Energy gap in layered superconductors

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The anisotropy of the gap in layered compounds is studied in the framework of the BCS model, taking into account the interlayer and different intralayer couplings in the system. The compounds with alternating normal (N) and superconducting (S) layers are studied in detail, the N layers being assumed to have zero pairing intralayer interaction. In S-N systems with Josephson coupling of layers, there are two quite different gaps and also different temperature scales of the superconducting properties of the S and N layers. The results are used to discuss the behavior of high-T_c superconductors and possible artificial superlattices based on these compounds.

I. INTRODUCTION

High-temperature superconductors based on copper oxides have layered crystal structures and strong electron anisotropy. This anisotropy was observed very clearly in the magnetic properties of the superconducting phase; i.e., the critical magnetic field and London penetration depth depend strongly on the orientation of the field with respect to the layers' plane (ab). Quantitatively the electron anisotropy is characterized by the ratio $2t/\varepsilon_F$ where t is the hopping integral between layers while ε_F is the Fermi energy of electrons inside the layers. The electron anisotropy is about 25 in $YBa_2Cu_3O_7$ and $La_{2-x}Sr_xCuO_4$ while it is as high as 10^4-10^5 in $Bi_2Sr_2CaCu_2O_8$.[1-4]

Recently, tunneling[5-8] and optical[9] measurements were made which showed the possible anisotropy of the energy gap in high-T_c layered superconductors. In the case of $YBa_2Cu_3O_7$ the ratio $2\Delta(0)/T_c$ is 8 for **E** polarized in the (ab) plane, while it is about 3 for the polarization along the c axis [perpendicular to the (ab) plane]. Here $\Delta(0)$ is the energy gap obtained at very low temperatures $T \ll T_c$. According to the Raman scattering measurements, the suppression of the electron scattering at low frequencies at temperatures below T_c is anisotropic in the same manner and, besides, gives some evidence in favor of gapless superconductivity in $YBa_2Cu_3O_7$ and $Tl_2Ca_2Ba_2Cu_3O_{10}$.[10,11]

The magnetic anisotropy of the layered superconductors is due directly to the anisotropy of the Fermi surface. The complete description of the anisotropic magnetic properties of superconductors near T_c is given by the Ginzburg-Landau model with anisotropic effective mass (if the anisotropy is not very strong) or by the Lowrence-Doniach model in the limit of very high anisotropy (in case of Josephson coupling of the layers).[12,13] However, the question concerning the anisotropy of the energy gap in the layered superconductors is still unresolved. The essential point is the fact that the standard BCS model with anisotropic Fermi surface but isotropic electron coupling gives an isotropic energy gap. So the key point here is the coupling but not the electron band structure itself.

The problem of the energy gap anisotropy in the layered superconductors is interesting also from the point of view of artificial superconducting superlattices with a superstructure on the atomic scale. Intercalated layered compounds and high-T_c superlattices of $YBa_2Cu_3O_7$/$DyBa_2Cu_3O_7$ and $(YBa_2Cu_3O_7)_n/(PrBa_2Cu_3O_7)_m$ type[14,15] are counted among these systems. In fact, the layered high-T_c superconductors can be considered as natural superlattices which consist of layers with different electronic properties. In $YBa_2Cu_3O_7$, there are layers with CuO chains as well as the standard CuO_2 layers. The electrons of CuO chains give also a contribution to the Fermi surface states. In Bi and Tl high-T_c compounds, the layers BiO and TlO can contribute to the Fermi surface states.[16]

We study below the possible anisotropy of the energy gap in layered compounds in the framework of a standard BCS model assuming isotropic coupling of the electrons inside the layers [in the (ab) plane]. We focus our attention on the anisotropy of the electron coupling which is inherent to the layered crystals (interlayer coupling and different coupling in different layers). We will show in Secs. II and III that in crystals with equivalent layers the ansiotropy of the gap is remarkable if the anisotropy of electron motion is not very strong (i.e., the ratio t/ε_F is not very small). In Secs. IV–VII we consider systems which consist of layers with different coupling of electrons inside the layers, i.e., the systems of S-S' or S-N type, according to the terminology of superlattices. We assume that different coupling is possible inside the layers CuO_2 and CuO in $YBa_2Cu_3O_7$, or inside the layers CuO_2 and BiO or TlO in Bi and Tl compounds. We assume that the pairing coupling in CuO_2 layers is strong (S lay-

ers) while in CuO or BiO and TlO layers it may be weaker or completely absent (S' or N layers). The behavior of the system with nonequivalent layers depends strongly on the ratio t/T_c. We will show that in the Josephson coupling limit ($t \ll T_c$) the optical properties do depend on the orientation of the electric field with respect to the layers if the pairing coupling inside the layers varies from layer to layer.

We consider in Secs. V and VI also the density of states at low temperatures as well as temperature dependence of London penetration depth in the Josephson S-N system. The obtained results allow us to choose the model (S-S, S-S', or S-N) which describes the properties of highly anisotropic Bi and Tl compounds as well as the artificial superlattices.

II. CRYSTALS WITH ONE LAYER IN THE UNIT CELL

We shall use a Hamiltonian in the discrete representation for the description of electron motion along the c axis and in momentum representation to describe the electron motion in the (ab) plane. For this purpose we introduce the Wannier functions $w_{ni}(\mathbf{r})$ for an atom i in the layer n while the electron Bloch functions with quasimomentum $\mathbf{p}$ are used inside the layers

$$\varphi_{n,\mathbf{p}}(\mathbf{r})=(1/\sqrt{S})\sum_i e^{i\mathbf{p}\cdot\boldsymbol{\rho}_i}w_{ni}(\mathbf{r}),\quad \mathbf{r}=(\boldsymbol{\rho},z)\ , \tag{1}$$

where $\boldsymbol{\rho}_i$ are the coordinates (x,y) of atom i in the layer and S is the area of the layer. We write the electron field operator using the states $(n,\mathbf{p})$

$$\psi_\sigma(\mathbf{r})=\sum_{\mathbf{p},n}\varphi_{n,\mathbf{p}}(\mathbf{r})a_{n,\mathbf{p},\sigma}\ . \tag{2}$$

The function $\varphi_{np}(\mathbf{r})$ is delocalized with respect to coordinates x,y inside the layer n but localized in space in the coordinate z (along the c axis).

We start with the standard Hamiltonian

$$H=\int d\mathbf{r}\left[\sum_\sigma \Psi_\sigma^+(\mathbf{r})\left[\frac{p^2}{2}+V_0(\mathbf{r})\right]\Psi_\sigma(\mathbf{r})+\tfrac{1}{2}\Psi_\sigma^+(\mathbf{r})\Psi_{-\sigma}^+(\mathbf{r}')V(\mathbf{r},\mathbf{r}')\Psi_{-\sigma}(\mathbf{r}')\Psi_\sigma(\mathbf{r})\right]\ , \tag{3}$$

where $V_0(\mathbf{r})$ is the lattice potential and $V(\mathbf{r},\mathbf{r}')$ is the pairing potential. We obtain, using the discrete representation (2), the BCS-type Hamiltonian

$$\begin{aligned}
&H=H_0+H_{\rm int},\quad H_{\rm int}=H_{\rm int}^{(0)}+H_{\rm int}^{(1)}+H_{\rm int}^{(2)}\ ,\\
&H_0=\sum_{n,\mathbf{p},\sigma}[\varepsilon(\mathbf{p})-\varepsilon_F]a_{n\mathbf{p}\sigma}^\dagger a_{n\mathbf{p}\sigma}\\
&\quad +t(a_{n+1,\mathbf{p},\sigma}^\dagger+a_{n-1,\mathbf{p},\sigma}^\dagger)a_{n\mathbf{p},\sigma}\ ,\\
&H_{\rm int}^{(0)}=\frac{\lambda_0}{2N(0)}\sum_{n,\mathbf{p},\mathbf{p}',\sigma}a_{n,\mathbf{p},-\sigma}^\dagger a_{n,-\mathbf{p},\sigma}^\dagger \times a_{n,\mathbf{p}',\sigma}a_{n,\mathbf{p}',-\sigma}\ ,\\
&H_{\rm int}^{(1)}=\frac{\lambda_1}{2N(0)}\sum_{n,\mathbf{p},\mathbf{p}',\sigma,\pm}a_{n,\mathbf{p},\sigma}^\dagger a_{n,-\mathbf{p},-\sigma}^\dagger a_{n,\mathbf{p}',-\sigma} \times a_{n\pm1,-\mathbf{p}',\sigma}+\text{c.c.}\ ,
\end{aligned} \tag{4}$$

