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Superconductive fluctuations in the density of states and tunneling resistance in high-T_c superconductors

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The effect of superconductive fluctuations on the density of states and tunneling resistance is revisited in the case of dirty superconductors and derived for clean samples with and without pair-breaking sources. For clean superconductors new features appear in the density of states. In particular, with respect to the commonly known dirty case, the fluctuation effects manifest themselves on a new scale of energy with a different power-law behavior as function of $T-T_c$. The relevance of these results for high-T_c superconductors is analyzed.

I. INTRODUCTION

The discovery of high-T_c superconductors[1] has rekindled interest in the effect of superconductive fluctuations on the various physical quantities.[2] The small value of the zero-temperature coherence distance ξ_0 makes the Ginzburg criterion for the validity of the mean-field theories much more stringent; accordingly the deviations from the mean-field behavior together with the effect of fluctuations should appear[3] in an accessible range of temperatures around T_c even for clean superconductors (where $\xi_0 < l$, l being the scattering mean free path of the electrons due to the impurities). Our main concern in this paper will be the analysis of the single-particle density of states. Before discussing the clean case, we shall revisit the case of dirty superconductors to provide a complete theoretical framework by which the still unclear situation of the high-T_c superconductors can be analyzed.

A semiphenomenological study of the fluctuation effects on the density of states of dirty superconducting material was first carried out while analyzing the tunneling experiments of granular Al in the fluctuating regime just above T_c.[4] The second metal of the junction was in the superconducting regime and its gap gave a bias voltage around which a structure associated with the superconductive fluctuations of Al appeared. The measured density of states has a deep depression at the Fermi level ($\varepsilon=0$), reaches its normal value at frequency $\varepsilon_0(T)$ and shows a maximum at frequency value several times ε_0, decreasing again towards its normal value at higher frequencies. The characteristic frequency ε_0 is of the order of the inverse of the Landau-Ginzburg relaxation time $\tau_{\rm LG}$ for the fluctuations of the order parameter

$$\tau_{\rm LG}^{-1}=\frac{8}{\pi}(T-T_c)\ . \tag{1.1}$$

Here and in the following we use units with $\hbar=1$ and $k_B=1$.

The presence of a depression at $\varepsilon=0$ and of the peak at $\varepsilon\sim\tau_{\rm LG}^{-1}$ in the density of states above T_c is the precursor of the appearance of the superconductive gap at temperatures below T_c.

Microscopic calculations[5,6] for dirty superconductors ($\xi_0 > l$), were carried out not too near the critical temperature in the so-called classical region, where deviations from the mean-field behavior are small. The theoretical results reproduce the main features of the experimental behavior. The strength of the depression (and of the peak) is proportional to different powers of the Landau-Ginzburg relaxation time, depending on the dimensions, i.e., if we define $\delta N(\varepsilon)=[N(\varepsilon)-N_0(\varepsilon)]/N_0(0)$, $N(\varepsilon)$ and $N_0(\varepsilon)$ being the density of states per spin in the presence and in the absence of the fluctuations, respectively, the relative depression in the density of states at zero frequency is given by

$$\delta N(0)=-(3\sqrt{2}-4)C_{3,d}(T_c\tau_{\rm LG})^{3/2}\ ,\qquad C_{3,d}=\frac{T_c^{1/2}}{\pi^2 N_0 D^{3/2}} \tag{1.2}$$

for $d=3$, and

$$\delta N(0)=-4(1-\ln 2)C_{2,d}(T_c\tau_{\rm LG})^2\ ,\qquad C_{2,d}=\frac{1}{\pi^2 N_0 LD} \tag{1.3}$$

for a quasi-two-dimensional sample. Here the subscript d stands for dirty, $N_0=(mp_F/2\pi^2)$ is the three-dimensional (3D) density of states, $D=(v_F^2\tau/3)$ is the diffusion constant, τ is the scattering time due to nonmagnetic impurities ($v_F\tau=l$), and L is the thickness of the film. For the strictly two-dimensional case we have to replace in Eq. (1.3) N_0L with the two-dimensional density of states. At large frequencies the density of states recovers its normal value according to the following behavior:

$$\delta N(\varepsilon)=\frac{C_{3,d}}{2\sqrt{2}}\left[\frac{T_c}{\varepsilon}\right]^{3/2}, \quad \varepsilon \gg \tau_{\rm LG}^{-1},\ d=3\ , \tag{1.4}$$

$$\delta N(\varepsilon)=C_{2,d}\left[\frac{T_c}{\varepsilon}\right]^{2}\ln(\varepsilon\tau_{\rm LG}), \quad \varepsilon \gg \tau_{\rm LG}^{-1},\ d=2\ . \tag{1.5}$$

When tunneling junctions are considered either with both electrodes in the fluctuating regime or with one electrode in the normal state and the other one in the fluctuating regime, the effect induced in the differential resistance by the anomalous behavior of the density of states is rather weak.[7] In the quasi-two-dimensional samples for instance, the differential resistance depends only logarithmically on $T-T_c$ at $eV \ll T_c$ and has a weak dependence on the voltage V itself at a scale $eV \sim T_c$. A stronger dependence on $T-T_c$ appears only when the interaction among fluctuations is considered in the higher-order expansion in $1/D$.

Despite the difficulties that arise in carrying out tunneling experiments[8] in high-T_c superconductors, these systems show some distinguishing features that need to be taken into account in the theoretical analysis. Indications that high-T_c superconductors behave, at least in the best samples, as clean systems are present in the literature, see for instance the values of the ratio between the gap Δ_0 and the scattering rate as derived from the reflectivity measurements in $YBa_2Cu_3O_x$.[9] The presence of a strong pair-breaking might be another important aspect of high-T_c superconductors. This has been advocated in discussing several physical situations. The conductivity fluctuations around T_c in yttrium and bismuth compounds can be fitted with the 3D and 2D Aslamazov-Larkin term only.[2] The absence of the Maki-Thompson term is an indication that either the system is clean or it is in the presence of a strong pair breaking. A strong pair breaking has been put forward also in the analysis of the magnetoresistance in 1-2-3 compounds.[10,11] This induces us to complete the above theoretical picture for the density of states by investigating the clean case and by introducing the possibility of pair-breaking effects.

