Flux flow in layered high-T_c superconductors

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(Received 11 June 1990)

For layered high-temperature superconductors we have calculated the current-voltage curve in the flux-flow regime when the magnetic field is parallel or slightly inclined to the plane of layers. We assume that there are no defects in the crystal and that the pinning is entirely intrinsic: It results from the interaction between vortices and the layered structure of the superconductor.

I. INTRODUCTION

Flux creep, flux flow, and pinning in high-temperature superconductors are now under intensive investigation (see, e.g., Refs. 1–7) since it was suggested[1] that flux creep plays an important role in magnetic measurements. Usually, pinning is produced by various defects and inhomogeneities of the material such as dislocations, twin boundaries, etc. Moreover, most high-temperature superconductors are layered compounds. In layered compounds there is an additional pinning, an intrinsic interlayer pinning, which works when the vortices are parallel to the layers and the Lorentz force produced by the transport current acts perpendicular to the layers. This intrinsic pinning results from the interaction between vortices and the layered structure and does not depend on a concentration of defects. It can be observed in sufficiently perfect single crystals.

The intrinsic pinning was first discussed in Ref. 8. In Refs. 9 and 10, the authors have calculated the depinning current for layered superconductors using the Ginzburg-Landau theory in two limiting cases, i.e., when $\xi_c(T) \gg s$ and $\xi_c(T) \cong s/\sqrt{2}$. [Here $\xi_c(T)$ is the coherence length along the crystal c axis and s is the interlayer distance.] In Ref. 11 we studied the flux creep due to the intrinsic pinning.

In Ref. 4 a strong anisotropy has been found of the depinning current as a function of the angle between the magnetic field and the plane of layers: The depinning current had narrow maxima near magnetic-field directions parallel to the layers (the Cu-O planes). This behavior can be certainly attributed to the intrinsic pinning. The fact that the intrinsic depinning current should be very sensitive to the magnetic-field orientation can be expected on the basis of rather general considerations. In the present paper, we will discuss this problem by considering the vortex motion in the flux-flow regime when the magnetic field is parallel or slightly inclined with respect to the layers. We show that the intrinsic depinning current decreases rapidly already at rather small tilting angles of the magnetic field, according to the angular dependence observed experimentally.[4] We calculate also the current-voltage curve for a layered superconductor in the flux-flow regime using the time-dependent Ginzburg-Landau (TDGL) theory for layered superconductors.

The analysis of experimental data shows[12,13] that the coherence length in the crystal c direction, $\xi_c(T)$, exceeds the interlayer distance s in a rather broad temperature region near T_c for the high-temperature $YBa_2Cu_3O_7$ compound. One can consider, therefore, the $YBa_2Cu_3O_7$ compound as an essentially three-dimensional anisotropic superconductor with a weakly layered structure [having $\xi_c(T) \gg s$] at least near T_c. On the other hand, Bi-Sr-Ca-Cu-O-type compounds are closer to two-dimensional superconductors and have more pronounced layered structure with $\xi_c(T) \sim s$ practically at all temperatures except for, maybe, a very narrow vicinity of T_c. The case with $\xi_c(T) \sim s$ we call the highly layered structure. In the present paper, we consider both structures.

Both limits can be described by the Ginzburg-Landau model which considers a layered superconductor as a system with the Josephson interaction between layers.[14,15] We use its time-dependent modification for nonstationary problems. To justify the TDGL approach, one can argue that, in high-temperature materials, superconductivity should be nearly gapless, at least close to T_c, because of a strong electron-phonon interaction.[16].

We restrict ourselves to magnetic fields close to H_{c2}. Strictly speaking, pinning effects can be suppressed considerably by thermal fluctuations in the region close to the H_{c2} curve on the phase diagram. As is known for the usual pinning by defects, the region, on the phase diagram, where vortices are pinned and the region where they are depinned by fluctuations are separated by the so-called depinning (or irreversibility) line. The depinning line is rather far from the H_{c2} curve. The intrinsic pinning, however, is not yet well studied, and it is difficult to establish, on the basis of experimental observations, where the depinning line lies in this case. On the other hand, theoretical predictions show[11] that the activation energy for the intrinsic pinning is quite high, especially for low currents. One can expect, therefore, that the depinning line should be very close to the H_{c2} curve. In any case, using the approximations pertaining to the limit

$H \rightarrow H_{c2}$ should not change qualitatively the results in the region where the intrinsic pinning is essential.