$$\begin{aligned}
&H_{\rm int}^{(2)}=\frac{\lambda_2}{2N(0)}\sum_{n,\mathbf{p},\mathbf{p}',\sigma,\pm}a_{n,\mathbf{p},\sigma}^\dagger a_{n\pm1,-\mathbf{p},-\sigma}^\dagger \times a_{n\pm1,\mathbf{p}',-\sigma}a_{n,-\mathbf{p}',\sigma}+\text{c.c.}\ ,\\
&\lambda_0=N(0)\int d\mathbf{r}\,d\mathbf{r}'\varphi_{n,p}^*(\mathbf{r})\varphi_{n,-\mathbf{p}}^*(\mathbf{r}')\varphi_{n,\mathbf{p}'}(\mathbf{r}') \times\varphi_{n,-\mathbf{p}'}(\mathbf{r})V(\mathbf{r},\mathbf{r}')\ ,\\
&\lambda_1=N(0)\int d\mathbf{r}\,d\mathbf{r}'\varphi_{n,\mathbf{p}}^*(\mathbf{r})\varphi_{n,-\mathbf{p}}^*(r')\varphi_{n,\mathbf{p}'}(\mathbf{r}') \times\varphi_{n+1,-\mathbf{p}'}(\mathbf{r}')V(\mathbf{r},\mathbf{r}')\ ,\\
&\lambda_2=N(0)\int d\mathbf{r}\,d\mathbf{r}'\varphi_{n,\mathbf{p}}^*(\mathbf{r})\varphi_{n,-\mathbf{p}'}(\mathbf{r}) \times\varphi_{n+1,-\mathbf{p}}^*(\mathbf{r}')\varphi_{n+1,\mathbf{p}'}(\mathbf{r}')V(\mathbf{r},\mathbf{r}')\ ,\\
&t=\int d\mathbf{r}\varphi_{n+1,\mathbf{p}}^*(r)\varphi_{n,\mathbf{p}}(\mathbf{r})[\mathbf{p}^2/2m+V_0(\mathbf{r})]\ ,
\end{aligned} \tag{4a}$$

where $N(0)$ is the density of states at Fermi energy inside the layer [$N(0)=m_\parallel/2\pi$, $m_\parallel$ is the effective mass along the layer]; in (4a) an average over $\mathbf{p},\mathbf{p}'$, with $\mathbf{p}=|\mathbf{p}'|=p_F$ is assumed; $\lambda_0,\lambda_1,\lambda_2$ are independent of n in this section, but they do depend on n in the case of many layers in a unit cell. Deriving the Hamiltonian (4), we use the tight binding approximation for the electron band structure along the c axis, so we preserve the terms which are of zeroth and first order in the overlap of the Wannier functions. The parameters of the model are shown schematically in Fig. 1. The term $H_{\rm int}^{(0)}$ describes the coupling inside the layers and it gives the standard BCS result (isotropic gap) without the other interaction terms. We can estimate λ_1/λ_0 as being roughly t/ε_F. The term $H_{\rm int}^{(2)}$ does not contain such an overlap of Wannier functions, but λ_2 is small in comparison with λ_0 due to the spatial separation of different layers, $\lambda_2/\lambda_0\sim\eta$ and $\eta<1$. The Hamiltonian (4) without the term $H_{\rm int}^{(1)}$ was studied in Refs. 17 and 18; the result for the energy gap was the dependence of the gap on momentum q along the c direction:

$$\Delta(q)=\Delta_0+\delta\cos q\ , \tag{5}$$

where $\delta/\Delta_0\sim(\lambda_2/\lambda_0)(t/\varepsilon_F)\sim\eta t/\varepsilon_F$. We will show below that the term $H_{\rm int}^{(1)}$ gives the same expression (5) with additional contribution in δ of the order of $(\lambda_1/\lambda_0)\Delta_0\sim(t/\varepsilon_F)\Delta_0$. For this reason, the term $H_{\rm int}^{(1)}$ is essential.

The Hamiltonian (4) gives the following Gor'kov equations (in Matsubara frequencies):

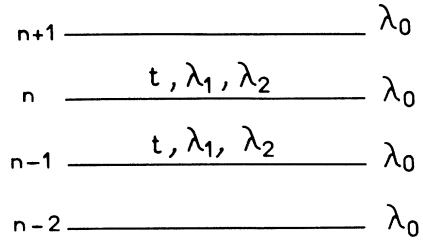

FIG. 1. The parameters of the model "one layer in unit cell."

$$(-i\omega+\xi)G(\omega,\mathbf{p},n-n')+\lambda_0F(0)F^{\dagger}(\omega,\mathbf{p},n-n')+\sum_{\alpha=\pm1}[tG(\omega\mathbf{p},n+\alpha-n')+\lambda_1F(\alpha)F^{\dagger}(\omega,\mathbf{p},n-n')$$
$$+\lambda_1F(0)F^{\dagger}(\omega,\mathbf{p},n+\alpha-n')+\lambda_2F(\alpha)F^{\dagger}(\omega,\mathbf{p},n+\alpha-n')]$$
$$=-\delta(n-n')\ ,$$
$$(i\omega+\xi)F^{\dagger}(\omega,\mathbf{p},n-n')-\lambda_0F(0)G(\omega,\mathbf{p},n-n')+\sum_{\alpha=\pm1}[tF^{\dagger}(\omega,\mathbf{p},n+\alpha-n')-\lambda_1F(0)G(\omega,\mathbf{p},n-n')$$
$$-\lambda_1F(\alpha)G(\omega,\mathbf{p},n-n')-\lambda_2F(\alpha)G(\omega,\mathbf{p},n+\alpha-n')]=0\ ,$$
$$\xi=\varepsilon(\mathbf{p})-\varepsilon_F\ ,$$
$$F(\alpha)=T\sum_{\omega,\mathbf{p}}F(\omega,\mathbf{p},\alpha)\ . \tag{6}$$

Making a Fourier transformation from the descrete variables n to the quasimomentum q, $-\pi\le q\le\pi$, we obtain

$$F^{\dagger}_{\omega p}(q)=\Delta(q)/[\omega^2+\bar{\xi}^2+\Delta^2(q)]\ ,$$
$$G_{\omega,p}(q)=-(i\omega+\bar{\xi})/[\omega^2+\bar{\xi}^2+\Delta^2(q)]\ ,$$
$$\bar{\xi}=\xi+2t\cos q\ , \tag{7}$$
$$\Delta(q)\equiv\Delta_0+\delta\cos q=\lambda_0F(0)+2\lambda_1F(1)+2\lambda_2F(1)+2[\lambda_1F(0)+\lambda_2F(1)]\cos q\ .$$

The self-consistency equations are

$$F(0)=[T/(2\pi)^3N(0)]\sum_{\omega}\int d\mathbf{p}\,dq\frac{\Delta(q)}{\omega^2+\bar{\xi}^2+\Delta^2(q)}\ ,$$
$$F(1)=F(-1)=[T/(2\pi)^3N(0)]\sum_{\omega}\int d\mathbf{p}\,dq\frac{\Delta(q)\cos q}{\omega^2+\bar{\xi}^2+\Delta^2(q)}\ . \tag{8}$$

According to (8), the value $F(1)$ is at least first order in (t/ε_F) because $F(1)=0$ at $t=0$ and $\lambda_1=0$. Thus we obtain estimates of the contributions of $H^{(1)}_{\rm int}$ and $H^{(2)}_{\rm int}$ to δ mentioned earlier. From (8) we get the value of T_c and the temperature dependence $\Delta_0(T)$; they coincide with standard BCS results with accuracy $(t/\varepsilon_F)^2$ at small values of the ratio (t/ε_F).