We recall that the coherence distance of the Landau-Ginzburg theory

$$\xi(T)=\xi_0\left[\frac{T_c}{T-T_c}\right]^{1/2}$$

as derived from microscopic calculations[12] in the clean case compares with the one for the dirty case via the different expressions for ξ_0:

$$\xi_{0,c}^2=\frac{7}{12}\frac{\zeta(3)}{\pi^2T_c^2}\frac{E_F}{2m},\quad \xi_{0,d}^2=\frac{\pi}{8T_c}D\ , \tag{1.6}$$

where the subscripts c and d stand for clean and dirty, respectively. In going from the dirty to the clean case one has to make the substitution $D\sim(p_Fl/m)\sim(E_F\tau/m)\rightarrow(E_F/mT_c)$. In Eq. (1.6) $\zeta(x)$ is the Riemann function and E_F is the Fermi energy.

The relevant energy scale in the dirty case is associated to the inverse of the time necessary for the particle to diffuse a distance equal to the coherence distance ξ. This energy scale coincides with the inverse relaxation time for the fluctuations given in Eq. (1.1)

$$t_\xi^{-1}=D\xi^{-2}\sim\tau_{\rm LG}^{-1}\sim T-T_c\ . \tag{1.7}$$

In the clean case instead the ballistic motion gives rise to a different characteristic energy

$$t_\xi^{-1}\sim v_F\xi^{-1}=(aT_c\tau_{\rm LG}^{-1})^{1/2}\sim|T-T_c|^{1/2}\ , \tag{1.8}$$

with $a=[6\pi^3/7\zeta(3)]$. Because of this fact, significant modifications in the expressions of the density of states and of the tunneling current-voltage characteristic will appear in the clean case in comparison with the dirty one.

Various sources of pair breaking can be hypothized in high-T_c superconductors, e.g., localized magnetic moments residing on Cu ions, electron-electron interaction, fluctuations as mediators of inelastic scattering,[13] and even phonons because of the high value of the critical temperature. In order to take into account a generic pair-breaking mechanism, we will simply introduce a pair-breaking scattering time τ_s that will shift the bare critical temperature and act as a novel energy scale in the calculations for the density of states.

Because of the uncertainty about the actual values of the pair breaking and of the ratio between l and ξ_0 we will consider in the following both the clean and the dirty samples with and without pair breaking. We shall work in two and three dimensions, while the more involved case of a three-dimensional layered structure will be the subject of future investigation.

The plan of the paper is the following. In Sec. II we will extend the known results for the density of states of dirty systems to include the pair-breaking term above T_c and in the gapless regime below T_c. In Sec. III the effects of the fluctuations on the density of states for clean systems are considered above T_c and later generalized to include a pair-breaking scattering. The tunneling current-voltage characteristics are discussed in Sec. IV together with some concluding remarks.

II. DENSITY OF STATES FOR DIRTY SUPERCONDUCTORS IN THE PRESENCE OF PAIR-BREAKING SCATTERING

The lowest-order correction to the density of states induced by the fluctuations is determined by the imaginary part of the diagram depicted in Fig. 1 for the one-electron Green's function. The calculations carried out in Refs. 5 and 6 for the dirty metal gave rise to the results summarized in Sec. I. We now extend these calculations to include a pair-breaking scattering process having a scattering time τ_s.

The correction to the density of states is given by

$$\delta N(\varepsilon)=-\frac{1}{\pi N_0}{\rm Im}R^R(\varepsilon)\ , \tag{2.1}$$

where $R^R(\varepsilon)$ is the retarded analytical continuation of

$$R(\varepsilon_n)=T\sum_{\Omega_k}\int\frac{d^dq}{(2\pi)^d}L(\mathbf{q},\Omega_k)\Lambda^2(\mathbf{q},\Omega_k,\varepsilon_n)\times\int\frac{d^dp}{(2\pi)^d}G_0^2(\mathbf{p},\varepsilon_n)G_0(\mathbf{q}-\mathbf{p},\,\Omega_k-\varepsilon_n)\ . \tag{2.2}$$

$\varepsilon_n=2\pi T(n+\frac{1}{2})$ and $\Omega_k=2k\pi T$ are the fermionic and the bosonic Matsubara frequencies, respectively, $G_0(\mathbf{p},\varepsilon_n)=(i\varepsilon_n-\xi_p+i\,\mathrm{sgn}\varepsilon_n/2\tau_t)^{-1}$ is the one-electron Green's function, Λ is the vertex due to the impurity ladder summation

$$\Lambda(\mathbf{q},\Omega_k,\varepsilon_n)=\tau_t^{-1}(Dq^2+|2\varepsilon_n-\Omega_k|+2\tau_s^{-1})^{-1}\ , \tag{2.3}$$

and $\tau_t^{-1}=\tau^{-1}+\tau_s^{-1}$ (τ_s^{-1} is usually much less than τ^{-1}). $L(\mathbf{q},\Omega_k)$ is the standard fluctuation propagator

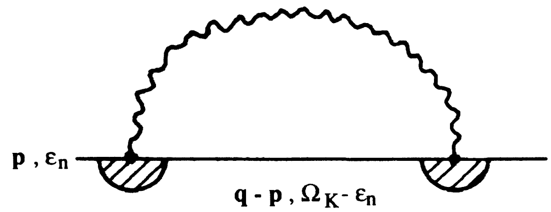

FIG. 1. Self-energy diagram for the correction to the one-electron Green's function. The wavy line indicates the fluctuation propagator $L(\mathbf{q},\Omega_k)$. The vertices are dressed with the impurity ladder.

$$L(\mathbf{q},\Omega_k)=-\frac{1}{N_0}\left[\ln\left[\frac{T}{T_c}\right]+\psi\left[\frac{1}{2}+\frac{Dq^2+|\Omega_k|+\tau_s^{-1}}{4\pi T}\right]-\psi\left[\frac{1}{2}\right]\right]^{-1}, \tag{2.4}$$

ψ being the digamma function. At small frequency and momentum, and near the critical temperature, $L(\mathbf{q},\Omega_k)$ can be written as

$$L(\mathbf{q},\Omega_k)\simeq-\frac{8T}{\pi N_0}(\tau_{\mathrm{LG}}^{-1}+Dq^2+|\Omega_k|)^{-1}\ . \tag{2.4$'$}$$

Notice that τ_s produces a shift in the critical temperature $T_c=T_c^0-(\pi/8)\tau_s^{-1}$.

