Theory of vortices in weakly-Josephson-coupled layered superconductors

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We present a theory for a vortex lattice in a layered superconductor, with very weak coupling between the layers. We use the Lawrence-Doniach model to show that only the component of the magnetic field that is perpendicular to the layers can give rise to an Abrikosov vortex lattice. The parallel component penetrates completely into the superconductor, which behaves as if it were magnetically transparent. The actions of these two components are very different, and almost completely decoupled. We find the lower critical fields, as well as the torque.

I. INTRODUCTION

It has been proposed recently[1] that the concept of a flux-line lattice breaks down when the magnetic field is parallel to the superconducting planes, for layered superconductors with very weak coupling between the layers, such as the high-T_C superconductor $Bi_2Sr_2CaCu_2O_8$ (Bi 2212). Several experiments on the properties of high T_C superconductors can be reconciled[1] only if the formation of an Abrikosov lattice is due to the field component $H_\perp$ alone, i.e., the component perpendicular to the layers.

In detail, the authors of Ref. 1 note that Bi 2212 becomes two dimensional half a kelvin below T_C, actually behaving as if the superconducting CuO_2 planes are decoupled. They propose that all of the experimental results can be explained if it is only the magnetic field component $H_\perp$ (perpendicular to the layers) that can create an Abrikosov vortex lattice. Both the order parameter zeros and the screening currents lie in the CuO_2 layers. The order parameter is finite only on these layers, and practically zero in between. The Bi 2212 material is magnetically transparent for the magnetic field component $H_\parallel$ (parallel to the layers). It seems thus that there is a certain *decomposition* taking place, in the sense that the superconductor responds more or less independently to the components of H along the layers and perpendicular to the layers.

This proposition is in conflict with the usual Ginzburg-Landau anisotropic model, which is presumed to be nonapplicable to these highly anisotropic superconductors. One is left then without a theory for a vortex lattice in a highly anisotropic superconductor.

It is the purpose of this paper to present precisely such a theory, derived from the standard Lawrence-Doniach model of layered superconductors.[2]

We show that indeed Abrikosov vortices arise only from the component $H_\perp$ perpendicular to the layers. In most of the experimentally interesting region, for an arbitrary field orientation, there are Abrikosov vortices due to $H_\perp$ alone, while $H_\parallel$ penetrates completely. There are two lower critical fields, $H_{c1}^\perp$ and $H_{c1}^\parallel$, quite independent of each other, and both of them very low in the case of Bi 2212. When $H_\parallel > H_{c1}^\parallel$, the parallel component of the field starts penetrating the material, which becomes magnetically transparent, and a Josephson vortex lattice with superconducting cores appears. While when $H_\perp > H_{c1}^\perp$, Abrikosov vortices begin to appear. These two conditions do not affect each other. It is thus possible to have a complete Meissner effect if $H_\parallel < H_{c1}^\parallel$, $H_\perp < H_{c1}^\perp$, or a usual Abrikosov vortex state with $B_\parallel = 0$, if $H_\parallel < H_{c1}^\parallel$, $H_\perp > H_{c1}^\perp$. It is also possible to have a *transparent state* with no Abrikosov vortices when $H_\parallel > H_{c1}^\parallel$ and $H_\perp < H_{c1}^\perp$, or a *transparent vortex state* when $H_\parallel > H_{c1}^\parallel$ and $H_\perp > H_{c1}^\perp$. The Abrikosov vortices are always perpendicular to the layers, and are independent of the Josephson vortices that are parallel to the layers.

We evaluate these lower critical fields in Sec. III, after presenting our model in Sec. II. In Sec. III we also calculate the torque for vortex lattices in fields $H_{c1} \ll H \ll H_{c2}$. Our conclusions are presented in Sec. IV.

II. THE MODEL

Consider a set of identical layers, separated by a distance d. Then the Gibbs free energy G in a uniform external magnetic field $\mathbf{H} = H_y\hat{\mathbf{y}} + H_z\hat{\mathbf{z}}$ is given by

$$G/d = \int dx \int dy \sum_n [\alpha|\Psi_n|^2 + \beta|\Psi_n|^4/2 + \hbar^2|-i\nabla_\parallel\Psi_n - 2e\,\mathbf{A}_n^\parallel\Psi_n/\hbar c|^2/2m + \eta|\Psi_{n+1} - \Psi_n \exp(2ie\,dA_{zn}/\hbar c)|^2 + (\mathbf{h}_n - \mathbf{H})^2/8\pi] \ . \quad (1)$$

Here $\Psi_n(x,y)$ is the order parameter on the nth layer, $\nabla_\parallel$ is the gradient along the layers, $A_{\parallel n}$ is the component of the vector potential of the nth layer that is parallel to the layers, and A_{zn} is the component of the vector potential of the nth layer that is perpendicular to the layers. All the constants are temperature independent, apart from $\alpha = \alpha_0(T - T_C)$, where $\alpha_0 > 0$. The constant η is the Josephson coupling between neighboring layers, and will

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be assumed to be very small throughout this work. We assume the z axis is perpendicular to the layers.