The quasiparticle density of states has the form

$$\rho(\varepsilon)=\pi^{-1}N(0)\mathrm{Re}\int_0^{\pi}\varepsilon/\sqrt{\varepsilon^2-(\Delta_0+\delta\cos q)^2}\ . \tag{9}$$

It is logarithmically singular at $\varepsilon=\Delta_0+\delta$ and becomes zero at $\varepsilon<\Delta_0-\delta$. With accuracy $(t/\varepsilon_F)^2$, the London penetration depth is given by the standard BCS expression and its anisotropy is determined by the anisotropy of the Fermi surface only. The anisotropy of the gap is proportional to $\delta\sim\Delta_0\,(t/\varepsilon_F)$ and it is small if t/ε_F is small (i.e., if the magnetic anisotropy is large). The term $\delta=0$ in an isotropic system (where t is order ε_F). We came to the conclusion that the gap anisotropy δ/Δ_0 in a system with one layer in the cell reaches its maximum at some intermediate values of the electron anisotropy $t/\varepsilon_F\sim\frac{1}{2}-\frac{1}{4}$. The effect of interlayer coupling may be essential for $YBa_2Cu_3O_7$ type compounds with intermediate magnetic anisotropy. In the highly anisotropic Bi and Tl compounds the anisotropy of the gap caused by interlayer coupling is negligible.

III. TWO EQUIVALENT LAYERS IN THE UNIT CELL

Let us consider a system which consists of equivalent layers with alternating spatial separation between neighboring layers. The resonance integral alternates $(t,t't,\ldots)$, as well as the parameter λ_1:$(\lambda_1,\lambda_1',\lambda_1,\ldots)$. The parameters of the system under consideration are shown schematically in Fig. 2. Again the remarkable difference from the results of the standard BCS model is obtained if the value t/ε_F is not very small. We consider below the case $t,|t-t'|\gg T_c$. We omit the term $H^{(2)}_{\rm int}$ because it does not change the qualitative behavior of the system.

Let us introduce the Gor'kov functions $G_{\alpha,\beta}(\omega,\mathbf{p},n,n')$ and $F^{\dagger}_{\alpha,\beta}(\omega,\mathbf{p},n,n')$ where indexes $\alpha,\beta=1,2$ are the numbers of the layers in a unit cell and n is the number of unit cells. We obtain then the equations for the Fourier components $G_{\alpha\beta}(\omega,\mathbf{p},q)$ and $F^{\dagger}_{\alpha,\beta}(\omega,\mathbf{p},q)$ [assuming $F(0)$ and $F(1)$ are real]:

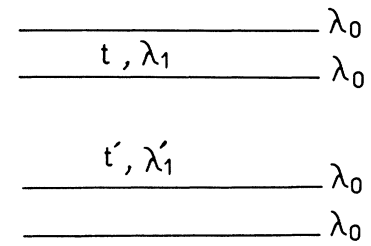

FIG. 2. The parameters of the model "two equivalent layers in unit cell."

$$\begin{bmatrix} \omega_- & -t(q) & \Delta_1 & \bar{\Delta}(q) \\ -t^*(q) & \omega_- & \bar{\Delta}^*(q) & \Delta_2 \\ \Delta_1 & \bar{\Delta}(q) & \omega_+ & t(q) \\ \bar{\Delta}^*(q) & \Delta_2 & t^*(q) & \omega_+ \end{bmatrix} \begin{bmatrix} G_{1\alpha}(\omega,\mathbf{p},q) \\ G_{2\alpha}(\omega,\mathbf{p},q) \\ F^{\dagger}_{1\alpha}(\omega,\mathbf{p},q) \\ F^{\dagger}_{2\alpha}(\omega,\mathbf{p},q) \end{bmatrix} = \begin{bmatrix} \delta_{1\alpha} \\ \delta_{2\alpha} \\ 0 \\ 0 \end{bmatrix}, \tag{10}$$

$$\omega_{\pm}=i\omega\pm\xi, \quad t(q)=t+t'e^{iq},$$

where

$$\Delta_1=\Delta_2=\Delta_0=[T/N(0)]\sum_{\omega,\mathbf{p},q}[\lambda_0 F_{11}(\omega,\mathbf{p},q)+(\lambda_1+\lambda_1'e^{iq})F_{21}(\omega,\mathbf{p},q)+(\lambda_1+\lambda_1'e^{-iq})F_{12}(\omega,\mathbf{p},q)],$$

$$\bar{\Delta}(q)=[T/N(0)](\lambda_1+\lambda_1'e^{iq})\sum_{\omega,\mathbf{p},q'}F_{11}(\omega,\mathbf{p},\mathbf{q}). \tag{10a}$$

The electron spectrum in the normal state has two branches

$$\varepsilon_{1,2}=\xi\pm|t(q)|, \tag{11}$$

i.e., there are two branches of the Fermi surface as determined by the equations $\varepsilon_{1,2}=0$. In the superconducting phase the energy gaps on these branches are the same if $\lambda_1=\lambda_1'=0$. For $\lambda_1\neq 0$ and $\lambda_1'\neq 0$, the gaps are different and both depend on q:

$$\Delta_{1,2}(q)=\Delta_0[1\pm\lambda_0^{-1}(\lambda_1^2+\lambda_1'^2+2\lambda_1\lambda_1'\cos q)^{1/2}\cos(\varphi_1-\varphi_2)],$$
$$\tan\varphi_1=t'\sin q/(t+t'\cos q), \quad \tan\varphi_2=\lambda_1'\sin q/(\lambda_1+\lambda_1'\cos q), \tag{12}$$

where Δ_0 is determined by the standard BCS expression with coupling parameter λ_0 to an accuracy $(t/\varepsilon_F)^2$. The gaps $\Delta_{1,2}(q)$ disappear simultaneously at temperature $T=T_c$. As a result, the system contains two groups of electrons (two branches of the Fermi surface) with different gaps. Its response to an external electromagnetic field is half of the sum of two superconductors with gaps $\Delta_1(q)$ and $\Delta_2(q)$. For example, the quasiparticle density of states is given by the expression

$$\rho(\varepsilon)=(\tfrac{1}{2}\pi)N(0)\mathrm{Re}\int_0^{\pi}dq(\varepsilon/\sqrt{\varepsilon^2-\Delta_1^2(q)}+\varepsilon/\sqrt{\varepsilon^2-\Delta_2^2(q)}). \tag{13}$$

At $\lambda_1'\ll\lambda_1$ and $t'\ll t$ the density of states has two narrow peaks at energies $\Delta_0(1\pm\lambda_1/\lambda_0)$ with width of order $2\lambda_1'/\lambda_0$. There are logarithmical singularities near these peaks. The ratios $2\Delta_{1,2}(T=0)/T_c$ are $3.52(1\pm 2\lambda_1/\lambda_0)$ if we take the positions of the peaks to be the "gaps."

The magnetic anisotropy in the system under consideration is determined by the value t'/ε_F at $t'\ll t\ll\varepsilon_F$. On the other hand the difference in the "gaps" is determined by the parameter t/ε_F if $t'\ll t$. We can see different gaps in such a system even in the case of very strong magnetic anisotropy. However, the anisotropy (q dependence) of the gaps is determined by the parameter t'/ε_F. So both gaps are isotropic in the case of strong magnetic anisotropy as in the system with one layer per unit cell.

We note that the system with two equivalent layers per unit cell at $t'=0$ and without the terms $H_{\mathrm{int}}^{(1)}, H_{\mathrm{int}}^{(2)}$ was studied by Hoffman *et al.*[19] We conclude that the effect of interlayer coupling inside the unit cell may be essential for all the compounds with several superconducting layers in the unit cell. This effect can smooth out remarkably the peak in the density of states near the gap and correspondingly suppress the enhancement of the NMR relaxation rate just below T_c.