Explicit calculations in the limit $\tau_s^{-1}\ll\tau_{\mathrm{LG}}^{-1}$ give rise at zero frequency to the results of Ref. 5 reported in Eqs. (1.2) and (1.3), whereas in the opposite limit $\tau_s^{-1}\gg\tau_{\mathrm{LG}}^{-1}$ we obtain

$$\delta N(0)=-\tfrac{1}{2}C_{3,d}(T_c\tau_s)^{3/2} \tag{2.5}$$

in three dimensions and

$$\delta N(0)=-C_{2,d}(T_c\tau_s)^2\ln\left[\frac{\tau_{\mathrm{LG}}}{\tau_s}\right] \tag{2.6}$$

in the quasi-two-dimensional systems. For a strictly two-dimensional case we have

$$\delta N(0)=-\frac{2}{\pi}\frac{1}{E_F\tau}(T_c\tau_s)^2\ln\left[\frac{\tau_{\mathrm{LG}}}{\tau_s}\right]\ . \tag{2.6$'$}$$

Equations (2.5)–(2.6$'$) show that τ_s^{-1} acts as a new cutoff saturating the effect of the fluctuations as $T\to T_c$.

Using Eq. (1.6) relating D to $\xi_{0,d}$, the coefficients $C_{2,d}$ and $C_{3,d}$ can be written as

$$C_{2,d}=\frac{1}{\pi^2N_0LD}=\frac{3}{p_FLE_F\tau}=\frac{1}{8\pi N_0L\xi_{0,d}^2T_c}\ ,$$

$$C_{3,d}=\frac{T_c^{1/2}}{\pi^2N_0D^{3/2}}=\frac{3^{3/2}}{2}\left[\frac{T_c}{E_F}\right]^{1/2}\frac{1}{(E_F\tau)^{3/2}}=\frac{1}{8^{3/2}\pi^{1/2}N_0\xi_{0,d}^3T_c}\ . \tag{2.7}$$

At large frequencies ($\varepsilon\gg\tau_s^{-1}\gg\tau_{\mathrm{LG}}^{-1}$) the normal value for the density of states is recovered according to the following behavior:

$$\delta N(\varepsilon)=\frac{1}{2\sqrt{2}}C_{3,d}\left[\frac{T_c}{\varepsilon}\right]^{3/2} \tag{2.8}$$

at $d=3$ and

$$\delta N(\varepsilon)=C_{2,d}\left[\frac{T_c}{\varepsilon}\right]^2\ln(\varepsilon\tau_{\mathrm{LG}}) \tag{2.9}$$

for a quasi-two-dimensional system.

The extension of the previous calculations to temperatures below the critical temperature is trivial in the gapless regime $[\Delta(T)\tau_s\ll1]$. There are no significant differences with respect to the case above T_c, except for a separation of the fluctuation propagator in two terms, one related to the amplitude fluctuation of the order parameter and still having τ_{LG}^{-1} as a mass cutoff and the other one due to the phase fluctuations, without τ_{LG}^{-1} as a mass. The resulting expression for $\delta N(\varepsilon)$ is

$$\delta N(\varepsilon)=-\frac{8T^2}{\pi N_0}\mathrm{Re}\int\frac{d^dq}{(2\pi)^d}\frac{1}{(2Dq^2+2\tau_s^{-1}-2i\varepsilon)^2}\times\left[\frac{1}{2\tau_{\mathrm{LG}}^{-1}+Dq^2}+\frac{1}{Dq^2}\right]\ . \tag{2.10}$$

Since the gapless regime also implies that $\tau_s^{-1}\gg\tau_{\mathrm{LG}}^{-1}$, in 3D the results are the same as for $T>T_c$, and δN behaves according to the formulas (2.5) and (2.8).

In the quasi-two-dimensional system the second term in parentheses of Eq. (2.10) leads to a logarithmic singularity. This has been cured in several ways. In particular in Ref. 13, the fluctuations are considered as a source of inelastic scattering and a cutoff is self-generated. A cutoff γ will in general appear together with Dq^2 in the denominator of the second term of Eq. (2.10). In this case for the quasi-two-dimensional system with $\tau_s^{-1} \gg \tau_{\rm LG}^{-1}$ we have

$$\delta N(\varepsilon) = -\tfrac{1}{2} C_{2,d} \frac{T^2}{(\varepsilon^2+\tau_s^{-2})^2} \left[(\tau_s^{-2}-\varepsilon^2) \ln \left[\frac{\varepsilon^2+\tau_s^{-2}}{\gamma \tau_{\rm LG}^{-1}} \right] + 4\varepsilon\tau_s^{-1} \arctan(\varepsilon\tau_s) + 2(\tau_s^{-2}-\varepsilon^2) \right] . \tag{2.11}$$