In the above discretized form of the Lawrence-Doniach model,[2] the magnetic field $\mathbf{h}_n$ is

$$\mathbf{h}_n=\hat{\mathbf{x}}\left[\frac{\partial A_{zn}}{\partial y}-\frac{A_{y,n+1}-A_{yn}}{d}\right]+\hat{\mathbf{y}}\left[\frac{A_{x,n+1}-A_{xn}}{d}-\frac{\partial A_{zn}}{\partial x}\right]+\hat{\mathbf{z}}\left[\frac{\partial A_{yn}}{\partial x}-\frac{\partial A_{xn}}{\partial y}\right]. \quad (2)$$

The Gibbs free energy is gauge invariant under the gauge transformations $\Psi_n \to \Psi_n e^{i\chi_n}$, $\mathbf{A}_n \to \mathbf{A}_n+\hbar c \nabla_{\parallel}\chi_n/2e+\hbar c(\chi_{n+1}-\chi_n)\hat{\mathbf{z}}/2ed$.

The field equations that minimize the Gibbs free energy are

$$\alpha\psi_n+\beta\psi_n^3+\hbar^2\psi_n(\nabla_{\parallel}\chi_n-2e\,\mathbf{A}_{\parallel n}/\hbar c)^2/2m+2\eta\psi_n -\eta\psi_{n-1}\cos\vartheta_{n-1}-\eta\psi_{n+1}\cos\vartheta_n=\hbar^2\nabla_{\parallel}^2\psi_n/2m\ , \quad (3)$$

where $\Psi_n=\psi_n e^{i\chi_n}$ and $\vartheta_n=\chi_{n+1}-\chi_n-2e\,dA_{zn}/\hbar c$,

$$(\hbar^2/2m)\nabla_{\parallel}\cdot[\psi_n^2(\nabla_{\parallel}\chi_n-2e\,\mathbf{A}_{\parallel n}/\hbar c)] =\eta\psi_n\psi_{n-1}\sin\vartheta_{n-1}-\eta\psi_n\psi_{n+1}\sin\vartheta_n\ , \quad (4)$$

$$\psi_n^2(2e\hbar/mc)(\partial\chi_n/\partial x-2eA_{xn}/\hbar c) =(\partial h_{zn}/\partial y-h_{yn}/d+h_{y,n-1}/d)/4\pi\ , \quad (5)$$

$$\psi_n^2(2e\hbar/mc)(\partial\chi_n/\partial y-2eA_{yn}/\hbar c) =(h_{xn}/d-h_{x,n-1}/d-\partial h_{zn}/\partial x)/4\pi\ , \quad (6)$$

$$(4e\,d\eta/\hbar c)\psi_{n+1}\psi_n\sin\vartheta_n =(\partial h_{yn}/\partial x-\partial h_{xn}/\partial y)/4\pi\ . \quad (7)$$

It is the solutions of Eqs. (3)–(7) that will give us the behavior of highly anisotropic superconductors in the presence of a uniform external magnetic field.

Let us examine some particular solutions of these equations.

(i) It is easy to verify that an exact solution of the field equations for any η, and for an arbitrary orientation of $\mathbf{H}$, is

$$\mathbf{h}_n=0,\quad \psi_n=\psi_0,\quad \mathbf{A}_n=0,\quad \chi_n=0\ , \quad (8)$$

where $\psi_0^2=-\alpha/\beta$. This is the familiar *Meissner state*, with Gibbs free-energy density

$$g_M=-\alpha^2/2\beta+\mathbf{H}^2/8\pi\ . \quad (9)$$

The magnetic field is excluded from the bulk of the material in this state.