Now we will discuss the problem in more detail. We suppose first that the magnetic field is parallel to the layers. We choose a coordinate system with the z axis perpendicular to the layers and the y axis along the magnetic field. The potential energy of the intrinsic pinning is a periodic function of the vortex displacement u_z in the direction perpendicular to the layers with the period equal to the interlayer distance. According to Ref. 11, the displacement u_z should be measured with respect to the equilibrium positions of vortices in a lattice whose configuration corresponds to the minimum of the pinning energy V_p, i.e., with respect to sites in a lattice commensurate with the layered structure. We call the lattice commensurate if one of its unit-cell vectors is parallel to the layers and the other has a z projection in a rational proportion to the interlayer distance. The lattice can be incommensurate when the pinning energy is small compared with the elastic energy of the deformation needed to make the lattice commensurate. In Ref. 11 we came to the conclusion that the depinning current should have a nonmonotonic dependence on the magnetic field with maxima corresponding to fields at which the vortex lattice is commensurate with the layered structure. If the lattice is commensurate, the vortex displacement in equilibrium is zero. On the other hand, if the lattice is incommensurate, the vortex displacements u_z are nonzero in equilibrium and depend on coordinates. As a result, the pinning energy $\langle V_p \rangle$ averaged over the sample volume would not depend on vortex positions. This implies that the intrinsic pinning vanishes.

A similar situation holds when the magnetic field (and thus the vortices) are inclined with respect to the layers. If the vortex displacement u_z due to an inclined magnetic field varies by more than one interlayer distance s on the sample width along the y axis, the averaged pinning potential $\langle V_p \rangle$ would not depend on the vortex positions and the pinning would vanish. In this case the vortex ends lie in different valleys of the potential V_p. A vortex will have inflections (kinks) in places where it passes from one valley of V_p to another. These kinks can slide easily in the y direction along the layers under the action of the Lorentz force produced by the transport current flowing in the x direction. The sliding of kinks results in a vortex motion across the layers (see Fig. 1).

This takes place, of course, if there is no pinning preventing from the motion of kinks along the layers. In practice, however, a sample may have defects, such as twin boundaries, which pin the vortex motion along the layers (i.e., perpendicular to twin boundaries), but do not affect much the vortex motion perpendicular to the layers (i.e., parallel to twin boundaries).[5] Under these conditions the flux creep across the layers would be determined by the intrinsic pinning even if the magnetic field is slightly inclined with respect to the layers.

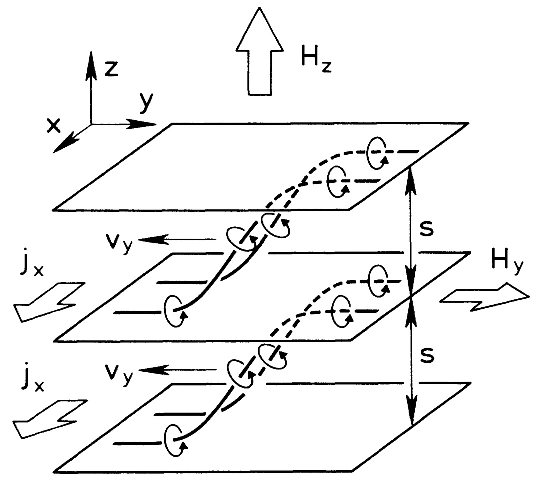

FIG. 1. Motion of vortex lines in an inclined magnetic field. Ends of each vortex lying in different valleys of the pinning potential are fixed, but the kinks are moving under the action of the Lorentz force produced by the transport current.

In the present paper, we will assume that there are no defects affecting the vortex motion. We calculate the current-voltage curve in the flux-flow regime in presence of the intrinsic pinning. We do not take into account the flux-creep effects, assuming that the activation probability is small according to Ref. 11, so that the resistivity of the superconductor is mainly due to the flux flow at currents higher than the depinning current. In Sec. III we consider a weakly layered structure. A similar problem was solved earlier[9,10] but for a different region of magnetic fields. In Sec. III we calculate the flux-flow conductivity for a highly layered structure.

II. VORTEX MOTION IN A WEAKLY LAYERED SUPERCONDUCTOR

A. Basic equations

We will use the time-dependent Ginzburg-Landau theory for layered superconductors with the Josephson interaction between layers in the form

$$\gamma \left[\frac{\partial}{\partial t} + 2ie\phi \right] \psi(n,\mathbf{r}) = -\frac{\delta F}{\delta \psi^*(n,\mathbf{r})} , \qquad (1)$$

where the free energy is[14]

$$F = \int d^2r\, s \sum_n \left[\alpha |\psi(n,\mathbf{r})|^2 + \tfrac{1}{2}\beta |\psi(n,\mathbf{r})|^4 + \frac{1}{2m} |(-i\nabla - \frac{2e}{c}\mathbf{A})\psi(n,\mathbf{r})|^2 + \frac{1}{2Ms^2} \left| \psi(n+1,\mathbf{r}) \exp\left[-\frac{2ie}{c} \int_{ns}^{(n+1)s} A_z dz \right] - \psi(n,\mathbf{r}) \right|^2 \right] + \frac{1}{4\pi} \int \left[\frac{\tilde{\mathbf{H}}^2}{2} - \mathbf{H}\tilde{\mathbf{H}} \right] d^3r \ . \qquad (2)$$