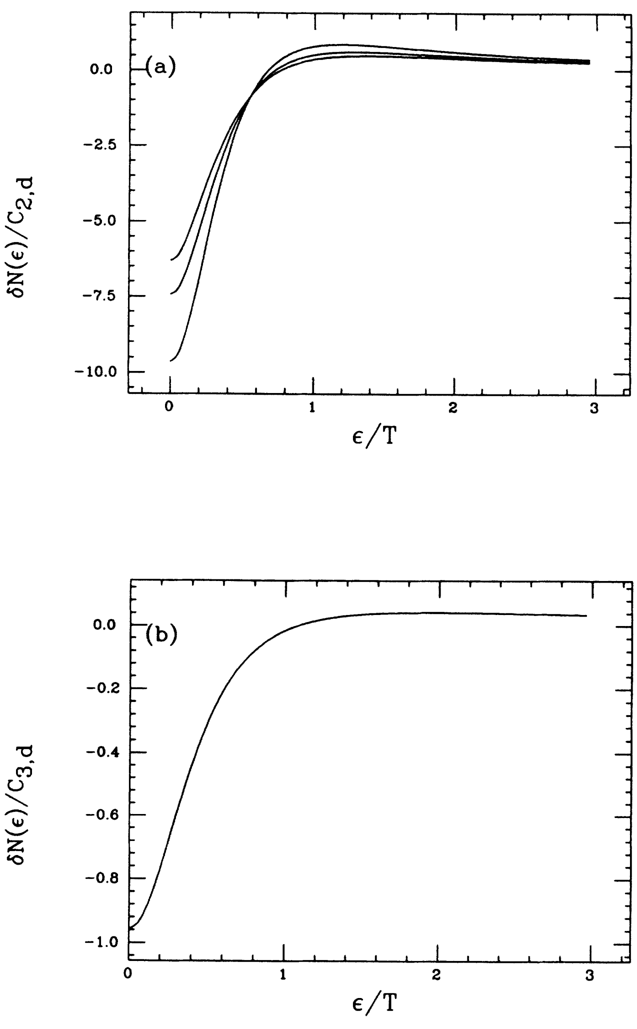

FIG. 2. The normalized correction $\delta N(\varepsilon)/C_{2\,(3),d}$ to the single-particle density of states vs the energy ε in units of T_c for a dirty superconductor with a pair-breaking term $\tau_s^{-1}=0.5T_c$. Three values of $\tau_{\rm LG}^{-1}=0.02T_c$, $0.04T_c$, $0.06T_c$ are shown. (a) refers to a quasi-two-dimensional sample with $C_{2,d}$ given by Eq. (2.7). (b) refers to a three-dimensional sample with $C_{3,d}$ given by Eq. (2.7). In this last case the three curves corresponding to the three values of $\tau_{\rm LG}^{-1}$ are superimposed.

In the two limits $\varepsilon=0$ and $\varepsilon \gg \tau_s^{-1}$ we obtain

$$\delta N(0) = -\tfrac{1}{2} C_{2,d} (T_c \tau_s)^2 \ln(\tau_{\rm LG} \gamma^{-1} \tau_s^{-2}), \quad \varepsilon=0 \tag{2.12}$$

$$\delta N(\varepsilon) = \tfrac{1}{2} C_{2,d} \left[\frac{T_c}{\varepsilon} \right]^2 \ln(\varepsilon^2 \tau_{\rm LG} \gamma^{-1}), \quad \varepsilon \gg \tau_s^{-1} . \tag{2.13}$$

When $\gamma \sim \tau_{\rm LG}^{-1}$ we recover the result (2.6) and (2.9) valid above T_c. The dependence of $\delta N(0)$ on $T-T_c$ is still logarithmic as for $T > T_c$.

Independent of being in $d=2$ or $d=3$, above or below the critical temperature, the fluctuation corrections to the density of states cease to depend significantly on temperature via powers of $\tau_{\rm LG}^{-1}$ because of the presence of τ_s^{-1} acting as a novel energy scale. The main final features of the density of states as T goes to T_c are determined by the parameter τ_s, which fixes, for instance, the strength of the depression and the position of the maximum as shown in Fig. 2(a) for $d=2$ and in Fig. 2(b) for $d=3$. This feature may be relevant in analyzing some characteristic saturation phenomena on the dependence of the differential resistance on $T-T_c$ revealed in tunneling experiments as $T \to T_c$.[8] While at $d=2$ the logarithmic dependence on $\tau_{\rm LG}$ still produces some variation of the depression (at $\varepsilon=0$) and of the maximum, at $d=3$ all curves for different values of $\tau_{\rm LG}$ are in this scale superimposed.

III. FLUCTUATION EFFECTS IN THE DENSITY OF STATES FOR CLEAN SUPERCONDUCTORS

The correction to the density of states for clean superconductors just above the critical temperature is given by the same diagram of Fig. 1 without the vertex Λ due to the impurity diffusion ladder. The expression for $\delta N(\varepsilon)$ is still given by Eq. (2.2) without the factor Λ^2. This fact does not make the problem easier, since the momentum dependence of the Green's function is now becoming important. By integrating over **p** we obtain

$$R(\varepsilon_n)=2\pi i N_0(\mathrm{sgn}\varepsilon_n)T\sum_{\Omega_k}\int\frac{d^dq}{(2\pi)^d}\frac{\Theta(\varepsilon_n(\varepsilon_n-\Omega_k))}{[\mathbf{v}_F\cdot\mathbf{q}+i(\Omega_k-2\varepsilon_n)]^2}L(\mathbf{q},\Omega_k)\ . \tag{3.1}$$

Here the fluctuation propagator $L(\mathbf{q},\Omega_k)$ is still given by Eq. (2.4′) with the substitution of D with $(8T_c/\pi)\xi_{0,c}^2$ according to Eq. (1.6).

Assuming $\varepsilon_n>0$ we evaluate the sum over bosonic frequencies by integrating over the contour C indicated in Fig. 3:

$$R(\varepsilon_n)=\frac{N_0}{2}\int\frac{d^dq}{(2\pi)^d}\oint_C\coth\left[\frac{z}{2T}\right]L(\mathbf{q},-iz)\frac{1}{(z-2i\varepsilon_n+\mathbf{v}_F\cdot\mathbf{q})^2}dz\ . \tag{3.2}$$

After shifting the variable of integration along the upper part of the contour, $R(\varepsilon_n)$ is an analytical function of $i\varepsilon_n$ and its analytical continuation is simply obtained by replacing $i\varepsilon_n$ with ε,

$$R^R(\varepsilon)=iN_0\int\frac{d^dq}{(2\pi)^d}\int_{-\infty}^{\infty}\coth\left[\frac{z}{2T}\right]\mathrm{Im}\frac{L^R(\mathbf{q},-iz)}{(z-2\varepsilon+\mathbf{v}_F\cdot\mathbf{q})^2}dz\ , \tag{3.3}$$

where we have neglected a less singular term that comes from the shifted integration and contains $\tanh(z/2T)$ instead of $\coth(z/2T)$. Carrying out the integration over z we find

$$R^R(\varepsilon)=-16iT_c^2\int\frac{d^dq}{(2\pi)^d}\left[\tau_{\mathrm{LG}}^{-1}+\frac{8T_c}{\pi}\xi_{0,c}^2q^2\right]^{-1}\left[\mathbf{v}_F\cdot\mathbf{q}-i\left[\tau_{\mathrm{LG}}^{-1}+\frac{8T_c}{\pi}\xi_{0,c}^2q^2\right]-2\varepsilon\right]^{-2}\ . \tag{3.4}$$

In addition to the substitution of D with $(8T_c/\pi)\xi_{0,c}^2$, the novel feature of the clean systems is the appearance of the term $\mathbf{v}_F\cdot\mathbf{q}$ in the denominator. This, as it will be clear in a moment, determines a change in the power-law behavior of the singular term of the density of states as function of τ_{LG} and introduces a scale of energy according to the formula (1.8) discussed in the Introduction.

