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K gaps for surface polaritons on gratings: Excitation by fast electrons

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We have derived the equations for light emission due to fast particles impinging on a metallic grating. We have solved these equations numerically to study the emission peaks due to the decay of surface polaritons (SPO's) on a Ag grating bombarded by 80-keV electrons. The SPO dispersion curve can be inferred from these peaks. Our calculations reproduce the anomalous behavior (k gap) of the SPO dispersion curve that was observed by Heitmann *et al.* near the $2\pi/a$ Brillouin zone boundary, a being the period of the grating.

I. INTRODUCTION

The dispersion curve of surface polaritons (SPO's) (Ref. 1) on a metallic grating has been measured by various experimental techniques. In optical reflectivity experiments, dips in the reflectivity of light trace out the SPO dispersion curve.[2,3] In experiments where the SPO is excited by fast electrons (50–80 keV) impinging on a thick metallic grating and the SPO subsequently radiates through the periodic grating, peaks in the emission trace out the SPO dispersion curve.[4] Another experiment that is of technological interest is light emission due to tunneling current in a corrugated metal-oxide-metal (MOM) junction. Again peaks in the emission spectrum trace out the SPO dispersion.[5,6] In all of these experiments the SPO dispersion curve is determined from the dips (peaks) through the kinematic condition

$$K_{sp}=K+nG=(\omega/c)\sin\theta+nG \ . \tag{1}$$

$G=2\pi/a$ is the basic reciprocal lattice wave vector, ω is the angular frequency of light, c is the speed of light, n is an integer, and θ (measured relative to the normal to the mean surface) is the direction of the outgoing photon that corresponds to minimum (maximum) intensity. We recall that the SPO wave vector for the flat surface of a medium of (complex) dielectric constant ϵ is

$$K_{sp}=\mathrm{Re}\left[\frac{\epsilon}{\epsilon+1}\right]^{1/2}\frac{\omega}{c} \ . \tag{2}$$

On a grating, the SPO dispersion relation is expected to depart from Eq. (2) by the appearance of energy gaps (ω gaps) at the Brillouin zone boundaries. The surprising fact is that all of the preceding methods have revealed anomalous behavior of the SPO dispersion near the $2\pi/a$ zone boundary.[1,7,8] The extreme form of this anomaly can be described as the appearance of a momentum gap (K gap) rather than an energy gap (ω gap) at the zone boundary. Recently Celli, Tran and co-workers[9,10] have shown that near the $2\pi/a$ zone boundary the interference between the forward and backward propagating SPO can alter the position of the dips in the reflectivity experiment in such a way that the dispersion curve inferred from these dips is anomalous. To compare with experiment a full calculation of the system's response to an external field is necessary; it is not enough to find the poles of the response function in the complex ω (or K) plane. Thus a separate calculation must be carried out for each experiment. In this paper we focus on the surface plasmon radiation excited by electrons (SPREE) experiment, in which fast electrons are used to excite the SPO. The coupled equations for the emitted light will be derived and solved numerically to study the SPO dispersion near the $2\pi/a$ zone boundary.

II. THE EQUATIONS FOR LIGHT EMISSION

In this section the general equations for the emitted fields from a two-dimensional grating will be derived. The thick film used in the experiments is replaced in the calculation by a semi-infinite medium bounded by the surface

$$z=\zeta(x,y) \ . \tag{3}$$

The medium, characterized by the dielectric function $\epsilon(\omega)$, occupies the region $z<\zeta$. Since the incident electrons have energy in the keV range, we can treat them classically and neglect recoil. The current due to one electron is then given by

$$\mathbf{j}(\mathbf{r},t) = -e\mathbf{v}\delta(\mathbf{r}-\mathbf{v}t-\mathbf{R}_0) , \tag{4}$$

where $\mathbf{v}=(\mathbf{V},-v_\perp)$ is the electron's velocity and $\mathbf{R}_0$ is the impact parameter. In this notation $v_\perp$ and e are positive quantities. Because the impact parameter for each electron is random we must average over $\mathbf{R}_0$ in the final analysis.