Here $\tilde{\mathbf{H}}$ is a microscopic magnetic field, $\mathbf{H}$ is an applied magnetic field, $\psi(n,\mathbf{r})$ is the order parameter at the nth layer, and γ is a phenomenological "viscosity" parameter. In case of a gapless superconductor, one can use the microscopic value $\gamma=1/2mD$, where $D=v_F^2\tau/2$ is the two-dimensional (in the plane of layers) diffusion coefficient. The electric current density in the plane of layers is

$$\mathbf{j}=\mathbf{j}^{(n)}-\frac{ie}{m}\left[\psi^*\left[\nabla-\frac{2ie}{c}\mathbf{A}\right]\psi-\text{c.c.}\right], \tag{3}$$

where $\mathbf{j}^{(n)}=\sigma_{ab}^{(n)}\cdot\mathbf{E}$ is a normal current. In Eqs. (2) and (3), $\mathbf{r}$, ∇, and $\mathbf{A}$ are two-dimensional vectors in the plane of layers.

In this section we restrict ourselves to the case of a weakly layered structure with $\xi_c(T)\gg s$. Let us take first the magnetic field parallel to the y axis, so that $\mathbf{A}=(0,0,-H_y x)$. The solution of Eqs. (1)–(3) for a stationary commensurate vortex lattice has been found in Ref. 10. It is

$$\psi_p^{(0)}(n,x)=\sum_{k=1}^{N}C_k\exp\left[\frac{2\pi ikn}{N}\right]\psi_p\left[x+\frac{\pi ck}{eHsN}\right]. \tag{4}$$

A commensurability of the lattice suggested by Eq. (4) comes from the fact that the number N (i.e., the size of the "Brillouin zone" with respect to the index k) is an integer. In Eq. (4), $\psi_p(x)$ is the Bloch function of the lowest-energy band $\varepsilon(p)$:

$$\hat{H}\psi_p(x)=\varepsilon(p)\psi_p(x), \tag{5}$$

of the Hamiltonian

$$\hat{H}=-\frac{1}{2m}\frac{\partial^2}{\partial x^2}+\frac{1}{Ms^2}\left[1-\cos\left[\frac{2eHsx}{c}\right]\right]. \tag{6}$$

The energy spectrum of the lowest-energy band is

$$\varepsilon(p)=\frac{h}{2Ms^2}-\frac{\Delta}{2}\cos\left[\frac{\pi cp}{eHs}\right],$$
$$\Delta=\frac{16h^{1/2}}{Ms^2\pi^{1/2}}\exp\left[-\frac{8}{h}\right]. \tag{7}$$

Here

$$h=\frac{2eHs^2}{c}\left[\frac{M}{m}\right]^{1/2}. \tag{8}$$

The quasimomentum p determines the supercurrent in the x direction:

$$j_x^{(s)}=2e\frac{\partial\varepsilon(p)}{\partial p}\langle|\psi_p^{(0)}|^2\rangle. \tag{9}$$

The order parameter normalization is given by nonlinear terms in the Ginzburg-Landau equation

$$\langle|\psi_p^{(0)}|^2\rangle=\sum_{k=1}^{N}|C_k|^2=\frac{m^2c^2h}{16\pi e^2\kappa^2Ms^2}\frac{H_{c2}-H}{\beta_A H_{c2}}. \tag{10}$$

Here $\kappa=\lambda_{ab}/\xi_{ab}$ is the Ginzburg-Landau parameter, m and M are the effective Ginzburg-Landau masses in the plane of layers and perpendicular to it, respectively. To the leading approximation in Δ, the upper critical field can be found from the equation $\alpha+h/2Ms^2=0$ and is

$$H_{c2}=\frac{c}{2e\xi_{ab}\xi_c}.$$

The factors C_k in Eq. (4) satisfy some periodicity conditions. In a weakly layered superconductor, the vortex lattice and, thus, the factors C_k differ only slightly from those for a three-dimensional anisotropic superconductor and the lattice parameter $\beta_A\cong 1.16$.[10]

Now we apply a small magnetic field H_z along z in addition to H_y. Vortices will be inclined with respect to the y axis, and this can be described by a displacement $u_z(y)$. One can use the perturbation theory if u_z is a slowly varying function of y (and also of t). When the vortex lattice is displaced by u_z, the vector potential $A_x=-H_y u_z$ arises due to flux quantization. It corresponds to the magnetic field with the induction

$$B_z=H_y\left\langle\frac{\partial u_z}{\partial y}\right\rangle. \tag{11}$$

When the vortices are displaced by $\mathbf{u}=(u_x,u_z)$, the order parameter transforms as

$$\psi=\sum_{k=1}^{N}C_k\exp\left[\frac{2\pi ik}{N}\left[n-\frac{u_z}{s}\right]-\frac{2ieH}{c}(x-u_x)u_z\right]\times\psi_{p-2eA_x/c}\left[x-u_x+\frac{\pi ck}{eHsN}\right]. \tag{12}$$