In fact performing the integration over $\mathbf{q}$ in 3D and taking the imaginary part according to the formula (2.1) we obtain

$$\delta N(\varepsilon)=-C_{3,c}f_3(\varepsilon)\ , \tag{3.5}$$

where

$$f_3(\varepsilon)=\tfrac{1}{2}\mathrm{Re}\frac{\left[\dfrac{\pi}{32T_c}\dfrac{v_F^2}{\xi_{0,c}^2}\right]^{1/2}}{\left[\tau_{\mathrm{LG}}^{-1}-2i\varepsilon+\dfrac{\pi}{32T_c}\dfrac{v_F^2}{\xi_{0,c}^2}\right]^{1/2}}\times\frac{T_c}{\tau_{\mathrm{LG}}^{-1}-i\varepsilon+\tau_{\mathrm{LG}}^{-1/2}\left[\tau_{\mathrm{LG}}^{-1}-2i\varepsilon+\dfrac{\pi}{32T_c}\dfrac{v_F^2}{\xi_{0,c}^2}\right]^{1/2}} \tag{3.5′}$$

and

$$C_{3,c}=\frac{1}{2\pi^2v_FN_0\xi_{0,c}^2}=\frac{2a}{\pi}\left[\frac{T_c}{E_F}\right]^2=\frac{1}{2(2\pi)^{3/2}a^{1/2}N_0\xi_{0,c}^3T_c}\ . \tag{3.5″}$$

Recalling via Eqs. (1.6) and (1.8) that $(\pi/32T_c)(v_F^2/\xi_{0,c}^2)=(a/4)T_c$, the depression at $\varepsilon=0$ is

$$\delta N(0)=-C_{3,c}\left[\frac{T_c\tau_{\mathrm{LG}}}{a}\right]^{1/2}\ , \tag{3.6}$$

with the novel feature of a square root of τ_{LG}.

In Fig. 4 the behavior of $\delta N(\varepsilon)$ as a function of ε is given according to the formula (3.5). The new scale of energy shows up clearly in determining the behavior of $\delta N(\varepsilon)$. The normal value of the density of states is in fact first recovered at ε of the order of t_ξ^{-1} [Eq. (1.8)],

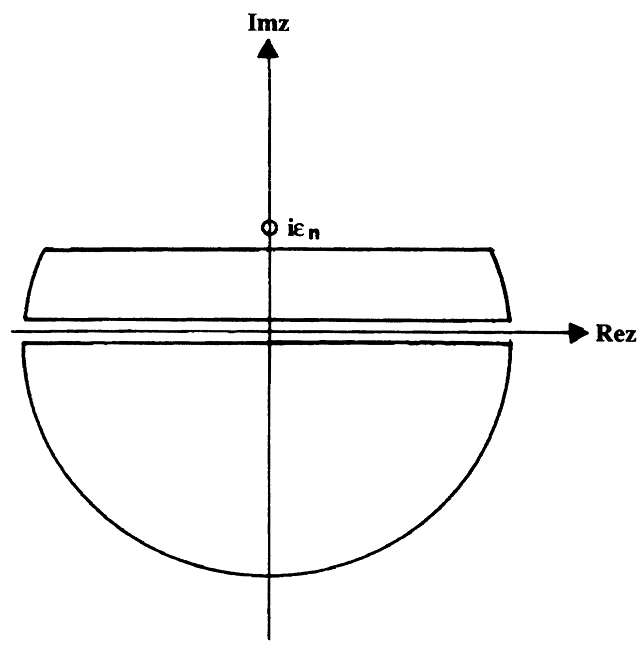

FIG. 3. Integration contour for the evaluation of the sum over the Matsubara frequencies in Eq. (3.2).

since $\delta N(\varepsilon)$ vanishes at $\varepsilon_0 \sim 2(aT_c\tau_{\rm LG}^{-1})^{1/2}$, as shown in insert. The positive deviation $\delta N(\varepsilon)$ for $\varepsilon > \varepsilon_0$ reaches a maximum at a frequency value $\varepsilon \sim 2\varepsilon_0$.

For the quasi-two-dimensional case, the general expression for $\delta N(\varepsilon)$ is

$$\delta N(\varepsilon) = -C_{2,c} f_2(\varepsilon) , \tag{3.7}$$

where

$$f_2(\varepsilon) = \frac{T_c^2}{(4\varepsilon^2 + aT_c\tau_{\rm LG}^{-1})}\left[1 - \frac{2\varepsilon}{(4\varepsilon^2 + aT_c\tau_{\rm LG}^{-1})^{1/2}} \ln\left[\frac{2\varepsilon + (4\varepsilon^2 + aT_c\tau_{\rm LG}^{-1})^{1/2}}{(aT_c\tau_{\rm LG}^{-1})^{1/2}}\right]\right] , \tag{3.7'}$$

$$C_{2,c} = \frac{8aT_c}{\pi^2 v_F^2 N_0 L} = \frac{8a}{p_F L}\frac{T_c}{E_F} = \frac{1}{\pi N_0 L \xi_{0,c}^2 T_c} . \tag{3.7''}$$

The depression in the density of states of $\varepsilon = 0$ in the region $(T - T_c)/T_c \ll 1$ reads

$$\delta N(0) = -C_{2,c}\frac{T_c \tau_{\rm LG}}{a} . \tag{3.8}$$

According to the formula (3.7), $\delta N(\varepsilon)$ vanishes again at a value ε_0 of the order of $(aT_c\tau_{\rm LG}^{-1})^{1/2}$, reaches a maximum at $\varepsilon \sim 1.8\varepsilon_0$, then the density of states approaches its normal value for higher ε as shown in Fig. 5.