IV. TWO NONEQUIVALENT LAYERS IN THE UNIT CELL

We consider now the crystal which consists of two different layers with intralayer pairing coupling λ_{01} and λ_{02} without taking into account the interlayer coupling λ_1 and λ_2; see Fig. 3. In the case $\lambda_{01}\neq 0$ and $\lambda_{02}\neq 0$ we deal with the *S-S'* superlattice while at $\lambda_{02}=0$ we label such a system *S-N*. For simplicity, we will assume that the layers 1 and 2 differ by the coupling parameter $\lambda_{0\alpha}$ only, i.e., their electronic spectra in the normal state are identical. The Gor'kov equations have the matrix form (10) where we put

$$\bar{\Delta}(q)=0, \tag{14}$$

neglecting the interlayer coupling. In the case $t,t'\ll T_c$, which will be of principal interest in the following, the in-

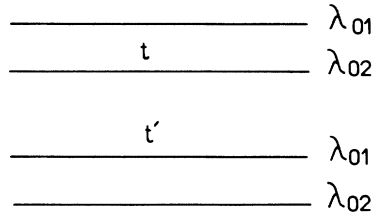

FIG. 3. The parameters of the model "two nonequivalent layers in unit cell."

terlayer coupling can be omitted. It should be taken into account at larger t,t' but the account of this coupling does not change the behavior of the system qualitatively.

The values Δ_α in (10) are assumed to be real; they are determined by the self-consistency equations

$$\Delta_\alpha=\lambda_{0\alpha}[(2\pi)^3N(0)]^{-1}T\sum_\omega\int d\mathbf{p}\,dq\,F^\dagger_{\alpha,\alpha'}(\omega,\mathbf{p},q)\ . \quad (15)$$

We consider first the quite interesting case $t=t'$ with a gapless spectrum of quasiparticles in the superconducting state. From Eqs. (10) and (14) the quasiparticle spectra are obtained as

$$E^2_{1,2}=\xi^2+4t^2\cos^2(q/2)+\tfrac{1}{2}(\Delta_1^2+\Delta_2^2)\pm L(\xi,q)\ ,$$
$$L^2(\xi,q)=16t^2\xi^2\cos^2(q/2)+\tfrac{1}{4}(\Delta_1^2-\Delta_2^2)^2 \quad (16)$$
$$+4(\Delta_1-\Delta_2)^2t^2\cos^2(q/2)\ .$$

Using (16) we can rewrite (15) as

$$\Delta_\alpha=(1/8\pi)\int d\xi\,dq\left[\left(\sum_{\beta=1}^{2}\frac{\tanh(E_\beta/2T)}{E_\beta}\right)\lambda_{0\alpha}\Delta_\alpha+\left(\sum_{\beta=1}^{2}(-1)^\beta\frac{\tanh(E_\beta/2T)}{E_\beta}\right)\right.$$
$$\left.\times4\lambda_{0\alpha}t^2\cos^2(q/2)(\Delta_1-\Delta_2)(-1)^\alpha-\tfrac{1}{2}\lambda_{0\alpha}\Delta_\alpha(\Delta_1^2-\Delta_2^2)(-1)^\alpha]/L(\xi,q)\right] \quad (17)$$

$\alpha=1,2$.

The equations (14)–(17) can be solved analytically in the limiting cases $t\ll T_c$ and $t\gg T_c$. The former corresponds to the Josephson coupling of the layers and the condition $t\ll T_c$ is equivalent to the condition $\xi_\perp(0)\ll d$ where $\xi_\perp(0)$ is the superconducting correlation length along the c axis extrapolated to $T=0$, and d is the interlayer distance. The proximity effect is weak in this limit.

We consider first the system with large t, i.e., with strong proximity effect. In the case $t\gg\omega_D$ the self-consistency equation takes the form

$$\Delta_\alpha=\lambda_{0\alpha}\frac{\Delta_1+\Delta_2}{2}\int_0^{\omega_D}d\xi\frac{\tanh\{[\xi^2+\frac{1}{4}(\Delta_1+\Delta_2)^2]^{1/2}/2T\}}{[\xi^2+\frac{1}{4}(\Delta_1+\Delta_2)^2]^{1/2}}\ , \quad (18)$$

where ω_D is the Debye frequency. We see now that $\Delta_1/\Delta_2=\lambda_{01}/\lambda_{02}$ and the value $(\Delta_1+\Delta_2)/2$ plays the role of effective order parameter while $(\lambda_{01}+\lambda_{02})/2$ is the effective average coupling. So at large t the strong proximity effect causes the effective averaging of the electron interaction and order parameter. However, the quasiparticle spectrum at small energies is more complicated than the BCS spectrum. According to (16) the minimal energy is reached at $\cos(q/2)=0$ and $\xi=0$. It is equal to Δ_2 (we assume $\lambda_{01}>\lambda_{02}$). At $q\neq\pi$ the energy gap is very close to $(\Delta_1+\Delta_2)/2$ because at a given $\cos(q/2)\gg\Delta_1/t$ the minimal energy is

$$\varepsilon^2_{\min}=\tfrac{1}{4}(\Delta_1+\Delta_2)^2-(\Delta_1^2-\Delta_2^2)^2/[64t^2\cos^2(q/2)]\ . \quad (19)$$

In the system with $\lambda_{02}=0$ (i.e., S-N system) superconductivity is gapless as seen from (16), the energy is zero at $\xi=0$ and $q=\pi$. For the energies $\varepsilon\ll\Delta_1$ we get in the S-N system the density of states

$$\rho_\alpha(\varepsilon)=(1/\pi)\int d\mathbf{p}\,dq(2\pi)^{-3}\mathrm{Im}[G_{\alpha\alpha}(\omega,p,q)|_{i\omega\to\varepsilon+i\delta}]\ , \quad (20a)$$
$$\rho_2(\varepsilon)=(2/\pi)N(0)(\varepsilon\Delta_1/2t^2)^{1/2}K(1\sqrt{2})\approx0.83N(0)(\varepsilon\Delta_1/t^2)^{1/2}\ , \quad (20b)$$
$$\rho_1(\varepsilon)\sim(\varepsilon/\Delta_1)\rho_2(\varepsilon)\ . \quad (20c)$$

The gapless character of the quasiparticle spectrum manifests itself in terms which are exponentially small in ordinary BCS theory at low temperatures. Thus at $T\ll\Delta$, the electron specific heat depends on temperature as

$$C\sim\gamma T_c(T_c/t)(T/T_c)^{3/2}\ , \quad (21)$$

where γ is the coefficient in specific heat in the normal phase.

We stress that the gapless character of the spectrum is due to the presence of electron eigenstates in the normal phase which are localized on N layers only. For this reason such electrons do not feel the pairing potential and have no gap in the superconducting phase. The system with identical transfer integrals $t=t'$ does have such states but they are absent if $t\neq t'$. To describe the spectrum in the case $t\neq t'$ we must substitute $(t^2+t'^2+2tt'\cos q)$ for $[4t^2\cos^2(q/2)]$ in (16). For $t,|t-t'|\gg T_c$ we obtain the usual BCS results with the gap $(\Delta_1+\Delta_2)/2$.

In the S-N system with transfer integrals $t,t'\gg T_c$ all the corrections to BCS results are at least of the order of T_c/t or T/t. Such a system is very similar to the three-dimensional anisotropic BCS superconductor with average parameter $\lambda=(\lambda_{01}+\lambda_{02})/2$.

V. NONEQUIVALENT LAYERS WITH JOSEPHSON COUPLING

At $t,t' \ll T_c$ the layers 1 and 2 are independent in the first approximation and the critical temperature of layer 1 is determined by the parameter λ_{01} in the usual way, i.e., $T_c = T_{c1}$ and $T_{c\alpha} = 1.14\omega_D \exp(-1/\lambda_{0\alpha})$. This result is valid in the framework of mean field theory; actually, the long-range superconducting order in the two-dimensional system is absent due to the fluctuations including the topological Kosterlitz-Thouless-Berezinskii excitations (Pearl vortices in the superconducting film).

However, in a layered compound the three-dimensional interaction of such topological excitations suppresses their formation. The first contribution to this interaction (of the short-range type) is due to the Josephson coupling of the layers and the second one (of the long-range type) is due to the electromagnetic interaction of the topological excitations (fluxons).[20] The latter can suppress the formation of fluxons quite effectively even in the case of very weak Josephson coupling and so the existence of the fluxons in the layered compounds has been under question until now.[20] In the absence of such excitations the Josephson coupling of the layers establishes the long-range order practically just below the mean field critical temperature T_c at any realistic values of t,t'.[21] According to Eq. (17) the value of T_c coincides with T_{c1} in an accuracy $(t^2+t'^2)/T_{c1}^2$ at $t,t' \ll T_{c1}$. If topological excitations do exist in the systems under study all of our following considerations will be valid below the critical temperature of the Kosterlitz-Thouless-Berezinskii transition; this temperature would be also quite close to T_{c1}.