Substituting Eq. (12) into the free energy [Eq. (2)], we get[11]

$$F=\int dV\left\{\frac{C_{11}-C_{66}}{2}\left[\frac{\partial u_x}{\partial x}+\frac{\partial u_z}{\partial z}\right]^2+\frac{C_{66}}{2}\left[\left[\frac{\partial u_x}{\partial x}\right]^2+\frac{M}{m}\left[\frac{\partial u_z}{\partial x}\right]^2+\frac{m}{M}\left[\frac{\partial u_x}{\partial z}\right]^2+\left[\frac{\partial u_z}{\partial z}\right]^2\right]\right.$$
$$\left.+\frac{C_{44}}{2}\left[\left[\frac{\partial u_x}{\partial y}\right]^2+\left[\frac{\partial u_z}{\partial y}\right]^2\right]-\frac{Hju_z}{c}+V_p(u_z)\right\}. \tag{13}$$

This free energy contains the elastic energy of the vortex lattice, the Lorentz force produced by the transport current and the pinning energy V_p. The latter is obtained from Eq. (8) by a substitution, $p \to p + 2eH_y u_z/c$:

$$V_p = [\varepsilon(p+2eH_y u_z/c) - h/2Ms^2]\langle|\psi|^2\rangle - \frac{H_{c2} j_c s}{2\pi c}\cos\left[\frac{\pi c p}{eHs} + 2\pi\frac{u_z}{s}\right] . \quad (14)$$

The critical current j_c has been calculated in Refs. 9 and 10:

$$j_c = \frac{4c(H_{c2}-H_y)m}{\sqrt{\pi}\beta_A \kappa^2 s M}\left[\frac{\xi_c}{s}\right]\exp\left[-\frac{8\xi_c^2}{s^2}\right] .$$

In presence of a vortex-lattice displacement, the transport current is

$$j = 2e\frac{\partial V_p}{\partial p} = j_c \sin\left[\frac{\pi c p}{eHs} + 2\pi\frac{u_z}{s}\right] . \quad (15)$$

As in Ref. 11, we will assume for simplicity that the sample thickness along z is less than the corresponding effective penetration depth $\lambda_c^{\rm eff} \sim \lambda_c(1-H/H_{c2})^{-1/2}$. In this case we may assume that u_z does not depend on z, and $u_x = 0$, and omit the term div**u** in Eq. (13).

To incorporate a time dependence of u_z, we present the TDGL equations in the form[17]

$$\frac{dF}{dt} = -\int W\, d^3r , \quad (16)$$

where the dissipation function is

$$W = 2\gamma\left|\frac{\partial\psi}{\partial t} + 2ie\phi\psi\right|^2 + \sigma_{ab}^{(n)}\mathbf{E}^2 . \quad (17)$$

The dissipation function can be calculated using Eq. (12). We employ the gauge with $\phi = 0$, so that

$$E_x = -\frac{1}{c}\frac{\partial A_x}{\partial t} = \frac{H_y}{c}\frac{\partial u_z}{\partial t} .$$

The Bloch function ψ_p can be expressed in terms of Wannier functions

$$\psi_p(x) = \sum_m \exp(ipx_0 m)\phi_0(x - x_0 m) ,$$

where $x_0 = \pi c/eHs$ is the period of the potential energy in the Hamiltonian (6). The Wannier function $\phi_0(x)$ for the lowest-energy band is close to the oscillator function when $x \ll x_0$. Calculations give

$$\langle W\rangle = \eta\left[\frac{\partial u_z}{\partial t}\right]^2 ,$$

where

$$\eta = \left[\frac{\sigma_{ab}^{(n)} H_{c2}^2}{c^2} + \gamma\frac{2eH_{c2}}{c}\left[\frac{M}{m}\right]^{1/2}\langle|\psi|^2\rangle\right] . \quad (18)$$

Using the microscopic expression $\gamma = 1/2mD$, we get, from Eqs. (16) and (18),

$$\eta = H_{c2}^2 \sigma_f / c^2 , \quad (19)$$

where

$$\sigma_f = \sigma_{ab}^{(n)}\left[1 + \frac{u_0}{2}\frac{H_{c2}-H}{\beta_A H_{c2}}\right]$$

is the flux-flow conductivity $[u_0 = \pi^4/14\zeta(3) \cong 5.79]$.[10] This expression is similar to that for an isotropic superconductor.[18]