The electric field on the vacuum side, $\mathbf{E}_v$, can be written as the sum of two parts, a homogeneous field $\mathbf{E}_h$ and an inhomogeneous field $\mathbf{E}_i$. Within the range of validity of the Rayleigh ansatz, $\mathbf{E}_h$ can be expanded in the form[11]

$$\mathbf{E}_h(r,t) = \sum_{\mathbf{K}} \int \frac{d\omega}{2\pi} \mathbf{E}_h(\mathbf{K},\omega) e^{i\mathbf{K}\cdot\mathbf{R}} e^{i(pz-\omega t)} , \tag{5}$$

where $\mathbf{K}$ is the momentum in the x-y plane, $\mathbf{R}\equiv(x,y)$, and p satisfies the equation

$$p^2 = \left[\frac{\omega}{c}\right]^2 - K^2 . \tag{6}$$

We can further decompose this field into s and p polarization as follows:

$$\mathbf{E}_h(\mathbf{K},\omega) = (\hat{\mathbf{K}} - \hat{\mathbf{z}}K/p) A_p(\mathbf{K},\omega) + (\hat{\mathbf{z}}\times\hat{\mathbf{K}}) A_s(\mathbf{K},\omega) . \tag{7}$$

The inhomogeneous field, generated by the current in Eq. (4), is

$$\mathbf{E}_i(\mathbf{r},t) = \sum_{\mathbf{K}} \int \frac{d\omega}{2\pi} \mathbf{E}_i(\mathbf{K},\omega) e^{i\mathbf{K}\cdot\mathbf{R}} e^{i(q_e z-\omega t)} , \tag{8}$$

where (see Appendix A) for a surface of area L^2

$$\mathbf{E}_i(\mathbf{K},\omega) = -\frac{4\pi i e}{v_\perp L^2} e^{-i\mathbf{K}\cdot\mathbf{R}_0} \left[\frac{\omega}{c}\frac{\mathbf{v}}{c} - \mathbf{K} - q_e\hat{\mathbf{z}}\right] \Big/ (q_e^2 - p^2) \tag{9}$$

with

$$q_e = \frac{\mathbf{K}\cdot\mathbf{V}-\omega}{v_\perp} . \tag{10}$$

The magnetic fields are given by

$$\mathbf{B} = -i\frac{c}{\omega}\nabla\times\mathbf{E} . \tag{11}$$

The vacuum fields, $\mathbf{E}_v$ and $\mathbf{B}_v$, must be matched on the boundary to the fields in the medium, $\mathbf{E}_m$ and $\mathbf{B}_m$. We avoid having to compute $\mathbf{E}_m$ and $\mathbf{B}_m$ explicitly by making use of the vector equivalent of the Kirchhoff integral[12] with sources, which for $z>\zeta(\mathbf{R})$ gives

$$\int dS' \left[i\frac{\omega}{c} G(\mathbf{n}'\times\mathbf{B}'_m) + (\mathbf{n}'\times\mathbf{E}'_m)\times\nabla' G + (n'\cdot\mathbf{E}'_m)\nabla' G \right] = \int_{z'<\zeta} d\mathbf{r}' G \left[i\frac{4\pi\omega}{c^2}\mathbf{j}' - \frac{4\pi}{\epsilon}\nabla'\rho' \right] + \frac{4\pi}{\epsilon}\int dS' \mathbf{n}' G\rho' . \tag{12}$$

The notation $a' \equiv a(\mathbf{r}')$ has been used; $\mathbf{n}'$ is the normal directed into the vacuum region; $G\equiv G(\mathbf{r}-\mathbf{r}')$ is the Green's function of the medium, obeying the equation

$$\left[\nabla'^2 + \epsilon\left[\frac{\omega}{c}\right]^2\right] G(\mathbf{r}-\mathbf{r}') = \delta(\mathbf{r}-\mathbf{r}') . \tag{13}$$

