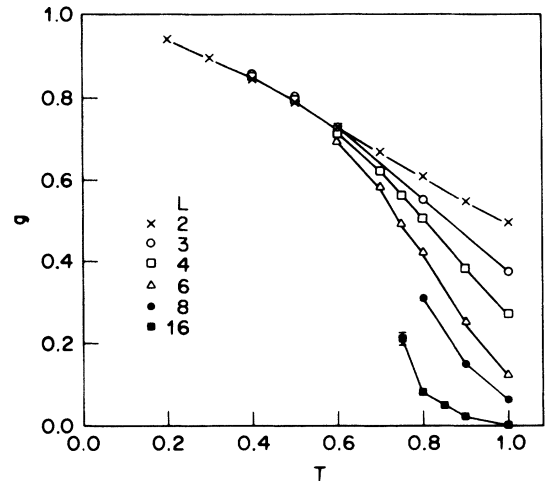

FIG. 2. "Renormalized coupling constant" $g(L,T)$ for samples of finite size L^3 of the random superconductor model (1). g is defined by Eq. (8). Up to a multiplicative temperature scale change, this figure is quantitatively very similar to those of Bhatt and Young (Ref. 11) for the three-dimensional $\pm J$ (their Fig. 7) and Gaussian (their Fig. 10) Ising spin glasses. The lines are guides to the eye and again the error bars, where not shown, are smaller than the symbols.

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