Now, from Eqs. (13) and (16) we can derive the equation for u_z. Assuming that u_z is a function only of y and t, we get

$$\eta\frac{\partial u_z}{\partial t} - C_{44}\frac{\partial^2 u_z}{\partial y^2} - \frac{H_y j}{c} + \frac{H_y j_c}{c}\sin\left[\frac{\pi c p}{eHs} + 2\pi\frac{u_z}{s}\right] = 0 . \quad (20)$$

Equation (20) is different from the time-dependent equation used in Refs. 9 and 10. Though both equations are valid near the upper critical magnetic field, their applicability regions are still different. The equation used in Refs. 9 and 10 is valid when the magnetic field is so close to H_{c2} that the nonlinear terms in the Ginzburg-Landau equation are small compared with the energy bandwidth [Eq. (7)]: $\beta\langle|\psi|^2\rangle \ll \Delta$. We would like also to draw attention to the fact that the statement in Ref. 10 concerning possible negative differential resistivity does not actually hold within the applicability region.

Equation (20), as well as the equation used in Ref. 11, is based on the order parameter representation in the form of Eq. (12). It is valid in the opposite limit when the energy bandwidth Δ is less than the nonlinear terms in the Ginzburg-Landau equations, i.e., when

$$\left[\frac{s}{\xi_c}\right]\exp\left[-\frac{8\xi_c^2}{s^2}\right] \ll \frac{H_{c2}-H_y}{H_{c2}} , \quad (21)$$

but still $H_{c2} - H_y \ll H_{c2}$. This region is more wide and more practical from the experimental point of view than that considered in Refs. 9 and 10. Slow variations of u_z suggested earlier require that

$$B_z \ll H_y \cong H_{c2} . \quad (22)$$

B. *I-V* curve of a weakly layered superconductor

In this section we consider the commensurate vortex lattice. For a principal commensurability, for example, when the period projection on the z axis is $Z_0 = Ks$ (K is an integer), the magnetic field H_y should satisfy the condition

$$H_y = \frac{\sqrt{3}}{2}\left[\frac{m}{M}\right]^{1/2}\frac{\Phi_0}{s^2 K^2} . \quad (23)$$

According to Ref. 11, commensurate configurations of the lattice have regions of attraction on the magnetic-field axis.

Let us consider first the vortex motion when the magnetic field is strictly parallel to the layers. One can put $\partial u_z/\partial y=0$ in this case. We get, from Eq. (20),

$$\frac{\eta c}{H_{c2}}\frac{\partial u_z}{\partial t}=j-j_c\sin\left[\frac{2\pi u_z}{c}\right] \tag{24}$$

(we have incorporated a constant p into u_z). When $j>j_c$, a nonzero averager velocity appears. To calculate it we find the time t_0 needed for the motion of a vortex by one interlayer distance. Integrating Eq. (24), we get

$$t_0=\frac{cs\eta}{H_{c2}}(j^2-j_c^2)^{-1/2} .$$

The average velocity is

$$\left\langle\frac{\partial u_z}{\partial t}\right\rangle=\frac{s}{t_0}=\frac{H_{c2}}{c\eta}(j^2-j_c^2)^{1/2} .$$

The electric field induced in the sample is

$$E=\frac{H_y}{c}\left\langle\frac{\partial u_z}{\partial t}\right\rangle=(j^2-j_c^2)^{1/2}/\sigma_f . \tag{25}$$

This current-voltage curve is similar to that for a resistively shunted Josephson junction.[19] The critical current j_c is just the depinning current.

If the magnetic field is inclined slightly to the plane of layers so that $B_z/H_y>s/L_y$, where L_y is the width of the sample along the y axis, the ends of each vortex lie in different valleys of the potential V_p and the depinning current vanishes. The vortex kinks formed where vortices pass from one valley of V_p to another can now move along the layers. We look for a solution of Eq. (20) in the form $u_z=u_z(y-v_yt)$. We have

$$\eta v_y\frac{\partial u_z}{\partial y}+C_{44}\frac{\partial^2 u_z}{\partial y^2}+\frac{H_{c2}}{c}\left[j-j_c\sin\left[\frac{2\pi u_z}{s}\right]\right]=0 .$$

Multiplying this equation by $\partial u_z/\partial y$ and integrating it over dy from $-L_y/2$ to y, we get

$$\frac{R_0^2}{2s^2}\left[\frac{\partial u_z}{\partial y}\right]^2+\frac{2\pi j}{j_c}\left[\frac{u_z}{s}\right]+\cos\left[\frac{2\pi u_z}{s}\right]+\frac{2\pi c\eta v_y}{sH_{c2}j_c}\int_{-L/2}^{y}\left[\frac{\partial u_z}{\partial y}\right]^2 dy=\text{const} , \tag{26}$$

where

$$R_0=\left[\frac{2\pi cC_{44}s}{H_{c2}j_c}\right]^{1/2} . \tag{27}$$

For the case of small tilting angles when the distance between kinks is much larger than R_0 (i.e., $B_z/H_y\ll s/R_0$), one can put $\partial u_z/\partial y=0$ far from kinks. For one kink we get, from Eq. (26),