The current $\mathbf{j}$ and the charge density ρ must satisfy the continuity equation,

$$\nabla\cdot\mathbf{j} - i\omega\rho = 0 . \tag{14}$$

We can eliminate $\mathbf{E}_m$ and $\mathbf{B}_m$ by using the boundary conditions

$$\mathbf{n}\times\mathbf{B}_v = \mathbf{n}\times\mathbf{B}_m , \tag{15}$$

$$\mathbf{n}\times\mathbf{E}_v = \mathbf{n}\times\mathbf{E}_m , \tag{16}$$

$$\mathbf{n}\cdot\mathbf{E}_v = \epsilon\mathbf{n}\cdot\mathbf{E}_m . \tag{17}$$

Using Eqs. (5)–(11) and Eqs. (15)–(17), we get, from Eq. (12), two coupled equations for the unknowns $A_p(\mathbf{K},\omega)$ and $A_s(\mathbf{K},\omega)$ (see Appendix B),

$$\sum_{\mathbf{K}'} \frac{(e^{i(p'-q)\zeta})_{\mathbf{K}-\mathbf{K}'}}{p'(p'-q)} [(KK' + \hat{\mathbf{K}}\cdot\hat{\mathbf{K}}'qp') A_p(\mathbf{K}',\omega) - qp'(\hat{\mathbf{K}}\times\hat{\mathbf{K}}'\cdot\hat{\mathbf{z}}) A_s(\mathbf{K}',\omega)]$$

$$= \frac{4\pi i e}{v_\perp L^2} \sum_{\mathbf{K}'} \frac{(e^{i(q'_e-q)\zeta})_{\mathbf{K}-\mathbf{K}'}}{(q'_e-q)(q'^2_e-p'^2)} \left\{ \left[\left[\frac{\omega}{c}\right]\frac{v_\perp}{c} + q'_e \right] K + \left[\frac{\omega}{c}\frac{\mathbf{V}\cdot\mathbf{K}}{c} - \mathbf{K}\cdot\mathbf{K}' \right] \frac{q}{K} \right\} e^{-i\mathbf{K}'\cdot\mathbf{R}_0} \tag{18}$$

and

$$\sum_{\mathbf{K}'} \frac{(e^{i(p'-q)\zeta})_{\mathbf{K}-\mathbf{K}'}}{p'(p'-q)}[\hat{\mathbf{K}}\cdot\hat{\mathbf{K}}'A_s(\mathbf{K}',\omega)+(\hat{\mathbf{K}}\times\hat{\mathbf{K}}'\cdot\hat{\mathbf{z}})A_p(\mathbf{K}',\omega)]$$

$$= \frac{4\pi i e}{v_\perp L^2}\sum_{\mathbf{K}'} \frac{(e^{i(q_e'-q)\zeta})_{\mathbf{K}-\mathbf{K}'}}{(q_e'-q)(q_e'^2-p'^2)}\left[\frac{\omega}{c}\hat{\mathbf{K}}\times\frac{\mathbf{v}}{c}\cdot\hat{\mathbf{z}}-(\hat{\mathbf{K}}\times\hat{\mathbf{K}}'\cdot\hat{\mathbf{z}})K'\right]e^{-i\mathbf{K}'\cdot\mathbf{R}_0}, \quad (19)$$

where

$$q^2=\epsilon\frac{\omega^2}{c^2}-K^2 \quad (20)$$

and, for a surface of area L^2,

$$(e^{i\alpha\zeta})_{\mathbf{K}}=\frac{1}{L^2}\int d\mathbf{R}\, e^{i\alpha\zeta(\mathbf{R})}e^{-i\mathbf{K}\cdot\mathbf{R}} . \quad (21)$$

Equations (18) and (19) are the main results of this section. They provide a starting point for numerical calculations in the case of a periodic grating. Perturbative solutions can also be generated from these equations and can be applied to a randomly rough surface. In either case the solution must still be averaged over $\mathbf{R}_0$. As a result, each term of the sum over $\mathbf{K}'$ on the right-hand side contributes incoherently to the total intensity. This is demonstrated explicitly in the next section for the particular geometry of interest, but is clearly true in general.