(ii) Another exact solution, for any η, is given by

$$\psi_n=\psi(\rho),\quad \mathbf{A}_n=A(\rho)\hat{\boldsymbol{\phi}}\ ,\qquad \chi_n=\phi,\quad \mathbf{h}_n=\hat{\mathbf{z}}\left[\frac{\partial A}{\partial\rho}+\frac{A}{\rho}\right], \quad (10)$$

where ϕ is the azimuthal angle, $\rho=\sqrt{x^2+y^2}$, and $\psi(\rho)$ and $A(\rho)$ satisfy the equations

$$\psi^2(\rho)\frac{2e\hbar}{mc}\left[\frac{2eA(\rho)}{\hbar c}-\frac{1}{\rho}\right]=\frac{1}{4\pi}\frac{\partial}{\partial\rho}\left[\frac{\partial A}{\partial\rho}+\frac{A}{\rho}\right], \quad (11)$$

$$\alpha\psi(\rho)+\beta\psi^3(\rho)+\frac{\hbar^2}{2m}\psi(\rho)\left[\frac{2eA(\rho)}{\hbar c}-\frac{1}{\rho}\right]^2 =\frac{\hbar^2}{2m}\left[\frac{\partial^2\psi}{\partial\rho^2}+\frac{1}{\rho}\frac{\partial\psi}{\partial\rho}\right]. \quad (12)$$

The solutions of these two equations describe a usual Abrikosov single vortex. Thus $A(\rho)\to\hbar c/2e\rho$ and $\psi(\rho)\to\psi_0$ as $\rho\to\infty$, while $A(\rho)\sim\rho$, $\psi(\rho)\sim\rho$, as $\rho\to 0$.

A very useful analytic approximation to the solution of these equations has been given by Clem.[3] The solution to Eq. (12) is approximated by

$$\psi(\rho)=\psi_0\rho/R\ , \quad (13)$$

with $R=\sqrt{\rho^2+b^2}$, in which case Eq. (11) can be solved exactly to give

$$A(\rho)=(\hbar c/2e\rho)[1-RK_1(R/\lambda)/bK_1(b/\lambda)]\ . \quad (14)$$

Here $\lambda^2=mc^2/16\pi e^2\psi_0^2$, where λ is the penetration depth. In the high-T_C materials, λ is much larger than the coherence length ξ. In particular, we shall take the number λ/ξ to be very large, from now on.

We insert these expressions into Eq. (1) and we thus evaluate G. The variational parameters ψ_0 and b, found by minimizing G, are given by $\psi_0^2=-\alpha/\beta$ and $b^2=-\hbar^2/m\alpha=2\xi^2$. If our area of integration in the x-y plane is $\pi\Lambda^2$, then the energy density for this vortex is

$$g_V=-\frac{\alpha^2}{2\beta}+\frac{\mathbf{H}^2}{8\pi}-\frac{H_z\hbar c}{4\pi e\Lambda^2} -\frac{\hbar^2\alpha}{m\beta\Lambda^2}[\ln(\sqrt{2}\lambda/\xi)+0.173]\ . \quad (15)$$

Note that α is negative everywhere, since $T<T_C$.

This solution describes the usual Abrikosov *vortex state*, and is valid for any η or $\mathbf{H}$.

(iii) A most interesting solution of the field equations is the following:

$$\psi_n=\psi_0[1+\eta/\alpha+\eta\cos(\omega x)/(e^2h_0^2d^2/mc^2-\alpha)]\ , \quad (16)$$

$$\chi_n=2eh_0z_nx/\hbar c-8\pi\eta\psi_0^2z_n\sin(\omega x)/h_0^2d\ , \quad (17)$$

$$\mathbf{A}_n=z_n\hat{\mathbf{x}}(h_0-8\pi\eta\psi_0^2\cos(\omega x)/h_0)\ , \quad (18)$$

$$\mathbf{h}_n=\hat{\mathbf{y}}(h_0-8\pi\eta\psi_0^2\cos(\omega x)/h_0)\ , \quad (19)$$