In the S-S' system the order parameter on layer 1 at $T < T_{c1}$ is given by the standard BCS result with accuracy of order of $(t^2+t'^2)/T_{c1}^2$. The order parameter of layer 2 at $T_{c2} < T < T_{c1}$ is proportional to the order parameter of layer 1 multiplied by the same small factor. At $T < T_{c2}$, the order parameter of layer 2 is given by the standard BCS result with the coupling parameter λ_{02}. Below, we consider in a more detailed manner the S-N system only, which is the limiting case of the S-S' system and is the most interesting due to the strong difference in the properties of the layers S and N in the superconducting phase.

In the S-N system ($\lambda_{02}=0$) the layers 2 are superconducting also below T_{c1} due to the proximity effect which mixes the properties of layers in accordance with the small parameter $(t^2+t'^2)/T_{c1}^2$. Using (10) and (14) we obtain the Gor'kov anomalous function $F_{22}^{\dagger}(\omega,p,q)$ which describes the superconductivity of the layers 2:

$$F_{22}^{\dagger}(\omega,\mathbf{p},q) = \Delta_1^2 \tilde{\Delta}_2(q)/(\omega^2+E_1^2)(\omega^2+E_2^2) ,$$
$$E_1^2 = \xi^2+\Delta_1^2, \quad E_2^2 = \xi^2+\tilde{\Delta}_2^2(q) , \tag{22}$$
$$\tilde{\Delta}_2(q) = (t^2+t'^2+2tt'\cos q)/\Delta_1 .$$

Here Δ_1 is the usual BCS order parameter of layers 1 which has the standard BCS dependence on T with critical temperature $T_c \approx T_{c1}$. We see from (22) that at frequencies $\omega \ll \Delta_1$ and energies $\xi \ll \Delta_1$ the value $\tilde{\Delta}_2(q)$ plays the role of effective gap of layers 2 which depends on q. At $T \gg \tilde{T}_2 \approx (t^2+t'^2)/\Delta_1$ the layers 2 are superconducting to a very small degree while at $T \lesssim \tilde{T}_2$ they behave as superconductors with the gap $\tilde{\Delta}_2(q)$. The value $\tilde{T}_2$ plays the role of effective critical temperature for layers 2, though real phase transition does not exist at this point. As we will see later the layers 1 determine mainly the Meissner effect in magnetic fields $\mathbf{H}\|c$ at temperatures $\tilde{T}_2 \ll T < T_{c1}$, while the layers 2 start to contribute strongly below $\tilde{T}_2$.

Now, we consider the quasiparticle density of states at low temperatures $T \ll \tilde{T}_2$. In the system with $t \neq t'$ a gap $\varepsilon_g = (t-t')^2/\Delta_1(0)$ exists for all the electrons. At $t=t'$ the spectrum is gapless and in the following part of this section we consider this interesting case.

For the gapless S-N system the density of states of the layers 2 is given by the expression (20b) at $\varepsilon \ll t^2/\Delta_1$. We note that the density of states of layer 1 is also nonzero at energy $\varepsilon \lesssim \Delta_1$ but in a very small degree:

$$\rho_1(\varepsilon) \sim \begin{cases} (\varepsilon/\Delta_1)\rho_2(\varepsilon), & \varepsilon \ll \tilde{T}_2 , \\ (t^2/\Delta_1^2)\rho_2(\varepsilon), & \tilde{T}_2 \ll \varepsilon \ll \Delta_1 . \end{cases} \tag{23}$$

The densities of states $\rho_1(\varepsilon)$ and $\rho_2(\varepsilon)$ are shown schematically in Fig. 4. The function $\rho_1(\varepsilon)$ differs from the standard BCS result at energies near Δ_1; it has logarithmic singularity here instead of the usual BCS square root one. Additionally, a weak peak is present in $\rho_1(\varepsilon)$ at $\varepsilon \approx \tilde{\Delta}_2(0)$. The function $\rho_2(\varepsilon)$ has logarithmic singularity at $\varepsilon = \tilde{\Delta}_2(0)$, weak peak near Δ_1, and $\rho_2(\varepsilon)$ is much smaller than the density of states in the normal state $N(0)$ at $\varepsilon \ll \tilde{\Delta}_2(0)$ in the gapless case while it vanishes at $\varepsilon < \varepsilon_g$ in the case with the gap. So two peaks occur in the density of states of the S-N system [at $\tilde{\Delta}_2(0)$ and Δ_1] at low temperatures $T \ll \tilde{T}_2$ while one peak exists (at Δ_1) at temperatures $\tilde{T}_2 \ll T < T_{c1}$.

The gapless character of superconductivity in the S-N system is suppressed by impurities which restore the gap. We will consider now this effect. As was mentioned early the gapless spectrum at $t=t'$ is explained by the presence of electrons which move along "normal" layers 2 where the pairing potential is absent. The impurities mix such trajectories with the trajectories of electrons that feel the pairing potential. For this reason a gap ε_g occurs in the presence of impurities in the system with $t=t'$. We will

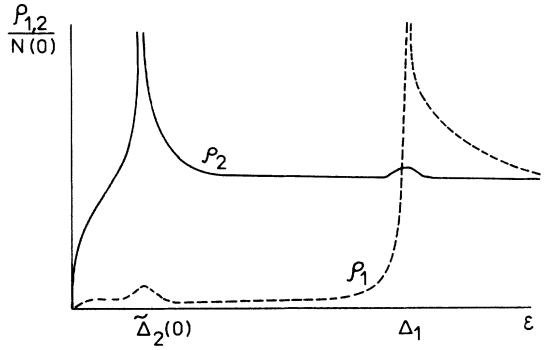

FIG. 4. The densities of states in a Josephson S-N system in the gapless case.

consider the dependence of ε_g on the electron scattering time τ in the simplest model of nonmagnetic scattering assuming that impurities' potential is isotropic inside the layers and diagonal in the indices n. We suppose also that disorder is the same in both S and N layers.

In the presence of impurities the Green's functions in Eq. (10) should be replaced by the average Green's functions $\bar{G}$ and $F^{\dagger}$, and the frequencies ω_α and order parameters Δ_α should be replaced by $\bar{\omega}_\alpha$ and $\bar{\Delta}_\alpha$:

$$\begin{aligned}\bar{\omega}_\alpha&=\omega-(2\pi)^{-1}nN(0)|U|^2\int \bar{G}_{\alpha\alpha}(\xi,q)d\xi\,dq\ ,\\ \bar{\Delta}_\alpha&=\Delta_\alpha+(2\pi)^{-1}nN(0)|U|^2\int \bar{F}^{\dagger}_{\alpha\alpha}(\xi,q)d\xi\,dq\ ,\end{aligned}\tag{24}$$

where U is the impurity potential and n is concentration of impurities, $\tau^{-1}=2\pi nN(0)|U|^2$. We obtain equations for $\bar{\omega}_\alpha$ and $\bar{\Delta}_\alpha$ by the standard method (see Ref. 22). At small t we get

$$\begin{aligned}&\bar{\omega}_2\left[1-\frac{1}{2\tau}\left\langle\frac{1}{a_2(q)}\right\rangle\right]=\omega,\quad \langle A(q)\rangle=\frac{1}{2\pi}\int_0^{2\pi}dq\ A(q)\ ,\\ &\bar{\Delta}_2\left[1-\frac{1}{2\tau}\left\langle\frac{1}{a_2(q)}\right\rangle\right]=\frac{2t^2}{\tau}\left\langle\frac{\cos^2(q/2)}{a_2(q)}\right\rangle\ ,\\ &a_2^2(q)=\bar{\Delta}_2^2+\bar{\omega}_2^2+8t^2\cos^2(q/2)\frac{\bar{\omega}_1\bar{\omega}_2+\bar{\Delta}_1\bar{\Delta}_2+\bar{\omega}_2^2+\bar{\Delta}_2^2}{\bar{\Delta}_2^2+\bar{\omega}_2^2}\\ &\qquad+t^4\cos^4(q/2)/(\bar{\Delta}_1^2+\bar{\omega}_1^2)\ ,\\ &\bar{\omega}_1=\omega\eta,\quad \bar{\Delta}_1=\Delta_1\eta,\quad \eta=1+2\tau\sqrt{\omega^2+\Delta_1^2}\ .\end{aligned}\tag{25}$$