III. NUMERICAL RESULTS

In this section we apply the results of the preceding section to the case of a one-dimensional grating. A typical SPREE setup is shown in Fig. 1. For a one-dimensional grating profile only Eq. (18) is needed. Its solution is of the form

$$A_p(\mathbf{K},\omega)=\sum_{\mathbf{K}'} A(\mathbf{K},\mathbf{K}')e^{-i\mathbf{K}'\cdot\mathbf{R}_0} , \quad (22)$$

where it is understood that $A(\mathbf{K},\mathbf{K}')$ depends on ω. The average over impact parameters gives then

$$\frac{1}{L^2}\int d\mathbf{R}_0|A_p(\mathbf{K},\omega)|^2=\sum_{\mathbf{K}'}|A(\mathbf{K},\mathbf{K}')|^2 . \quad (23)$$

Because the grating is one-dimensional, modes with different momentum along the groove do not mix, and Eq. (18) reduces to a set of coupled equations for modes with different momentum perpendicular to the groove but with the same momentum along the groove.

Numerical calculations were carried out for a one-dimensional sinusoidal Ag grating with a period $a=433.6$ nm and a height $h=17$ nm ($2h$ is the peak to valley height) to correspond to the work of Heitmann *et al.*[1] The electron energy is 80 keV and the incident angle is 45°. The plane formed by the incident electron and the outgoing photon is perpendicular to the grating groove as shown in Fig. 1. From Eqs. (18) and (20), with a slight change of notation,[13] the equation for $A(K,K')$ reduces to

$$\sum_{K''}\frac{(e^{i(p''-q)\zeta})_{K-K''}}{p''(p''-q)}(KK''+qp'')A(K'',K')=\frac{4\pi i e}{v_\perp L^2}\frac{(e^{i(q_e'-q)\zeta})_{K-K'}}{(q_e'-q)(q_e'^2-p'^2)}\left\{\left[\left(\frac{\omega}{c}\right)\frac{v_\perp}{c}+q_e'\right]K+\left[\frac{\omega}{c}\frac{V}{c}-K'\right]q\right\} . \quad (24)$$

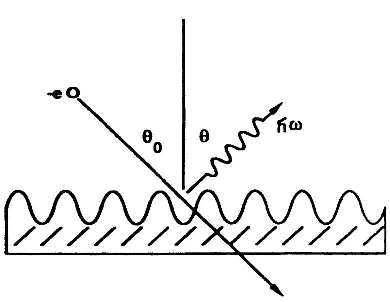

FIG. 1. Illustration of surface plasmon radiation excited by electrons (SPREE) experiment.

K,K',K'' are of the form $2\pi n/a$, with n integer. This equation is solved for N values of K' by truncating in each case the infinite series after N terms; N is increased until no change occurs to three significant digits in the averaged emitted intensity (23).

The quantity to be compared with experiment is the power emitted per unit solid angle and unit wavelength

$$\frac{dP}{d\Omega\, d\lambda}=\frac{L^2\omega^4}{32c^2\pi^4}\sum_{K'}|A(K,K')|^2 . \quad (25)$$

In Fig. 2 the calculated emission peaks at fixed λ are plotted along with the corresponding experimental data.