As foreseen in the Introduction, the expressions for the density of states for the clean case differ from the dirty samples in three major aspects: (1) The appearance of the expansion parameter T_c/E_F instead of $1/E_F\tau$. Apart from a numerical factor this amounts to substitute $\xi_{0,d}$ with $\xi_{0,c}$ as it is transparent from the explicit expressions (2.7), (3.5''), and (3.7'') of the coefficients C. (2) Different powers in $\tau_{\rm LG}^{-1}$ and therefore in $T - T_c$. (3) The change of the characteristic energy scale from $\tau_{\rm LG}^{-1}$ to $(aT_c\tau_{\rm LG}^{-1})^{1/2}$, which was already recognized as a result of the ballistic rather than the diffusive motion of the electrons. In the clean case, the depression of the density of states around $\varepsilon = 0$ is therefore broader than in the dirty limit.

From Eqs. (3.5'') and (3.7'') it is clear that due to the small value of the coherence distance ξ_0 (~ 10 Å) in the high-T_c superconductors, with respect to the ordinary superconductors, we gain 9 orders of magnitude of fluctuation effects in the density of states at $d = 3$ and 6 orders of magnitude at $d = 2$. The choice between the formulas valid in the clean or in the dirty limit has to be done by a direct comparison between l and ξ_0 (or τ and the gap Δ_0) for each given sample. A rough estimate of the depression of the density of states according to the formulas (1.2) at $d = 3$ and (1.3) at $d = 2$ for the dirty limit and (3.5) at $d = 3$ and (3.7) at $d = 2$ for the clean limit, with some typical values of the parameters ($\xi_0 \sim 10$ Å, $L \sim 100$ Å, $N_0 \sim 10^{22}$ cm^{-3}eV^{-1}, $T_c \sim 100$ K, $E_F \sim 0.2$ eV, and $\tau \sim 10^{-14}$ s) always gives a coefficient of the order of several percents times the corresponding power law of $T_c/(T - T_c)$ coming from $\tau_{\rm LG}$. Even at temperature that deviates as much as 10% from the critical temperature, the depression at $\varepsilon = 0$ is a sizable percentage of the bare density of states for any of the expressions considered above.

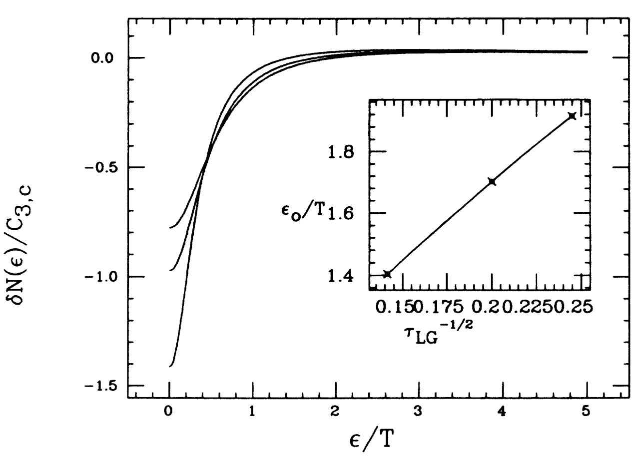

FIG. 4. The normalized correction $\delta N(\varepsilon)/C_{3,c}$ to the single-particle density of states vs the energy ε in units of T_c for a three-dimensional sample in the case of a clean superconductor above T_c, according to formula (3.5). $\tau_{\rm LG}^{-1}$ assumes the values $0.02T_c$, $0.04T_c$, and $0.06T_c$. In the inset the behavior of $\varepsilon_0(T)$ vs $\tau_{\rm LG}^{-1}$ is shown.

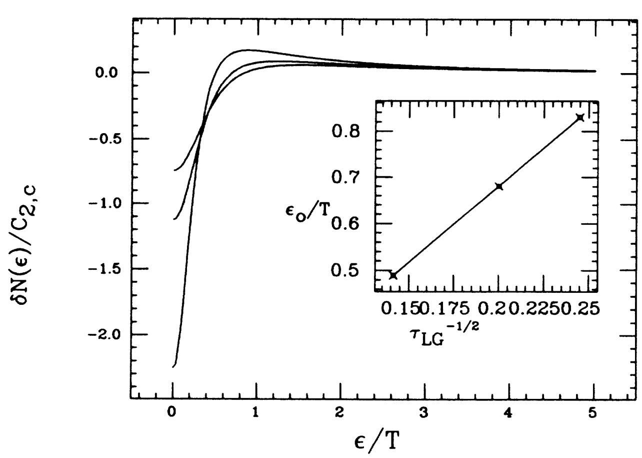

FIG. 5. The normalized correction $\delta N(\varepsilon)/C_{2,c}$ to the single-particle density of states vs the energy ε in units of T_c for a two-dimensional sample in the case of a clean superconductor above T_c, according to formula (3.7). $\tau_{\rm LG}^{-1}$ assumes the values $0.02T_c$, $0.04T_c$, and $0.06T_c$. In the inset the behavior of $\varepsilon_0(T)$ vs $\tau_{\rm LG}^{-1}$ is shown.

We qualitatively discuss the effect of a pair-breaking term in the clean case. Together with the energy scale $t_\xi^{-1} \sim v_F \xi^{-1}$, we now have the energy scale due to τ_s^{-1}. The behavior of the density of states due to fluctuations is determined by the largest between the two energies t_ξ^{-1} and τ_s^{-1}. Approaching T_c a crossover in the behavior of $\delta N(\varepsilon)$ will appear when $\tau_s^{-1} \sim t_\xi^{-1}$, with a saturation in the temperature dependence.

IV. TUNNELING CURRENT

We discuss in this section how the anomalies in the density of states reflect themselves in the tunneling current. This problem was previously considered for tunneling junctions with dirty superconductors above[7] and below[14] the critical temperature. We shall consider here the junctions with clean superconducting electrodes at $T > T_c$.