We admit the following ansatz for $a_2(q)$ which is useful to get the analytical continuation for energies near gap ε_g:

$$\begin{aligned}&\langle a_2^{-1}(q)\rangle\\ &\qquad=\min[(\bar{\omega}_2^2+\bar{\Delta}_2^2)^{-1/4}(t^2/\bar{\Delta}_1)^{-1/2},(\bar{\omega}_2^2+\bar{\Delta}_2^2)^{-1/2}]\ ,\\ &\langle a_2^{-1}(q)\cos^2(q/2)\rangle=\min[\Delta_1/t^2,(\bar{\omega}_2^2+\bar{\Delta}_2^2)^{-1/2}]\ .\end{aligned}\tag{26}$$

Then the density of states can be obtained using analytical continuation for $\bar{\omega}_\alpha$, i.e., $\rho_\alpha(\varepsilon)=\mathrm{Im}2\tau\bar{\omega}_\alpha(i\omega\to\varepsilon+i\delta),\delta\to0$. As a result we get the gap ε_g which depends on the relations between the scattering rate τ^{-1} and the values $\tilde{T}_2,\Delta_1$:

$$\varepsilon_g\approx\begin{cases}\tau^{-1}, & \tilde{T}_2\tau\gg1, & (27a)\\ t^2/\Delta_1, & \tilde{T}_2\ll\tau^{-1}\ll\Delta_1, & (27b)\\ t^2\tau, & \Delta_1\tau\ll1\ . & (27c)\end{cases}$$

For $\tau^{-1}\ll\tilde{T}_2$, the states at $\varepsilon<\varepsilon_g$ are absent; for $\varepsilon\gg\varepsilon_g$, we obtain he same density of states (20) as in the clean S-N system. For the intermediate scattering rate (27b) the states of layers 1 and 2 at low energies are mixed completely, there is gap $\varepsilon_g\approx\tilde{T}_2$; but at energies $\varepsilon\gg\varepsilon_g$ the layers 2 are normal while the layers 1 have a suppressed density of states up to the energy Δ_1. The behavior of functions $\rho_\alpha(\varepsilon)$ at $\tau^{-1}\gg\Delta_1$ is similar. Thus ε_g reaches a maximum as τ^{-1} grows, the decrease in ε_g at small τ being due to the suppression of electron hopping between the layers (the diffusion hopping rate of electrons is $t^2\tau$ at $t\tau\ll1$).

The conclusion is that the gapless character of the spectrum in the S-N system with Josephson coupling ($t\ll T_c$) at $t=t'$ is preserved in the presence of impurities as far as $\tau^{-1}\ll\tilde{T}_2$. Our analysis of the S-N system with $t\gg T_c$ shows that the gapless type of spectrum at $t=t'$ is preserved for $\tau^{-1}\ll T_c$.

VI. THE JOSEPHSON S-N SYSTEM IN THE MAGNETIC FIELD

We consider first the Meissner effect in the field along the c axis, i.e., superconducting currents flow in (ab) plane. The electromagnetic kernel $Q_\parallel(k)$ for the vector potential (in the Coulomb gauge) parallel to the layers may be obtained by the standard way using perturbation theory in the vector potential; the zeroth order functions are defined by Eq. (10). At $t,t'\ll T_{c1}$ with accuracy $\tilde{T}_2/T_{c1}$ we obtain for $k\to0$

$$\begin{aligned}Q_\parallel(k=0)/(e^2Nm_\parallel^{-1})&=N_{s,\parallel}(T)/N\\ &=N_{s\parallel}^{(1)}(T)/N+N_{s,\parallel}^{(2)}(T)/N\\ &=(\pi/2)T\sum_\omega\frac{\Delta_1^2}{(\omega^2+\Delta_1^2)^{3/2}}\\ &\quad+\tfrac{1}{2}T\sum_\omega\int_0^\pi dq\frac{\tilde{\Delta}_2^2(q)}{[\omega^2+\tilde{\Delta}^2(q)]^{3/2}}\ ,\end{aligned}\tag{28}$$

where N is the density of the conducting electrons. The first term in the right side describes the contribution of layers 1 which is of the usual BCS form. The second one gives the contribution of layers 2 and it is of the same order of magnitude as the first term at $T\lesssim\tilde{T}_2$. At temperatures $\tilde{T}_2\ll T\lesssim T_{c1}$ the contribution of the layers 2 is of the order of $\tilde{T}_2^2\Delta_1^2(T)/T_{c1}^4$, i.e., smaller by the factor $\tilde{T}_2^2/T_{c1}^2$ than the contribution of layers 1.

For the gapless case $t=t'$ at $T\ll\tilde{T}_2$ the concentration of superconducting electrons of layers 2 is given by the expression

$$\begin{aligned}N_{s,\parallel}^{(2)}(T)/(N/2)&=1-\int_0^{2\pi}dq[\tilde{\Delta}_2(q)/T]\exp[-\tilde{\Delta}_2(q)/T]\\ &=1-(T\Delta_1/\pi t^2)^{1/2}\ .\end{aligned}\tag{29}$$

In the system with gap (at $t\neq t'$ or in the presence of impurities at $t=t'$) the value $N_{s,\parallel}^{(2)}$ approaches to zero temperature value $(N/2)$ exponentially.

The temperature dependence of the corresponding London penetration depth (inverse and squared) $\lambda_\parallel^{-2}$ is shown schematically in Fig. 6. Far below T_{c1} but above $\tilde{T}_2$ it is determined by layers 1; the concentration of superconducting electrons is $(N/2)$. Below $\tilde{T}_2$ this concentration increases up to N.

We remark that calculating (28) we use local approximation $k\to0$ (i.e., London limit) for both layers N and S.

For N layers this approach is correct at condition $\lambda_{\parallel} \gg V_F/\tilde{T}_2$ which can be invalid in systems with very small t.

The behavior of the electromagnetic kernel $Q_{\perp}$ for the magnetic field parallel to the layers (and the vector potential $A_{\perp}$ perpendicular to the layers) is quite different. We consider here the case $t=t'$ and introduce the vector potential $A_{\perp}(\rho)$ in the Hamiltonian in the following way: The resonance integral t in (4) has to be multiplied by $\exp[\pm\chi(\rho)]$ with $\chi(\rho)=ed/\hbar c A_{\perp}(\rho)$; a sign depends on the direction of electron transfer. Thus in the presence of $A_{\perp}(\rho)$, we get the additional term in the Hamiltonian

$$H_{\mathrm{EM}}=\sum_{n,\mathbf{pp}',\sigma} t(a^{\dagger}_{n+1,\mathbf{p},\sigma} f_{\mathbf{p},\mathbf{p}'} + a^{\dagger}_{n-1,\mathbf{p},\sigma} f^{*}_{-\mathbf{p},-\mathbf{p}}) a_{n,\mathbf{p}',\sigma} ,$$
$$f_{\mathbf{p},\mathbf{p}'}=\int d\rho (e^{i\chi}-1) e^{i(\mathbf{p}-\mathbf{p}')\cdot\rho} . \tag{30}$$