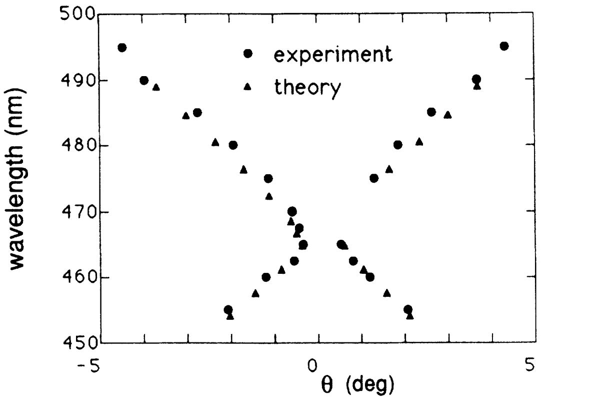

FIG. 2. Plot of the emission peaks from constant frequency scans. The experimental data are from Ref. 1.

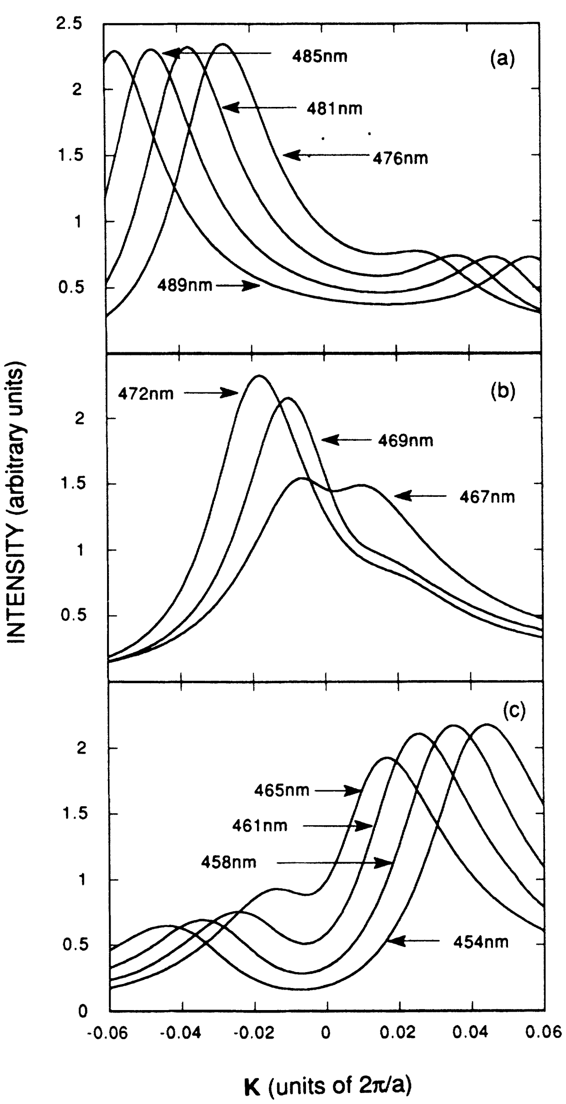

FIG. 3. Constant frequency scans corresponding to Fig. 2.

The theoretical calculation is a little higher in energy, but the agreement with experiment can be improved by decreasing the real part of the dielectric constant, which is interpolated from the data of Johnson and Christy,[14] by about 5%. Another possible explanation of the small discrepancy is that random roughness, which is always present, has not been taken into account. The size of the k gap, which is at $2\pi/a$, shows excellent agreement. Both theory and experiment show the absence of a peak for $\theta>0$ around $\lambda=470$ nm. However, the theory predicts that the intensity of the peaks increases only slightly as one goes to higher frequency, while Heitmann *et al.*[1] report that the intensity of the peaks increases sharply at higher frequency. Figure 3 shows the computed constant frequency scans.

In conclusion, we have obtained a set of coupled equations that can be used as a starting point for a perturbative or numerical solution of a general SPREE problem on a shallow grating. We have solved them numerically and found the anomalous dispersion of surface polaritons near the $2\pi/a$ zone boundary on a one-dimensional Ag grating that has been observed experimentally.