where $\omega=2eh_0d/\hbar c$, $\psi_0^2=-\alpha/\beta$, h_0 is arbitrary, and z_n is the location of the nth layer along the z axis. Consequently, $z_{n+1}-z_n=d$. We can easily verify that these equations constitute an exact solution of the field equations (3)–(7), provided we neglect terms of order η^2. It is correct up to $O(\eta)$. It should also be noted that this expansion in η becomes invalid very close to T_C, since there $|\alpha|$ can actually become less than η, no matter how small η is. Since the coupling η in the Bi 2212 is very small, the region of temperatures where this solution fails is very close to T_C, and therefore of not too much experimental interest. It is expected anyway that this two-dimensional solution should fail very close to T_C, where the coherence length becomes infinite, rendering the system three dimensional. This region is about 0.5 K for the case of Bi 2212, as mentioned in Ref. 1. Indeed, if we use the estimates $d \approx 1.2$ nm, $\xi_{ab}(0) \approx 3.2$ nm, $T_C \approx 85$ K, $m_z/m \equiv \hbar^2/2m\eta d^2 \approx 3000$ of Ref. 1, then at half a kelvin below T_C the ratio $\eta/|\alpha|$ is about 0.4. At 82 K, this ratio has dropped to 0.067, and is small enough to justify our perturbative expansion.

This solution also fails if $h_0=0$. This will be understood later.

The parameter h_0 is determined variationally, by minimizing G. Thus we find

$$h_0 = H_y \ , \tag{20}$$

where H_y is the magnetic field component parallel to the layers. The Gibbs free-energy density of this state is

$$g_T = -\alpha^2/2\beta + H_z^2/8\pi - 2\eta\alpha/\beta + O(\eta^2) \ . \tag{21}$$

We see then that, even though there may exist a component of the external magnetic field perpendicular to the layers, only the parallel field can come through. The perpendicular field is screened. It is, in fact, as if the material is magnetically transparent along the layers. We shall call this state then the *transparent state*.

Note that the field H_y can be as large as one wishes. This indicates that the parallel upper critical field is infinite for this state. It is indeed known[2] that $H_{c2}^{\parallel}$ in layered superconductors is infinite once the dimensional crossover from 3D to 2D has occurred. Such is the case here.

A most interesting feature of the *transparent state* is the undulation of the magnitude of the order parameter and of the magnetic field $\mathbf{h}_n$. In particular, the maxima of h_{yn} coincide with the minima of the order parameter. Thus, each of the maxima of h_{yn} is in fact the core of a Josephson vortex along the layers, but the order parameter is nowhere zero. Note also that the distance along the x axis between neighboring maxima is $2\pi/\omega$.

Let us evaluate the flux through the superconductor, over the distance $2\pi/\omega$ along the x axis. It is $d\sum_n \int_0^{2\pi/\omega} h_{yn}\,dx$. But our solution satisfies the relations $\partial\chi_n/\partial x = 2eA_{xn}/\hbar c$ and $h_{yn} = (A_{x,n+1} - A_{xn})/d$, as can be seen from Eqs. (17)–(19). Thus the flux is

$$\sum_n (\hbar c/2e)[(\chi_{n+1}-\chi_n)|_{2\pi/\omega} - (\chi_{n+1}-\chi_n)|_0] = \sum_n (hc/2e) \ .$$

It is clear then that each maximum of h_{yn} is indeed the core of a Josephson vortex, since the associated flux is one quantum of flux, equal to $hc/2e$. This solution is simply a *lattice of Josephson vortices* parallel to the layers, where the order parameter remains nonzero at the core. Such Josephson vortices have also been studied previously by Bulaevskii.[4] His solution, however, is not valid for highly anisotropic layered superconductors with $\eta \to 0$, because the nonlinear cores of the vortices overlap, and the approximations he made no longer apply. Our solution (iii) takes explicitly into account the nonlinear behavior of the vortex cores.

(iv) We now present one last solution. This is in essence the combination of solutions (ii) and (iii), i.e., of the Abrikosov *vortex* state and the *transparent* state. We shall call it the *transparent vortex* state. We can easily verify that

$$\begin{aligned} &\psi_n = \psi(\rho), \quad \chi_n = 2eh_0 z_n x/\hbar c + \phi \ , \\ &\mathbf{A}_n = h_0 z_n \hat{\mathbf{x}} + A(\rho)\hat{\boldsymbol{\phi}} \ , \\ &\mathbf{h}_n = h_0\hat{\mathbf{y}} + \hat{\mathbf{z}}(\partial A/\partial\rho + A/\rho) \ , \end{aligned} \tag{22}$$

is an exact solution of the field equations (3)–(7), *for* $\eta=0$, provided $\psi(\rho)$ and $A(\rho)$ are given by (11) and (12). This is clearly an Abrikosov vortex perpendicular to the layers that is pierced by a magnetic field parallel to the layers. It is a *transparent vortex*, the superposition of a usual Abrikosov vortex parallel to the z axis and of the absence of screening of the magnetic field along the layers.