We next calculate the contribution of (30) to the free energy of the system and obtain then the superconducting current $J_{\perp}$ calculating derivatives of the free energy with respect to $A_{\perp}(,\rho)$. The diagram of perturbation theory for H_{EM} are shown in Fig. 7. At low tempertures $T \ll T_{c1}$ with accuracy $(t/T_{c1})^6$ we get the expressions

$$Q_{\perp}(k\to 0)/(e^2 N m_{\perp}^{-1}) = N_{s,\perp}(T)/N$$
$$= (1/\pi) T \sum_{\omega} \int d\xi \int_0^{\pi} dq \frac{\Delta_1 \tilde{\Delta}_2(q)}{(\omega^2+\xi^2+\Delta_1^2)[\omega^2+\xi^2+\tilde{\Delta}_2^2(q)]} , \tag{31}$$

$$N/m_{\perp} = 2t^2 dN(0) .$$

We obtain with logarithmic accuracy the low-temperature behavior $N_{s,\perp}$ as

$$N_{s,\perp}(T)/N = (4t^2/\pi\Delta_1^2)\ln(\Delta_1/T) , \quad \tilde{T}_2 \ll T \ll T_{c1} ,$$
$$N_{s,\perp}(T)/N = (4t^2/\pi\Delta_1^2)\ln(\Delta_1/\tilde{T}_2) , \quad T \ll \tilde{T}_2 , \tag{32}$$

while near T_{c1} we get

$$N_{s,\perp}(T)/N \sim t^2\Delta_1^2(T)/T_{c1}^4 . \tag{33}$$

According to (32) and (33) the inverse squared perpendicular London penetration length [for the magnetic field along the (ab) plane] $\lambda_{\perp}^{-2}(T)$ decreases gradually with temperature and saturates at $T<\tilde{T}_2$. In contrast, the inverse squared parallel London penetration length ($\mathbf{H}\perp c$) has two upturns: at T_{c1} due to the layers 1 and at $\tilde{T}_2$ due to the layers 2; see Fig. 5. It is worth noting that according to (28)–(33) the magnetic anisotropy $\lambda_{\perp}^2(T)/\lambda_{\parallel}^2(T)$ grows with temperature down to the temperature $\tilde{T}_2$.

Due to the condition of Josephson coupling of the layers $t \ll T_c$ the value $\lambda_{\perp}=(4\pi Q_{\perp})^{-1/2}$ is very large and the parallel lower magnetic critical field $H_{c1,\perp}$ is very small, $H_{c1,\parallel}=(\phi_0/4\pi\lambda_{\perp}\lambda_{\parallel})\ln(\lambda_{\parallel}/d)$.[21] So the parallel magnetic field $H \gg H_{c1,\parallel}$ penetrates almost completely into the sample. We estimate now the value of parallel magnetic field which affects remarkably the Josephson coupling of the layers. For this purpose we present first another method for studying the gap function $\tilde{\Delta}_2(q)$ which allows us to calculate this function in a simple way in more general systems (finite set of S and N layers, different transfer integrals, or in the presence of magnetic field).

To obtain $\tilde{\Delta}_2(q)$, i.e., the gap function of N layers, we use the perturbation expansion for the off-diagonal part of self-energy. At $t=t'=0$ we have normal layers N with Green's functions $G_{22}^{(0)}(\omega,\mathbf{p})=(i\omega-\xi)^{-1}, F_{22}^{(0)}=0$ and superconducting S layers with standard Gor'kov function $F_{11}^{(0)}=\Delta_1/(\omega^2+\xi^2+\Delta_1^2)$. Now the Gor'kov function of the layers 2, i.e., $F_{22}^{\dagger}(\omega,\mathbf{p},m,n)$ for $T \ll T_{c1}$ to the second order in t,t', is given by the expression

$$F_{22}^{\dagger}(\omega,\mathbf{p},m,n)=G_{22}^{(0)}(\omega,\mathbf{p})F_{11}^{(0)}(\omega;\mathbf{p})G_{22}^{(0)}(-\omega,-\mathbf{p}) \times[(t^2+t'^2)\delta_{mn} + tt'(\delta_{m,n+1}+\delta_{m,n-1})] . \tag{34}$$

Comparing this expression with the expansion of $F_{22}^{\dagger}$ in the off-diagonal part of self-energy $\Delta_{mn}(\omega,\mathbf{p})$

$$F_{22}^{\dagger}(\omega,\mathbf{p},m,n)=G_{22}^{(0)}(\omega,\mathbf{p}) \times\Delta_{mn}(\omega,\mathbf{p})G_{22}^{(0)}(-\omega,-\mathbf{p}) , \tag{35}$$

we get the "order parameter" in a discrete representation at $T \ll T_{c1}$ as

$$\Delta_{mn}=[(t^2+t'^2)\delta_{mn}+tt'(\delta_{m,n+1}+\delta_{m,n-1})]/\Delta_1$$

and the gap function

$$\tilde{\Delta}_2(q)=(t^2+t'^2+2tt'\cos q)/\Delta_1$$

in momentum representation. At $t=t'$ we get

$$\tilde{\Delta}_2(q)=(2t^2/\Delta_1)(1+\cos q)$$

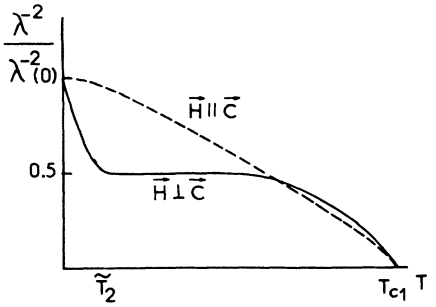

FIG. 5. The temperature dependence of the London penetration length in a Josephson S-N system.

in a complete analogy with results (5) of Sec. II. The difference now is that in the S-N system $\Delta_0=\delta$ and this is a reason for the gapless nature of the spectrum.

Now we can obtain the gap function in the presence of parallel magnetic field $H_\parallel$ which penetrates completely into the sample. To take into account such a field we multiply t by the factor $\exp(\pm\chi)$ where $\chi=edHx/\hbar c$. Now the hopping term H_t in the Hamiltonian has the form

$$H_t=\sum_{n,\mathbf{p},s} t(a^\dagger_{n+1,\mathbf{p}+\mathbf{k},\sigma}+a^\dagger_{n-1,\mathbf{p}-\mathbf{k},\sigma})a_{n,\mathbf{p},\sigma}\ , \tag{36}$$

where $|\mathbf{k}|=edH/\hbar c$. Using (36) we get the off-diagonal part of self-energy $\Delta_{m,n}(\omega,\mathbf{p},\mathbf{p}')$:

$$\begin{aligned}\Delta_{n,n}(\omega,\mathbf{p},\mathbf{p})&=\frac{\Delta_1 t^2}{\omega^2+(\xi+\mathbf{v}\cdot\mathbf{k})^2+\Delta_1^2}\\&\quad+\frac{\Delta_1 t^2}{\omega^2+(\xi-\mathbf{v}\cdot\mathbf{k})^2+\Delta_1^2}\ ,\\ \Delta_{n,n\pm1}(\omega,\mathbf{p},\mathbf{p}\pm2\mathbf{k})&=\frac{\Delta_1 t^2}{\omega^2+(\xi+\mathbf{v}\cdot\mathbf{k})^2+\Delta_1^2}\ ,\end{aligned} \tag{37}$$

where $\mathbf{v}$ is the velocity of electrons on the two-dimensional Fermi surface. We see from (37) that parallel magnetic field changes the proximity effect if $H_\parallel \gtrsim \phi_0/d\xi_\parallel(0)$, i.e., the only very strong parallel fields affect the superconductivity of layers 2.

VII. OPTICAL PROPERTIES OF THE SYSTEMS S-S' AND S-N

At $t\gg T_{c1}$ the systems S-S' and S-N are very similar to the homogeneous system with average coupling and thus optical conductivity can be determined by the usual BCS expression. There the optical conductivities for $\mathbf{E}$ parallel and perpendicular to the layers have similar frequency dependence [with gap $(\Delta_1+\Delta_2)$] and only the magnitudes depend on $\mathbf{E}$ orientation.

In the Josephson S-S' and S-N systems the frequency dependence of the conductivity is anisotropic. In the first approximation (at $t=0$) the system is an insulator for $\mathbf{E}\|\mathbf{c}$, and for $\mathbf{E}$ along (ab) plane the conductivity is the sum of conductivities $\sigma_{1,\parallel}$ and $\sigma_{2,\parallel}$, which are characterized by the gaps $2\Delta_1$ and $2\Delta_2$. At $t\neq0$ the conductivity $\sigma_\perp(\omega)$ for $\mathbf{E}\|\mathbf{c}$ is determined by the transfer of electrons between neighboring layers S and S'. It does not vanish for $\omega>(\Delta_1+\Delta_2)$. So the gap $2\Delta_1$ and $2\Delta_2$ can be seen in $\sigma_\parallel(\omega)$ while the "gap" $(\Delta_1+\Delta_2)$ occurs in $\sigma_\perp(\omega)$. In the S-N system we get the gap $2\Delta_1$ in $\sigma_\parallel(\omega)$ and Δ_1 in $\sigma_\perp(\omega)$.