$$\frac{2\pi j}{j_c}\left[\frac{u_+-u_-}{s}\right]+\cos\left[\frac{2\pi u_+}{s}\right]-\cos\left[\frac{2\pi u_-}{s}\right]=-\frac{2\pi c\eta v_y}{sH_{c2}J_c}\int_{-\infty}^{\infty}\left[\frac{\partial u_z}{\partial y}\right]^2 dy , \tag{28}$$

where $u_\pm=u_z(y=\pm\infty)$. If B_z is positive, then $u_+-u_-=s$ and

$$\frac{j}{j_c}=-\frac{c\eta v_y}{sH_{c2}j_c}\int_{-\infty}^{\infty}\left[\frac{\partial u_z}{\partial y}\right]^2 dy . \tag{29}$$

At low currents, $j\ll j_c$, the form of the kink can be found from EQ. (26) with $j=v_y=0$. To the first approximation, we get

$$\frac{R_0^2}{2s^2}\left[\frac{\partial u_z}{\partial y}\right]^2=1-\cos\left[\frac{2\pi u_z}{s}\right] .$$

For the kink velocity we have

$$v_y=-\frac{\pi R_0H_{c2}j}{4cs\eta} . \tag{30}$$

Since the average electric field is

$$\langle E_x\rangle=-B_zv_y/c , \tag{31}$$

we find that the effective conductivity is

$$\sigma_{\text{eff}}=\sigma_f\frac{4s}{\pi R_0}\frac{H_{c2}^{(ab)}}{B_z} . \tag{32}$$

This conductivity is much larger than σ_f due to a small density of kinks (small B_z).

If now we increase the z component of the magnetic field so that $B_z/H_y>s/R_0$, the effect of the intrinsic pinning will decrease. When $B_z/H_y\gg s/R_0$, the layered structure would not affect the vortex motion any more. Indeed, in this case $\partial u_z/\partial y=B_z/H_y$ and is constant. According to Eq. (26), we get $\mathbf{j}=\sigma_f\mathbf{E}$, i.e., just as for a three-dimensional superconductor.

III. VORTICES IN A HIGHLY LAYERED SUPERCONDUCTOR

Materials with $\xi_c(T)\sim s$ we call highly layered compounds. This criterion is satisfied, for example, by Bi-Sr-Ca-Cu-O-based superconductors. The case of $\xi_c\sim s$ can also be considered on the basis of Eqs. (1)–(3). The most simple situation is when the coherence length $\xi_c(T)\cong s/\sqrt{2}$. This possibility has been pointed out in Ref. 15. We will also consider this limit in the present paper. The upper critical field in the direction parallel to the layers is now large and diverges as $\xi_c(T)\to s/\sqrt{2}$. One can understand this as if the vortex cores fit just in between the layers so that the order parameter taken at the layers is not affected.

In this section we discuss the vortex behavior in an inclined magnetic field and calculate the effective flux-flow conductivity.

A. Model

We recall first the static solution.[10,15] The limit $\xi_c(T)\to s/\sqrt{2}$ corresponds to $h\gg 1$ in Eq. (8). The solution of Eqs. (5) and (6) for the lowest energy $\varepsilon_H(0)=(1-h^{-2})/Ms^2$ is

$$w(x)=1+2h^{-2}\cos(2eHsx/c)+(h^{-4}/2)\cos(4eHsx/c)\ . \tag{33}$$

The condition $\varepsilon_h(0)=|\alpha|$ gives the upper critical field[15]

$$H_{c2}=\frac{\phi_0}{2\pi s^2}\left[\frac{m}{M}\right]^{1/2}\frac{\xi_c(T)}{[\xi_c^2(T)-s^2/2]^{1/2}}\ . \tag{34}$$

With Eq. (33) one can construct the function

$$\psi(n,x)=\sum_q c_q\exp(iqsn)w\left[x+\frac{cq}{2eH}\right], \tag{35}$$

similar to Eq. (4), which is to describe the vortex lattice. The lattice parameter

$$\beta_L=\frac{\langle|\psi|\rangle^4}{\langle|\psi|^2\rangle^2}$$

is then equal to

$$\beta_L=\left[\sum_{q_1q_2q_3}c_{q_1}c^*_{q_2}c_{q_3}c^*_{q_1-q_2+q_3}\right]\Big/\left[\sum_q|c_q|^2\right]^2, \tag{36}$$

since $w(x)\cong 1$. Thus the lattice parameter β_L is proportional to the number of terms in the sum (35). Since the lowest β_L value is the most favorable one, we can conclude that there should be only one term in the sum (35). A n-dependent phase factor in Eq. (35) can be incorporated into the vector potential A_z. Therefore, we have

$$\psi(n,x)=Cw(x)\ , \tag{37}$$

and the lattice parameter $\beta_L=1$.