APPENDIX A

In this appendix we derive the fields due to a current in a medium characterized by a dielectric constant $\epsilon(\omega)$. The current is of the form

$$\mathbf{j}(\mathbf{r},t)=-e\mathbf{v}\delta(\mathbf{r}-\mathbf{R}_0-\mathbf{v}t)\ . \tag{A1}$$

The Fourier transform of $\mathbf{j}$ with respect to $\mathbf{R}$ and t is

$$\mathbf{j}(\mathbf{K},\omega,z)=\int d\mathbf{R}\,dt e^{i\omega t}e^{-i\mathbf{K}\cdot\mathbf{R}}\mathbf{j}(\mathbf{r},t)=-\frac{e\mathbf{v}}{v_\perp}e^{iq_e z}e^{-i\mathbf{K}\cdot\mathbf{R}_0}\ , \tag{A2}$$

where

$$q_e=\frac{\mathbf{K}\cdot\mathbf{V}-\omega}{v_\perp}\ . \tag{A3}$$

The Hertz vector potential $A(\mathbf{K},\omega,z)$ obeys the equation

$$\left[\frac{d^2}{dz^2}+\epsilon\left[\frac{\omega}{c}\right]^2-K^2\right]\mathbf{A}(\mathbf{K},\omega,z)=-\frac{4\pi i}{\omega\epsilon}\mathbf{j}(\mathbf{K},\omega,z)\ . \tag{A4}$$

The solution to (A4) when $\mathbf{j}(\mathbf{K},\omega,z)$ is given by (A2) is

$$\mathbf{A}(\mathbf{K},\omega,z)=(-e)\frac{\mathbf{v}}{v_\perp}\frac{4\pi i}{\omega\epsilon}\frac{e^{iq_e z}e^{-i\mathbf{K}\cdot\mathbf{R}_0}}{q_e^2-q^2}\ , \tag{A5}$$

where

$$q^2=\epsilon\left[\frac{\omega}{c}\right]^2-K^2\ . \tag{A6}$$

The $\mathbf{E}$ field is given by

$$\mathbf{E}(\mathbf{r},\omega)=\nabla[\nabla\cdot\mathbf{A}(\mathbf{r},\omega)]+\epsilon\left[\frac{\omega}{c}\right]^2\mathbf{A}(\mathbf{r},\omega) \tag{A7}$$

with

$$\mathbf{A}(\mathbf{r},\omega)=\frac{1}{L^2}\sum_{\mathbf{K}}e^{i\mathbf{K}\cdot\mathbf{R}}\mathbf{A}(\mathbf{K},\omega,z)\ . \tag{A8}$$

Inserting (A5) into (A7) we get

$$\mathbf{E}(r,\omega)=-\frac{4\pi ie}{v_\perp L^2}\sum_{\mathbf{K}}e^{i\mathbf{K}\cdot(\mathbf{R}-\mathbf{R}_0)}e^{iq_ez}\left[\frac{\omega}{c}\frac{\mathbf{v}}{c}-\frac{\mathbf{K}+q_e\hat{\mathbf{z}}}{\epsilon}\right]\Big/(q_e^2-q^2)\ . \tag{A9}$$

To obtain the vacuum solution, put $\epsilon=1$ and replace q by p, Eq. (6). The result is given as $\mathbf{E}_i$ in Eqs. (8) and (9).