We can verify further that, in the region far from the core, the expressions

$$\begin{aligned} &\psi_n \approx \psi_0[1 + \eta/\alpha + \eta\cos(\omega x)/(e^2h_0^2d^2/mc^2 - \alpha)] \ , \\ &\chi_n \approx 2eh_0 z_n x/\hbar c - 8\pi\eta\psi_0^2 z_n \sin(\omega x)/h_0^2 d + \phi \ , \\ &\mathbf{A}_n \approx z_n\hat{\mathbf{x}}(h_0 - 8\pi\eta\psi_0^2\cos(\omega x)/h_0) + \hbar c\hat{\boldsymbol{\phi}}/2e\rho \ , \\ &\mathbf{h}_n \approx \hat{\mathbf{y}}(h_0 - 8\pi\eta\psi_0^2\cos(\omega x)/h_0) \ , \end{aligned} \tag{23}$$

with $\psi_0^2 = -\alpha/\beta$, constitute a solution of the field equations (3)–(7) far from the core, correct to $O(\eta)$. Just as in case (iii), the solution is invalid if $h_0=0$ or if we are too close to T_C.

We can see very clearly from these expressions that the vortex can screen the magnetic field H_z, but not the field H_y. So the parallel field can penetrate the material, whether there is an Abrikosov vortex [as in (iv)] or not [as in (iii)]. The existence of an Abrikosov vortex perpendicular to the layers cannot influence the magnetic transparency along the layers, and vice versa. This is precisely the claim of Ref. 1, and it is most aptly demonstrated by Eqs. (23).

The parameter h_0 will be determined by minimizing G. There is some difficulty, however, in doing this, because the solution is not easy to find in the region of the core. In that region ψ is not a simple function of x and y, and there is some interplay between the nature of the Josephson vortex solution (iii) parallel to the layers and the nature of the Abrikosov vortex solution perpendicular to the layers.

It is best then to use a method similar to the one used in case (ii). For simplicity we shall omit all terms of $O(\eta^2)$ in G. Then we can obtain G correct up to $O(\eta)$ by inserting the *unperturbed* solution into our general expression for the Gibbs free energy; Clem's ansatz[3] comes in quite handy then. So we shall insert the *unperturbed* solution (22) into (1), using the $\psi(\rho)$ and $A(\rho)$ of (13) and (14). The variational parameters are h_0, ψ_0, and b. Note that unlike cases (i) and (iii), where ψ_0 was determined exactly from the field equations, here it will be determined variationally. We find $\psi_0^2=(-\alpha-2\eta)/\beta$, $h_0=H_y$, and $b^2=-\hbar^2/m\alpha$. If our region of integration is $\pi\Lambda^2$, and we neglect terms of order η/Λ^2, then the Gibbs free-energy density is

$$g_{TV}=-\frac{\alpha^2}{2\beta}-\frac{2\eta\alpha}{\beta}+\frac{H_z^2}{8\pi}-\frac{H_z\hbar c}{4\pi e\Lambda^2}-\frac{\hbar^2\alpha}{m\beta\Lambda^2}[\ln(\lambda\sqrt{2}/\xi)+0.173]\ . \tag{24}$$

We have thus seen that there are four possible states of equilibrium for highly anisotropic layered superconductors. Apart from the usual Meissner and vortex states, there is also the *transparent* state and the *transparent vortex* state. These differ from the other two in that they allow the parallel component of the magnetic field to penetrate unhindered into the material. There is no screening of the magnetic field along the layers for these *transparent* solutions, even though flux is still quantized, with each Josephson vortex having one flux quantum. In the next section we shall examine the conditions under which each one of the four states mentioned above is the actual state of thermal equilibrium.

III. LOWER CRITICAL FIELDS AND TORQUES

Comparison of g_M and g_V gives the lower critical field for the direction perpendicular to the layers,

$$H_{c1}^{\perp}=(\hbar c/4e\lambda_0^2)[\ln(\lambda_0\sqrt{2}/\xi)+0.173]\ , \tag{25}$$

with $\lambda_0^2=-mc^2\beta/16\pi e^2\alpha$. If $H_z>H_{c1}^{\perp}$, Abrikosov vortices will appear.

Comparison of g_V and g_{TV} gives the lower critical field parallel to the layers,

$$H_{c1}^{\parallel}=\sqrt{-16\pi\eta\alpha/\beta}\ . \tag{26}$$

If $H_y>H_{c1}^{\parallel}$, then the parallel field penetrates completely into the material, and a Josephson vortex lattice appears. For the Bi 2212 material both of these critical fields are very small.