We consider now in a more detailed fashion the dependence $\sigma_\perp(\omega)$ in a clean and dirty S-N system at $t\ll T_{c1}$. To obtain $\sigma_\perp(\omega)$ we calculate the electromagnetic kernel $Q_\perp(\omega)$ for the vector potential $A_\perp(\rho,\omega)$ using the standard technique (see Ref. 22). The contributions to $Q_\perp(\omega)$ are given by all the diagrams shown in Fig. 6 except diagram (a). In the presence of impurities the kernel is determined by the average of the product of the two Green's functions. To the lowest order in t, these two

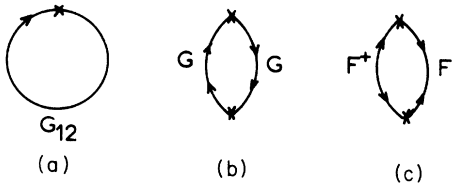

FIG. 6. The diagrams for the free energy (up to second order in the vector potential).

Green's functions are for neighboring layers. We take into account the scattering of electrons by impurities inside the layers only. Thus calculating the average value of the product we obtain the product of the average Green's functions omitting the diagrams shown in Fig. 7. As a result we obtain in the normal state

$$\sigma_\perp(\omega)=\frac{1}{\omega}\mathrm{Im}Q_\perp(\omega)=\frac{\omega_{p\perp}^2\tau}{1+\omega^2\tau^2}\ ,\quad \omega_{p\perp}^2=\frac{4\pi e^2N}{m_\perp}\ . \tag{38}$$

In the superconducting phase at low temperatures ($T\ll T_{c1}$) we get in the clean system at $k\to0$.

$$\frac{\sigma_{\perp,s}(\omega)}{\sigma_{\perp,n}(\omega)}=\frac{\pi\Delta_1^2}{2\omega^3\tau}\Theta(\omega-\Delta_1)\ , \tag{39}$$

where $\Theta(x)$ is the step function. In the dirty system at $\Delta_1\tau\ll1$ we get

$$\frac{\sigma_{\perp,s}(\omega)}{\sigma_{\perp,n}(\omega)}=\frac{\omega-\Delta_1}{\omega}\Theta(\omega-\Delta_1)\ . \tag{40}$$

VIII. CONCLUSIONS

We can summarize the main results on the S-N systems as follows.

(1) The atomic scale layered S-N systems with large transfer integrals do not differ practically from the usual BCS superconductors. The pairing interaction is averaged there; the layers S and N lose their individual properties due to the strong proximity effect.

(2) The individuality of S and N layers preserves to a great extent in the limiting case $t\ll T_{c1}$. The S layers be-

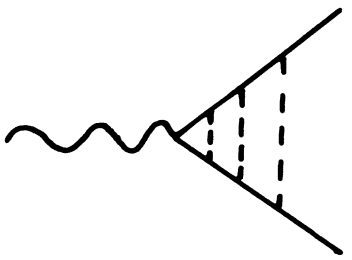

FIG. 7. The diagrams omitted in calculations of perpendicular conductivity.

come superconducting there below T_{c1} and the gap Δ_1 of order of T_{c1} forms at temperatures $T \ll T_{c1}$. However, only the very small superconducing order parameter is induced inside the layers N down to the temperature $\tilde{T}_2 \sim t^2/\Delta_1$. At temperatures $T \ll \tilde{T}_2$ almost all the electrons of N layer become superconducting and the gap here is of order of $\tilde{T}_2$.

(3) The tunneling density of states in the Josephson S-N system has the normal contribution as well as the superconducting one at $T \gtrsim \tilde{T}_2$ while at $T \ll \tilde{T}_2$ two gaps $\tilde{\Delta}_2$ and Δ_1 can be observed.

(4) The remarkable growth of superconductivity below $\tilde{T}_2$ in N layers manifests itself also in the temperature behavior of London penetration depth, specific heat, as well as Knight shift and NMR relaxation rate measured on nuclei of N layers.

(5) The magnetic anisotropy $\lambda_1^2/\lambda_\parallel^2$ grows with temperature in the Josephson S-N and S-S' systems while it is temperature independent in the case $t \gg T_c$ or in the Josephson S-S system (for more details, see Ref. 23).

(6) The frequency dependence of optical conductivity in the Josephson S-N system is remarkably anisotropic; different "gaps" characterize the conductivity for $\mathbf{E}\|c$ and $\mathbf{E}\|(ab)$; the ratio of the "gaps" is about two.

It is worthwhile to compare the atomic scale S-N systems with those made of S and N thick films.[24,25] The systems which consist of alternating S and N layers with thickness d_N, d_S larger than atomic scale but smaller than the superconducting correlation length $\xi(0)$ behave very similar to the microscopic ones. All the macroscopical parameters of such systems are determined by average pairing interaction due to the strong proximity effect, and the possible gapless character of the excitation spectrum in clean systems reminds us of the presence of N layers.[25]

The properties of the systems with thick metallic layers $d_N \gg \xi(0)$ can be compared with those of Josephson atomic scale S-N systems. In both systems the induced superconductivity of N layers is characterized by the small gap at very low temperatures. However, the systems under consideration behave quite differently in the parallel magnetic field due to the remarkable difference in the thickness of N layers. In the systems with thick N layers $d_N \gg \lambda_\parallel$ the N layer alone can screen the magnetic field due to the induced superconductivity. Thus, two phases are possible inside the N layer in the presence of rather weak parallel magnetic field (normal and superconducting). In systems with atomic N layers such screening is impossible and the very strong magnetic fields are necessary to suppress the induced superconductivity of N layers.

Considering now high-T_c superconductors we conclude that, in principle, some optical and tunneling data on high-T_c layered superconductors can be explained in the framework of the Josephson S-N model. For example, normal contribution in tunneling current was observed until now in all high-T_c compounds. The anisotropy of the optical properties in such superconductors is also in agreement with predictions of the Josephson S-N model. However, up to now we do not know if such a model is appropriate for these crystals. First, we do not know certainly if the layers BiO and TlO are conducting or insulating. Secondly, we do not know if the transfer integral t' between suspicious "N" layers and superconducting CuO_2 layers fulfills the condition of Josephson coupling. In Bi and Tl compounds the smallest hopping integral does fulfill such criteria. Here the anisotropy $A = \lambda_1^2/\lambda_\parallel^2 = \xi_\parallel^2/\xi_1^2$ is as high as 10^3–10^4 near T_c obtained from torque measurements.[26–28] So $\xi_1^2(0)/d^2 = [\xi_\parallel(0)/d]^2 A^{-1} \approx 10^{-3}$ at $\xi_\parallel(0) \approx 20$ Å and $d \approx 10$ Å. However, we get the Josephson S-N system if all the transfer integrals between S and N layers are small enough in comparison with T_c. If transfer integral t between S and N layers in the unit cell is large ($t \gg T_c$) and intercell integral t' is small ($t' \ll T_c$) we obtain the Josephson S-S system, each sheet S consisting of layers S and N with average pairing interaction. So the Bi and Tl high-T_c superconductors can be described by the Josephson S-S system if all the layers except CuO_2 are insulating or if BiO or TlO layers are metallic but intracell hopping integral t is large in comparison with T_c. Alternatively these compounds are described by the Josephson S-S' or S-N system.

The anisotropy A was found to be independent of temperature in $Tl_2Ba_2CaCu_2O_x$ in the interval 90–100 K.[28] Thus, we can think that we get the system S-S in this case, the TlO layers being nonconducting or alternately the intracell integral t being large in comparison with T_c.

We used the BCS model to study the proximity effect in the S-N and S-S' layered compounds. Of course, such a model does not give an accurate quantitative description of high-T_c superconductors. However, we think that this model is useful to understand in a qualitative way the proximity effect in any kind of superconductors which can be described by the superconducting order parameter.

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