APPENDIX B

In this appendix Eqs. (18) and (19) will be derived. First, to express Eq. (12) in terms of the fields in the vacuum region the boundary conditions [Eqs. (15)–(17)] are used:

$$\int dS'\left[i\frac{\omega}{c}G(\mathbf{n}'\times\mathbf{B}'_v)+(\mathbf{n}'\times\mathbf{E}'_v)\times\nabla' G+\frac{1}{\epsilon}(\mathbf{n}'\cdot\mathbf{E}'_v)\nabla' G\right]=\int_{z'<\zeta}d\mathbf{r}'G\left[i\frac{4\pi\omega}{c^2}\mathbf{j}'-\frac{4\pi}{\epsilon}\nabla'\rho'\right]+\frac{4\pi}{\epsilon}\int dS'\,\mathbf{n}'G\rho'\ . \tag{B1}$$

$G(\mathbf{r}-\mathbf{r}')$, which is the solution to Eq. (13), is

$$G(\mathbf{r}-\mathbf{r}')=\frac{1}{L^2}\sum_{\mathbf{K}}e^{i\mathbf{K}\cdot(\mathbf{R}-\mathbf{R}')}e^{iq|z-z'|}\frac{-i}{2q}\ . \tag{B2}$$

The current can be written as (see Appendix A)

$$\mathbf{j}(\mathbf{r},\omega)=\frac{-e}{L^2}\sum_{\mathbf{K}}\frac{\mathbf{v}}{v_\perp}e^{i\mathbf{K}\cdot(\mathbf{R}-\mathbf{R}_0)}e^{iq_ez}\ . \tag{B3}$$

By the continuity equation, the charge density is

$$\rho(\mathbf{r},\omega)=\frac{-e}{v_\perp L^2}\sum_{\mathbf{K}}e^{i\mathbf{K}\cdot(\mathbf{R}-\mathbf{R}_0)}e^{iq_ez}\ . \tag{B4}$$

Inserting Eqs. (B2)–(B4) into the first term on the right-hand side of Eq. (B1) we get

$$\int_{z'<\zeta}d\mathbf{r}'G\left[i\frac{4\pi\omega}{c^2}\mathbf{j}'-\frac{4\pi}{\epsilon}\nabla'\rho'\right]=-e\frac{4\pi}{L^2}\sum_{\mathbf{K}}e^{i\mathbf{K}\cdot\mathbf{R}}e^{iqz}\sum_{\mathbf{K}'}\frac{(e^{-i(q_e'-q)\zeta})_{\mathbf{K}-\mathbf{K}'}}{2iqv_\perp(q_e'-q)}\left[\frac{\omega\mathbf{v}}{c^2}-\frac{\mathbf{K}'+q_e'\hat{\mathbf{z}}}{\epsilon}\right]e^{-i\mathbf{K}'\cdot\mathbf{R}_0}\ . \tag{B5}$$

Similarly

$$\frac{4\pi}{\epsilon}\int dS'\,\mathbf{n}'G\rho'=-e\frac{4\pi}{L^2}\sum_{\mathbf{K}}e^{i\mathbf{K}\cdot\mathbf{R}}e^{iqz}\sum_{\mathbf{K}'}\frac{(e^{-i(q_e'-q)\zeta})_{\mathbf{K}-\mathbf{K}'}}{2iqv_\perp(q_e'-q)}\frac{1}{\epsilon}(\mathbf{K}'+q_e'\hat{\mathbf{z}}-\mathbf{K}-q\hat{\mathbf{z}})e^{-i\mathbf{K}'\cdot\mathbf{R}_0}\ , \tag{B6}$$

where the identity

$$\mathbf{n}=(\hat{\mathbf{z}}-\nabla\zeta)/[1+(\nabla\zeta)^2]^{1/2} \tag{B7}$$

has been used and integration by parts carried out. The integrand on the left-hand side of Eq. (B1) can be written as

$$i\frac{\omega}{c}G(\mathbf{r}-\mathbf{r}')(\mathbf{n}'\times\mathbf{B}'_v)+(\mathbf{n}'\times\mathbf{E}'_v)\times\nabla' G(\mathbf{r}-\mathbf{r}')+\frac{1}{\epsilon}(\mathbf{n}'\cdot\mathbf{E}'_v)\nabla' G(\mathbf{r}-\mathbf{r}')$$