Equation (37) suggests that the supercurrent flows only along the z axis: $j_x=0$ and

$$\begin{aligned} j_z&=\frac{2e}{Ms}\mathrm{Im}\left[\psi^*(n+1,x)\psi(n,x)\exp\left[\frac{2ie}{c}\int_{ns}^{(n+1)s}A_z dz\right]\right]\\ &=\frac{2e}{Ms}|C|^2\sin\left[\frac{2eHsx}{c}\right]\ . \end{aligned} \tag{38}$$

Such distribution of currents corresponds to a rectangular vortex lattice. The fact that $\psi(n,x)$ does not actually depend on n implies that the vortex lattice is always commensurate with the layered structure, and its period along z is just $Z_0=s$. The commensurability takes place over the whole region of magnetic fields where nonlinear terms in the Ginzburg-Landau equations are small, i.e., when $H_{c2}-H\ll H_{c2}$.[20]

We assume now that, in addition to the current j_z and the magnetic field H_y, there are small currents in the (x,y) plane and a magnetic field H_z along z. The order parameter will have the form of Eq. (37), but the factor C will now be a slow function of x and y (and also of t). To obtain the equation for this function, let us put

$$\psi(n,r,t)=\psi(x,y,t)w(x)\ . \tag{39}$$

The free energy [Eq. (2)] then becomes

$$\begin{aligned} F=L_z\int dx\,dy\Bigg[&[\alpha+\varepsilon_{H_y}(0)]|\psi|^2+\frac{\beta\beta_L}{2}|\psi|^4\\ &+\frac{1}{2m}\left|\left[\nabla-\frac{2ie}{c}\mathbf{A}\right]\psi\right|^2\\ &+\frac{1}{8\pi}(\tilde{\mathbf{H}}^2-2\tilde{\mathbf{H}}\mathbf{H})\Bigg]\ . \end{aligned} \tag{40}$$

Equation (1) will have the same form also for the function ψ since $w(x)\cong 1$. It is just the usual TDGL equation, but with the characteristic length of variations of ψ replaced by

$$\begin{aligned} \xi_{\rm eff}(T)&=\{2m[|\alpha|-\varepsilon_{H_y}(0)]\}^{-1/2}\\ &=\frac{h\xi_{ab}}{\sqrt{2}}\left[1-\frac{H}{H_{c2}}\right]^{-1/2}\ . \end{aligned} \tag{41}$$

The length $\xi_{\rm eff}$ is considerably longer than the period x_0 of $w(x)$.

B. Vortices in an inclined field: Dissipation

The static equation which results from Eqs. (1) and (40) has the form

$$\frac{\xi_{\rm eff}^{-2}}{2m}\psi-\beta|\psi|^2\psi+\frac{1}{2m}\left[\nabla-\frac{2ie}{c}\mathbf{A}\right]^2\psi=0\ . \tag{42}$$

Equation (42) together with Eq. (3) for the current in the (x,y) plane are completely similar to the usual set of the Ginzburg-Landau equations with $\xi_{\rm eff}$ instead of ξ. This implies, in particular, that the problem of vortices in an inclined magnetic field for highly layered superconductors in our model reduces to the problem of vortices in usual superconductors, but in a field H_z. On the basis of this similarity, one can conclude that there exist the lower $H_{c1}^{(z)}$ and the upper $H_{c2}^{(z)}$ critical fields. The lower critical field is

$$H_{c1}^{(z)}=\left[\frac{\Phi_0}{4\pi\lambda_{\rm eff}}\right]\ln\kappa\ , \tag{43}$$

where

$$\lambda_{\rm eff}^2=\frac{mc\beta}{16\pi e^2[|\alpha|-\varepsilon_{H_y}(0)]}\ .$$

To find the upper critical field, one can take $A_x=-H_z y$. The solution of the linearized equation (42) has the usual form

$$\begin{aligned} &\psi\sim\exp\left[-\frac{e|H_z|y^2}{c}\right],\\ &|H_z|=H_{c2}^{(z)}=\frac{\Phi_0}{2\pi\xi_{\rm eff}^2}\ . \end{aligned} \tag{44}$$

If $H_{c2}(\Theta)$ is the upper critical field for the angle Θ between the magnetic field and the plane of layers, then $H_{c2}^{(z)}=H_{c2}(\Theta)\sin\Theta$. One has, from Eq. (44) for small Θ,

$$H_{c2}(\Theta)-H_{c2}(0)=-\frac{H_{c2}^2(0)h^2}{2H_{c2}(\pi/2)}|\Theta| , \qquad (45)$$

where $H_{c2}(0)$ and $H_{c2}(\pi/2)$ are the critical fields in directions parallel and perpendicular to the layers, respectively. Equation (45) coincides with the result of Ref. 15 and resembles the angular dependence of the critical field for thin films.[21]