It is easy then to see that

(i) If $H_y<H_{c1}^{\parallel}$, $H_z<H_{c1}^{\perp}$, we have the Meissner state.

(ii) If $H_y<H_{c1}^{\parallel}$, $H_z>H_{c1}^{\perp}$, we have the usual Abrikosov vortex state.

(iii) If $H_y>H_{c1}^{\parallel}$, $H_z<H_{c1}^{\perp}$, we have the *transparent state* (i.e., a lattice of Josephson vortices parallel to the layers).

(iv) If $H_y>H_{c1}^{\parallel}$, $H_z>H_{c1}^{\perp}$, we have the *transparent vortex* state (i.e., a lattice of Abrikosov vortices perpendicular to the layers, and a Josephson vortex lattice parallel to the layers).

In fact, since both of these fields are so small for Bi 2212, and most experiments use fields $H_{c1}\ll H\ll H_{c2}$, highly anisotropic high-T_C superconductors are mostly in the *transparent vortex* state.

Note that at the point $h_0=H_y=0$, where solutions (iii) and (iv) break down, they are no longer energetically favorable. Therefore the limit $h_0\to 0$ presents no problem.

Typically then, if we start with a magnetic field $H\gg H_{c1}^{\perp}$ perpendicular to the layers, we have the usual Abrikosov vortex lattice. As we begin rotating the field direction away from the normal to the layers, the material becomes magnetically transparent along the layers, and a Josephson vortex lattice appears parallel to the layers. The angle θ that the field **H** makes with the normal to the layers is $\sin^{-1}(H_{c1}^{\parallel}/H)$ at that point, and is quite small. As we keep rotating, the density of the Abrikosov vortices decreases, because the perpendicular component of the field decreases, and this is the only component that affects the Abrikosov vortices. Eventually the Abrikosov vortices disappear, when $\cos\theta=H_{c1}^{\perp}/H$, and we get the *transparent* state of our solution (iii). In this state, there are no Abrikosov vortices. There is only the lattice of Josephson vortices along the layers, which does not screen the parallel magnetic field.

We have already described the structure of a single Abrikosov vortex coexisting with a Josephson vortex lattice in terms of (22) and (23). However, most experiments are done in regimes with many Abrikosov vortices. In particular, torque magnetometry experiments[5] probe the superconducting anisotropy precisely in such regimes, where demagnetization effects are very small. However, no formal treatment of the torque in the two-dimensional case has been presented as yet. It is precisely this theoretical vacuum that we shall now attempt to fill, especially in view of the educated guess that has been made in this regard in Ref. 1.

In other words, we shall calculate the torque in a region where it is a good approximation to neglect the Abrikosov vortex cores, and to set ψ_n equal to a constant. Such a calculation has only been done so far in the context of the Ginzburg-Landau model.[6] We present below the corresponding calculation for the two-dimensional case.

Let us calculate then the torque for the *transparent vortex* state. We use the standard methods of Ref. 6. We assume the absence of demagnetization effects for simplicity. This assumption is justified when $H_{c1}\ll H_z\ll H_{c2}$ by the smallness of the magnetization.[6] In that case the torque is given by $T_\theta=-V\partial g/\partial\theta$, where θ is the angle between the field orientation and the normal to the planes, and V is the sample volume. The torque is shape independent, at least to the extent that terms involving the square of the magnetization have been neglected. We shall only keep terms in G up to $O(\eta)$, thus we need simply use the $\eta=0$ solution of the field equations. Then G will automatically be correct to $O(\eta)$.

We shall let ψ_n be constant almost everywhere. Thus let $\psi_n\approx\psi_0$, with $\psi_0^2=-\alpha/\beta$. Let also

$$\chi_n=2eh_0z_nx/\hbar c+\chi(x,y)\ , \tag{27}$$

$$\mathbf{A}_n=h_0z_n\hat{\mathbf{x}}+A_x(x,y)\hat{\mathbf{x}}+A_y(x,y)\hat{\mathbf{y}}\ , \tag{28}$$

$$\mathbf{h}_n=h_0\hat{\mathbf{y}}+\hat{\mathbf{z}}h_z(x,y)\ , \tag{29}$$

where $h_z=\partial A_y/\partial x-\partial A_x/\partial y$. The parameter h_0 is found variationally, by minimizing G, to be $h_0=H_y$.