$$=\sum_{\mathbf{K}}[\mathbf{n}'\times(\nabla'\times\mathbf{E}'_v)+(\mathbf{n}'\times\mathbf{E}'_v)\times(-i\mathbf{k})+\frac{1}{\epsilon}(\mathbf{n}'\cdot\mathbf{E}'_v)(-i\mathbf{k})]G(\mathbf{K},\omega,z-z')e^{i\mathbf{K}\cdot(\mathbf{R}-\mathbf{R}')}\ , \tag{B8}$$

where

$$\mathbf{k}=\mathbf{K}+q\hat{\mathbf{z}}\ , \tag{B9}$$

$$G(\mathbf{K},\omega,z-z')=\frac{-i}{2qL^2}e^{iq|z-z'|}\ . \tag{B10}$$

Following Ref. 15 we get for the homogeneous fields,

$$\int dS' \left[i\frac{\omega}{c} G(\mathbf{n}'\times\mathbf{B}_h') + (\mathbf{n}'\times\mathbf{E}_h')\times\nabla' G + \frac{1}{\epsilon}(\mathbf{n}'\cdot\mathbf{E}_h')\nabla' G \right] = \sum_{\mathbf{K}} e^{i\mathbf{K}\cdot\mathbf{R}} e^{iqz} \sum_{\mathbf{K}'} \frac{(e^{i(p'-q)\zeta})_{\mathbf{K}-\mathbf{K}'}}{2q(p'-q)} (1-\epsilon) \left[\frac{\mathbf{k}}{\epsilon}\mathbf{k}\cdot\mathbf{E}_h(\mathbf{K}') - \left[\frac{\omega}{c}\right]^2 \mathbf{E}_h(\mathbf{K}') \right] . \quad \text{(B11)}$$

For the inhomogeneous fields we get

$$\int dS' \left[i\frac{\omega}{c} G(\mathbf{n}'\times\mathbf{B}_i') + (\mathbf{n}'\times\mathbf{E}_i')\times\nabla' G + \frac{1}{\epsilon}(\mathbf{n}'\cdot\mathbf{E}_i')\nabla' G \right] = -\frac{4\pi i e}{v_\perp L^2} \sum_{\mathbf{K}} e^{i\mathbf{K}\cdot\mathbf{R}} e^{iqz} \sum_{\mathbf{K}'} \frac{(e^{i(q_e'-q)\zeta})_{\mathbf{K}-\mathbf{K}'}}{2q(q_e'-q)(q_e'^2-p^2)} \left[\left[\frac{1-\epsilon}{\epsilon}(\mathbf{k}\cdot\tilde{\mathbf{k}}') - \frac{1}{\epsilon}\mathbf{k}_e'\cdot\tilde{\mathbf{k}} \right] \mathbf{k} + (\mathbf{k}_e'\cdot\tilde{\mathbf{k}}')\mathbf{k}_e' - (k_e'^2 - k^2)\tilde{\mathbf{k}}' \right] e^{-i\mathbf{K}'\cdot\mathbf{R}_0} , \quad \text{(B12)}$$

where

$$\mathbf{k}_e' = \mathbf{K}' + q_e \hat{\mathbf{z}} \quad \text{(B13)}$$

and

$$\tilde{\mathbf{k}}' = \left[\frac{\omega}{c}\right] \frac{\mathbf{v}}{c} - \mathbf{k}_e' . \quad \text{(B14)}$$

Substituting Eqs. (B5), (B6), (B11), and (B12) into Eq. (B1) will give the vector equation from which Eqs. (18) and (19) are obtained. First, we Fourier transform with respect to $\mathbf{R}$ to eliminate the sum over $\mathbf{K}$. Then taking the dot product with two vectors $(q\hat{\mathbf{K}}, -K\hat{\mathbf{z}})$ and $(\hat{\mathbf{z}}\times\hat{\mathbf{K}})$, Eqs. (18) and (19) follow.

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