The quasiparticle current through the tunnel junction is determined, at first order of barrier transparency, by the imaginary part of the analytically continued expression of the diagram of Fig. 6(a)

$$I(V) = -e\, \mathrm{Im} K^R(\omega_\nu)|_{i\omega_\nu = eV} , \qquad (4.1)$$

where

$$K(\omega_\nu) = T \sum_{\varepsilon_n} \sum_{\mathbf{p},\mathbf{k}} |T_{\mathbf{p},\mathbf{k}}|^2 G_{\mathrm{I}}(\mathbf{p},\varepsilon_n) G_{\mathrm{II}}(\mathbf{k},\varepsilon_n+\omega_\nu) . \qquad (4.2)$$

The quantities $T_{\mathbf{p},\mathbf{k}}$ are the matrix elements of the tunneling Hamiltonian, ε_n and ω_ν are the fermionic and the bosonic Matsubara frequencies, and the subscripts I and II specify the electron Green's function for the "left-hand" and "right-hand" electrodes.

Starting from the expression (4.2) of $K(\omega_\nu)$, it is straightforward to evaluate the fluctuation contributions to the tunneling current. As a first approximation it is sufficient to take into account the corrections to the one-electron Green's function according to the diagram of Fig. 6(b). In the case of a symmetric junction it is necessary to include the diagram of Fig. 6(c) as well. Physically these contributions originate from the fluctuations in the density of states of the two electrodes. When one writes down the expression for the diagram of Fig. 6(b), the correction $\delta K(\omega_\nu)$ reads

$$\delta K(\omega_\nu) = T \sum_{\varepsilon_n} \sum_{\mathbf{p},\mathbf{k}} |T_{\mathbf{p},\mathbf{k}}|^2 G_0^2(\mathbf{p},\varepsilon_n) G_0(\mathbf{k},\varepsilon_n+\omega_\nu) \times T \sum_{\Omega_k} \int \frac{d^dq}{(2\pi)^d} L(\mathbf{q},\Omega_k) \times G_0(\mathbf{q}-\mathbf{p},\Omega_k-\varepsilon_n) . \qquad (4.3)$$

Here $L(\mathbf{q},\Omega_k)$ is the fluctuation propagator introduced in Secs. II and III.

Since the matrix elements $T_{\mathbf{p},\mathbf{k}}$ depend weakly on the momenta near the Fermi surface, we can substitute the summation over $\mathbf{p}$ and $\mathbf{k}$ in Eq. (4.3) with the integration over the energies, with the help of the formula for the normal resistance

$$\sum_{\mathbf{p},\mathbf{k}} |T_{\mathbf{p},\mathbf{k}}|^2(\cdots) = \frac{1}{4\pi e^2 R_N} \int d\xi_p \int d\xi_k (\cdots) , \qquad (4.4)$$

where $R_N^{-1} = 4\pi e^2 N_{\mathrm{I}}(0) N_{\mathrm{II}}(0) \langle |T_{\mathbf{p},\mathbf{k}}|^2 \rangle$ is the resistance of the junction in the normal state and $N_{\mathrm{I}}(0)$, $N_{\mathrm{II}}(0)$ are the densities of states of the two electrodes at the Fermi surface. Once these integrations are performed, one finds

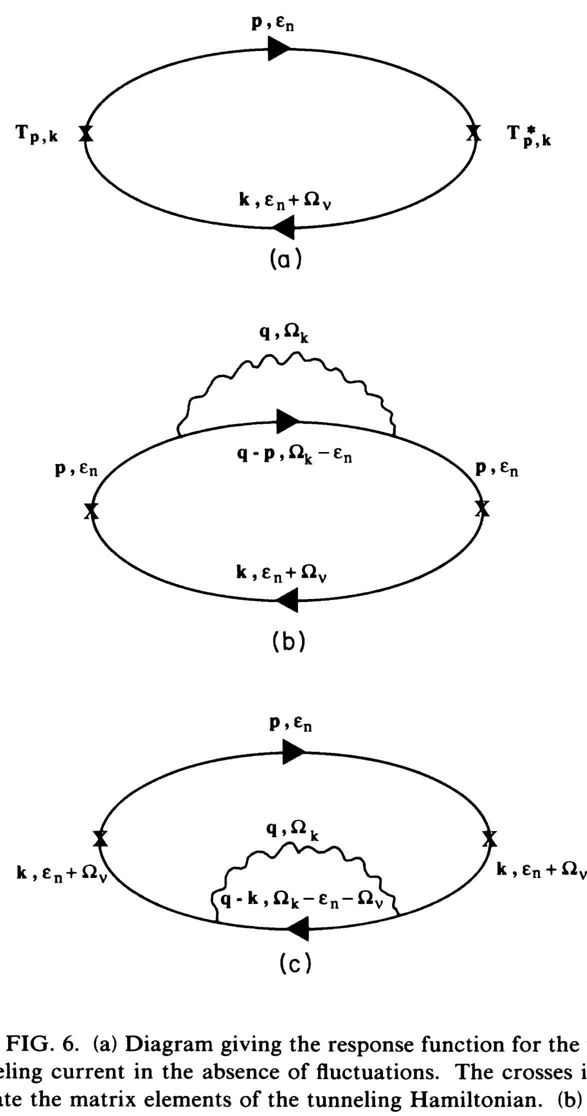

FIG. 6. (a) Diagram giving the response function for the tunneling current in the absence of fluctuations. The crosses indicate the matrix elements of the tunneling Hamiltonian. (b) and (c) corrections to the tunneling current due to the superconductive fluctuations of electrode I and of electrode II, respectively.

$$\delta K(\omega_\nu)=-\frac{\pi}{2e^2R_N}T\sum_{\varepsilon_n}\mathrm{sgn}(\varepsilon_n)\mathrm{sgn}(\varepsilon_n+\omega_\nu)T\sum_{\Omega_k}\Theta(\varepsilon_n(\varepsilon_n-\omega_\nu))\int\frac{d^dq}{(2\pi)^d}L(\mathbf{q},\Omega_k)(i\Omega_k-2i\varepsilon_n+\mathbf{v}_F\cdot\mathbf{q})^{-2}\ . \tag{4.5}$$