When the magnetic field H_z inside the superconductor is less than $H_{c1}^{(z)}$, there are no vortices parallel to the z axis which could be produced by H_y, and ψ nowhere turns to zero. This can be understood as if the vortices produced by the total field $\mathbf{H}=(0,H_y,H_z)$ remain parallel to the layers when H_z is small enough. When H_z inside the superconductor is larger than H_{c1}, the vortices parallel to z are created, and there appear points where $\psi=0$. One can say that the vortices produced by the total field now cross the layers. The vortices exist until the magnitude of the total field reaches $H_{c2}(\Theta)$. A qualitatively similar picture of a vortex penetration has been considered in Ref. 7. The existence of vortices parallel to the z axis at small tilting angles of the applied magnetic field depends very much on the sample shape: For example, for a thin plate with $L_z \ll L_y$, a vortex penetration begins for very small angles, since H_z outside the sample is equal to B_z inside due to the boundary conditions.

The dynamic equation for the new order parameter results from Eqs. (1) and (40) and has the form

$$-\gamma\left[\frac{\partial\psi}{\partial t}+2ie\phi\psi\right]+\frac{\xi_{\rm eff}^{-2}}{2m}\psi-\beta|\psi|^2\psi+\frac{1}{2m}\left[\nabla-\frac{2ie}{c}\mathbf{A}\right]^2\psi=0 . \qquad (46)$$

The problem of a vortex motion in fields strictly parallel to the layers seems to be more complicated in this case than for a weakly layered superconductor. It resembles probably the problem of the resistive state in thin superconducting films (see, for example, Ref. 22 and references therein). We will not consider here this problem which, surely, deserves a special study. One can find the critical depairing current from Eq. (46) assuming a homogeneous distribution of the current:

$$j_c=\frac{c^2}{es^3}\left[\frac{m}{M}\right]^{3/2}\frac{2}{3\sqrt{3}\pi\kappa^2h^3}\left[1-\frac{H}{H_{c2}}\right]^{3/2} .$$

It coincides with the result obtained in Refs. 9 and 10.

The vortex motion in an inclined magnetic field can be considered on the basis of Eqs. (3) and (46) in a way similar to that for usual superconductors. Using the results obtained in Refs. 18 and 23, we get the effective conductivity

$$\sigma_{\rm eff}=\begin{cases}1.45\sigma_{ab}^{(n)}\dfrac{H_{c2}^{(z)}}{B_z} , & B_z \ll H_{c2}^{(z)} ,\\[2ex] \sigma_{ab}^{(n)}\left[1+\dfrac{u_0}{2}\,\dfrac{H_{c2}^{(z)}-B_2}{\beta_A H_{c2}^{(z)}}\right] , & B_z \to H_{c2}^{(z)} .\end{cases}$$

This result does not depend on the orientation of the transport current with respect to the magnetic field within the plane of layers. The reason is that the vortices created by the magnetic-field component H_y and parallel to the layers fit completely in between the layers. Therefore, the energy dissipation does not depend on whether or not the vortices are moving along the x axis parallel to the layers. The dissipation is mainly due to the motion of vortices produced by the magnetic-field component perpendicular to the layers.

IV. CONCLUSION

We have considered the flux flow in layered superconductors when the magnetic field is parallel or slightly inclined to the plane of layers. It was assumed that there are no defects in the crystal so that the pinning is entirely due to an intrinsic mechanism, resulting from an interaction between vortices and the layered structure of the superconductor. We discussed the cases of $\xi_c(T)\gg s$ (weakly layered) and $\xi_c(T)\sim s$ (highly layered structures). A weakly layered structure is realized in the $YBa_2Cu_3O_7$ compound near the critical temperature, and a highly layered structure is the one which is present in materials like Bi-Sr-Ca-Cu-O-type compounds.

For a weekly layered superconductor, we have calculated the current-voltage curve for fields parallel and slightly inclined with respect to the layers. The intrinsic depinning current has very sharp maxima at small angles between the magnetic field and the layers. At higher angles, the vortices move in a flux-flow regime. The effective flux-flow conductivity decreases with an increase in the magnetic-field component perpendicular to the layers, and finally, the effect of the layered structure on the vortex motion vanishes at still rather small tilting angles.

In a highly layered material, the magnetic field parallel to the layers does not affect much the superconductivity. Vortices are created mainly by the magnetic-field component perpendicular to the layers. In an inclined magnetic field, the vortex motion is a flux flow. The corresponding flux-flow conductivity is calculated on the basis of effective TDGL equations.

ACKNOWLEDGMENTS

The authors are grateful to V. L. Pokrovsky for valuable discussions. N.B.K. expresses his thanks to Low Temperature Laboratory at Helsinki University of Technology for hospitality.

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