These equations solve exactly the field equations (4)–(7), for $\eta=0$, provided

$$2e\hbar\psi_0^2(\nabla\chi-2e\,\mathbf{A}/\hbar c)/mc=\nabla\times(h_z\hat{\mathbf{z}}/4\pi)\ . \quad (30)$$

Taking the curl leads to the London equation

$$h_z-\lambda^2\nabla^2 h_z=\phi_0\sum_i \delta_2(\mathbf{r}-\mathbf{r}_i)\ , \quad (31)$$

where $\lambda^2=mc^2/16\pi e^2\psi_0^2$, $\phi_0=hc/2e$, and $\mathbf{r}_i$ is the location of a vortex.

This equation can be solved, since h_z is periodic, in terms of Fourier transforms.[6] We find $h_z=\sum_{\mathbf{q}} h_{\mathbf{q}} e^{i\mathbf{q}\cdot\mathbf{r}}$, where the $\mathbf{q}$'s run over the reciprocal lattice of the triangular vortex array, and where $h_q=B_z/(1+\lambda^2\mathbf{q}^2)$.

We can now evaluate G. Unlike the standard treatment,[6] though, we do not drop the 1 from the factor $1+\lambda^2\mathbf{q}^2$. For evaluating the sums, we approximate them to integrals, with cutoffs $q_{\max}=1/b=1/\xi\sqrt{2}$ and $q_{\min}=(\phi_0/\pi B_z)^{-1/2}$. We obtain the Gibbs free-energy density

$$g=-\frac{\alpha^2}{2\beta}-\frac{2\eta\alpha}{\beta}+\frac{(B_z-H_z)^2}{8\pi} -\frac{B_z\phi_0}{32\pi^2\lambda^2}\ln\left[\frac{2\xi^2}{\lambda^2}+\frac{2\xi^2\pi B_z}{\phi_0}\right]\ . \quad (32)$$

B_z can be found from $\partial g/\partial B_z=0$:

$$B_z-H_z\approx(\phi_0/8\pi\lambda^2)\ln(2\xi^2/\lambda^2+2\xi^2\pi B_z/\phi_0)\ . \quad (33)$$

Thus we see that the magnetization is of order $H_{c1}^{\perp}$, and hence small. Our neglect of demagnetization effects is then justified, at least to the extent that $O(H_{c1}^2)$ terms may be neglected.

We now note that g is a function of H_z, not only explicitly, but also through B_z, which is itself a function of H_z. Thus $dg/dH_z=(\partial g/\partial B_z)(dB_z/dH_z)+(\partial g/\partial H_z)$. Since $\partial g/\partial B_z=0$, this leaves us with $dg/dH_z=(H_z-B_z)/4\pi$. Thus the torque is $VH\sin\theta(H_z-B_z)/4\pi$, where V is the sample volume, and $H_z=H\cos\theta$. This is precisely the expression used in Ref. 1.

In the region $H_{c1}^{\perp}\ll H\cos\theta\ll H_{c2}$ we can replace B_z by H_z inside the logarithm, since $H_z-B_z\ll H_z$, and we end up with

$$T_\theta\approx-VH\sin\theta\frac{\phi_0}{32\pi^2\lambda^2}\ln\left[\frac{2\xi^2}{\lambda^2}+\frac{2\xi^2\pi H\cos\theta}{\phi_0}\right]\ . \quad (34)$$

Closer to $H_{c1}^{\perp}$,

$$T_\theta=VH\sin\theta(H_z-B_z)/4\pi\ , \quad (35)$$

where B_z is defined implicitly by (33). Note that if $B_z=0$, then this equation implies that $H_z\approx(\phi_0/4\pi\lambda^2)\ln(\lambda/\xi\sqrt{2})\approx H_{c1}^{\perp}$. Therefore, when the angle θ has rotated so closely to 90° that there are very few Abrikosov vortices left ($\cos\theta\approx H_{c1}^{\perp}/H$), T_θ becomes approximately $VHH_{c1}^{\perp}/4\pi$. Once the angle θ gets even closer to 90° than that, we have a transition from the *transparent vortex* state to the *transparent* state. Therefore, Eq. (35) becomes invalid, and we have to use $T_\theta=-V\partial g_T/\partial\theta$, assuming the absence of demagnetization effects. Using (21) we obtain

$$T_\theta\approx VH^2\sin(2\theta)/8\pi\ . \quad (36)$$

This holds, of course, only in the *transparent* state ($\cos\theta<H_{c1}^{\perp}/H$). Observe that, at the transition, T_θ takes again the value $VHH_{c1}^{\perp}/4\pi$.