The sum over the Matsubara frequency Ω_k can be performed with the same procedure we used in Sec. III for the density of states. For $(T-T_c)/T_c \ll 1$ we obtain

$$\delta K(\omega_\nu)=-\frac{4T^2}{e^2R_NN_0}T\sum_{\varepsilon_n}\mathrm{sgn}(\varepsilon_n)\mathrm{sgn}(\varepsilon_n+\omega_\nu)\int\frac{d^dq}{(2\pi)^d}\left[\tau_{\mathrm{LG}}^{-1}+\frac{8T_c}{\pi}\xi_{0,c}^2q^2\right]^{-1}\left[\mathbf{v}_F\cdot\mathbf{q}+i\left[\tau_{\mathrm{LG}}^{-1}+\frac{8T_c}{\pi}\xi_{0,c}^2q^2\right]-2i\varepsilon_n\right]^{-2}\ . \tag{4.6}$$

Notice that an additional summation over the fermionic Matsubara frequency ε_n is present in Eq. (4.6), with respect to the expression for the density of states. This will introduce a cutoff of order T in the main singularity of the density of states, thus reducing the effect of the fluctuations in the tunneling current.

In fact, by performing the summation over ε_n and restricting to the most diverging terms, we obtain

$$\delta K(\omega_\nu)=\frac{T}{2\pi^2e^2R_NN_0}\int\frac{d^dq}{(2\pi)^d}\left[\tau_{\mathrm{LG}}^{-1}+\frac{8T_c}{\pi}\xi_{0,c}^2q^2\right]^{-1}\psi'\left[\frac{1}{2}+\frac{\omega_\nu}{2\pi T}\right]\ , \tag{4.7}$$

where $\psi'(x)$ is the trigamma function.

It is apparent from the expression (4.7) that the tunneling current may exhibit singular behavior only for a sufficiently thin junction [when thickness $L<\xi(T)$], i.e., for a quasi-two-dimensional electrode. In this case, carrying out the integration of Eq. (4.7), we obtain for the correction to the resistance of the junction a weak (logarithmic) singularity:

$$\frac{\delta R_{\mathrm{fl}}}{R_N}=-\frac{3\pi}{112\zeta(3)}\frac{1}{(p_FL)}\left[\frac{T_c}{E_F}\right]\ln\left[\frac{T_c}{T-T_c}\right]\mathrm{Re}\psi''\left[\frac{1}{2}-\frac{ieV}{2\pi T}\right]\ . \tag{4.8}$$

The expression (4.8) for the resistance is valid for a nonsymmetrical junction. If the junction is symmetrical, the diagram of Fig. 6(c) must be considered as well, leading to an addition factor of 2 in Eq. (4.8).

In the case of a tunnel junction with dirty superconducting electrodes, the voltage dependence of the differential resistance has the same form as Eq. (4.8), provided the prefactors for the two-dimensional dirty case are used:[7,14]

$$\frac{\delta R_{\mathrm{fl}}}{R_N}=-\frac{3}{2\pi^2}\frac{1}{(p_FL)}\frac{1}{(E_F\tau)}\ln\left[\frac{T_c}{T-T_c}\right]\mathrm{Re}\psi''\left[\frac{1}{2}-\frac{ieV}{2\pi T}\right]\ . \tag{4.9}$$

We now briefly discuss our results. As it can be seen from the expressions (4.8) and (4.9) for the resistance, the energy dependence is developed at the scale $eV\sim T$ for both clean and dirty samples. No structure is present at the energy scales that characterize the density of states, i.e., $eV\sim(aT_c\tau_{\mathrm{LG}}^{-1})^{1/2}$ and $eV\sim\tau_{\mathrm{LG}}^{-1}$ for the clean and the dirty case, respectively. The singularities present in the density of states do not appear in the differential resistance. This result is more transparent by writing the differential resistance as convolution over the density of states. In fact, assuming again a weak dependence of the matrix elements $T_{\mathbf{p},\mathbf{k}}$ on the momenta, Eqs. (4.1) and (4.2) for the tunneling current can be written in terms of the densities of states of the two electrodes

$$I(V)\sim\int_{-\infty}^{\infty}d\varepsilon\left[\tanh\left[\frac{\varepsilon+eV}{2T}\right]-\tanh\left[\frac{\varepsilon}{2T}\right]\right]\times N_{\mathrm{I}}(\varepsilon+eV)N_{\mathrm{II}}(\varepsilon)\ . \tag{4.10}$$

For $N_{\mathrm{I}}(\varepsilon)\sim$const, the differential resistance reads

$$\frac{\delta R_{\mathrm{fl}}}{R_N}\sim\int_{-\infty}^{\infty}d\varepsilon\cosh^{-2}\left[\frac{\varepsilon+eV}{2T}\right]\delta N_{\mathrm{II}}(\varepsilon)\ . \tag{4.11}$$

According to Eq. (4.11), whenever the density of states $\delta N_{\mathrm{II}}(\varepsilon)$ has a structure at energy scale $\varepsilon>T$, the temperature dependent factor in the integral would not mask it

$$\delta R(V)\sim\delta N_{\mathrm{II}}(eV)\ .$$

However, in the case under consideration, the typical energy scales of the density of states are at $\varepsilon\ll T$, and if we now perform the integration over ε in Eq. (4.11), using the expressions for the $\delta N_{\mathrm{II}}(\varepsilon)$ obtained in Sec. III, we get the less-structured behavior for the differential resistance given in Eqs. (4.8) and (4.9).

Coming back to Eq. (4.10), we note that if the electrode I is in the superconducting regime, with energy gap $\Delta_{\mathrm{I}}(T)$, the effects of the fluctuations in the electrode II can emerge as a structure around the bias voltage $eV\sim\Delta_{\mathrm{I}}(T)$. As already stated high-T_c superconductors are good candidates to observe such phenomena. However, in order to observe such an effect, we need a junction where the electrode I is a traditional superconductor below its critical temperature T_{cI} and the electrode II is a high-T_c superconductor just above its critical temperature T_{cII}. This experimental setting could be achieved by using a traditional sample with the maximum possible T_c and a novel superconductor that, while showing the physical characteristics of the high-T_c superconductivity, has a sufficiently low critical temperature $T_{cII} < T_{cI}$.

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