Finally we have to calculate the torque for angles very close to $\theta=0$. There, $H_y<H_{c1}^{\parallel}$ and there is the usual Abrikosov vortex lattice, with no magnetic transparency. In this case, g is given again by (32), after replacing the $-2\eta\alpha/\beta$ term by $H_y^2/8\pi$. This change fully takes into account the disappearance of the magnetic transparency. Thus B_z is still given by (33), but now the torque is $-VHB_z\sin\theta/4\pi$. Since H_z and B_z differ by terms of $O(H_{c1}^{\perp})$, according to Eq. (33), the demagnetization torques, which are proportional to the square of the magnetization, are $O(H_{c1}^2)$, and have been correctly neglected. We may use Eq. (33) to approximate the torque for this case of $H_y<H_{c1}^{\parallel}$ by

$$T_\theta\approx-VH^2\sin(2\theta)/8\pi -VH\sin\theta\frac{\phi_0}{32\pi^2\lambda^2}\ln\left[\frac{2\xi^2}{\lambda^2}+\frac{2\xi^2\pi H\cos\theta}{\phi_0}\right]\ . \quad (37)$$

We have now completed the calculation of the torques in the two-dimensional case, the case of highly anisotropic layered superconductors. Let us summarize our results.

We assumed that $H\gg H_{c1}^{\parallel}$, $H\gg H_{c1}^{\perp}$. At $\theta=0$, the external field is normal to the layers and there is the usual Abrikosov vortex lattice. The torque vanishes at $\theta=0$, and is given by Eq. (37) in the vicinity. But soon after, when H_y becomes greater than $H_{c1}^{\parallel}$, it becomes energetically favorable to go into a *transparent vortex* state. The torque is then given by (34). As θ keeps increasing, the torque keeps changing. In fact, the best way to express it in most of the region from 0° up to almost 90° is through (33) and (35). Note that T_θ increases monotonically in all of this region. Finally, at $\theta\approx 90°$, when H_z becomes smaller than $H_{c1}^{\perp}$, the Abrikosov vortex lattice disappears, and we are left with a *transparent state* of Josephson vortices with no Abrikosov vortices, where the torque is given by (36). Note that at $\theta=90°$, the torque is zero. Therefore there is a very acute maximum in the torque when $\cos\theta\approx H_{c1}^{\perp}/H$, at the point where the Abrikosov vortices disappear. This is precisely the behavior typically observed in torque magnetometry experiments.[5]

IV. CONCLUSIONS

In this paper we have presented the description of highly anisotropic layered superconductors in terms of a Lawrence-Doniach type of model. Our model differs from that of Ref. 2 in that the vector potential, as well as the order parameter, has been discretized in the direction normal to the layers.

We have found four possible states of thermal equilibrium. The state of the system is determined by the components of the external magnetic field. If $H_z<H_{c1}^{\perp}$ there are no Abrikosov vortices, and $B_z=0$ (Meissner state in the z direction). If $H_z>H_{c1}^{\perp}$, we have an Abrikosov vortex lattice. On the other hand, if $H_y<H_{c1}^{\parallel}$, the parallel field component is screened, but if $H_y>H_{c1}^{\parallel}$, it penetrates completely and the material becomes magnetically transparent. These two distinct behaviors do not affect each other, except possibly at the cores. In other words, the magnetic transparency along the layers and the creation of Abrikosov vortex lines perpendicular to the layers are independent of each other. Abrikosov vortices exist only if $H_z > H_{c1}^{\perp}$, they are always perpendicular to the layers and they are created by the field component H_z. No Abrikosov vortices exist if the external field is strictly parallel to the layers.

This theoretical description fully justifies the proposition of Ref. 1, and therefore their interpretation of apparently conflicting experimental results.

We have further calculated the torque, which is given for most of the angles by (34). Unlike the expression used in Ref. 1, this is still well defined at $\theta = 90°$, but, of course, it is no longer relevant very close to $\theta = 90°$. In that tiny region, there are no Abrikosov vortices anymore, so we have to use Eq. (36). As a result, the behavior of the torque changes from monotonically increasing to monotonically decreasing, and so the torque must be maximum very close to $\theta = 90°$, as is indeed observed.[5] This maximum of the torque is a very pronounced feature of the experimental results and denotes, as we said, the disappearance of the Abrikosov vortices at the point where H_z is too small to be able to give rise to them.

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