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Mutual drag of two- and three-dimensional electron gases in heterostuctures

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Mutual drag of two-dimensional (2D) and 3D electron gases in GaAs/$Al_xGa_{1-x}As$/GaAs heterostructures is considered. The main purpose of the paper is to explain the recent experiment of Solomon *et al.* [Phys. Rev. Lett. **63**, 2508 (1989)] in which a current in the channel induced a current in the gate and the latter changed its sign when temperature decreased. It is shown that Coulomb mutual scattering gives rise to two mechanisms of the current induction. The first of them is the direct momentum transfer between the 2D and 3D electron gases and induces a current in the gate in the same direction as that in the channel. The second mechanism is connected with the presence of a temperature gradient in the channel due to the Peltier effect on the contacts. Energy exchange between the electron gases transfers the temperature gradient to the gate where it induces a thermocurrent in the direction opposite to the current in the channel. At high temperatures the temperature gradient in the gate is small because of efficient energy exchange between the 3D electron gas and the lattice so that the current due to the direct momentum transfer dominates. When temperature decreases, the number of phonons participating in scattering processes with 3D electrons drops off (the Bloch-Grüneisen regime sets in), and the energy transferred from the 3D electron gas to the lattice decreases. As a result the thermocurrent dominates and the total current changes its sign. The theory explains the principal features of the experiment.

I. INTRODUCTION

An advanced molecular-beam-epitaxy (MBE) technology which allows fabrication of heterostructures between GaAs and (Al,Ga)As not only contributes to the design of new useful devices but also leads to observation of unexpected physical phenomena. Recently Solomon, Price, Frank, and La Tulipe[1] observed the effect of three-dimensional electron gas (3D EG) drag in a GaAs gate electrode caused by an electric current through 2D EG in a GaAs channel separated from the gate by a 300-Å insulating of $Al_{0.5}Ga_{0.5}As$ (Fig. 1). The electron drag effect in heterostructures due to Coulomb mutual scattering (CMS) was predicted by Price,[2] and the experiment[1] was designed for the observation of this effect. The most striking feature discovered in Ref. 1 was that the effect changed its sign when the temperature decreased, i.e., the drift in the 3D EG had the opposite direction to that in the 2D EG. The reciprocal effect in the channel also was observed when the signal was applied to the gate. The first attempts to explain this effect with mechanisms involving transmitted phonon drag, CMS, and thermoelectric effects did not lead to any success.[1] The purpose of the present paper is to give such an explanation based on the Peltier effect, very weak inelastic electron scattering, and CMS.

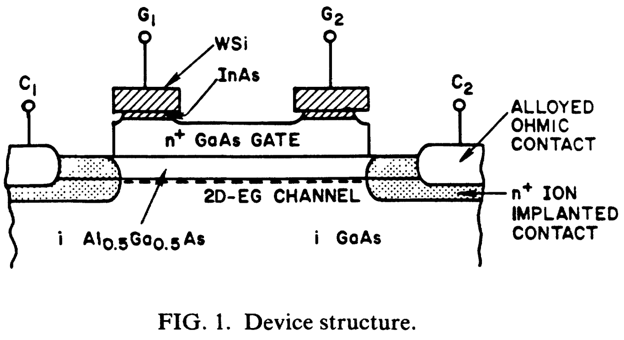

FIG. 1. Device structure.

It is known that CMS leads to the exchange of an energy (Price,[2] Jacoboni and Price[3]) and momentum (Price,[2] Boiko and Sirenko[4]) between electron gases separated by a thin insulating layer. Under conditions of electric current flow, the electron distribution in the 2D EG has a nonzero net momentum. The transfer of momentum to the gate induces there a current of the same direction as that in the channel. The origin of an oppositely directed current in the gate is nonuniform heating of the 2D EG due to the Peltier effect. This effect leads to a cooling of the 2D EG near the contact where electrons come into the gate and to a heating of the 2D EG near the other contact. At temperatures below 50 K (where the current reversal was observed[1]), the main electron scattering mechanism in the 2D EG is elastic impurity scattering. Electrons give their energy to the lattice by acoustical phonon scattering which is rather weak. So, rather lengthy regions of 2D EG near the contacts have temperatures different from that of the lattice. The energy exchange between 2D EG and 3D EG induces a temperature gradient in the 3D EG which eventually gives rise to thermocurrent. The magnitude of the temperature gradient in the 3D EG, and therefore of the thermocurrent, depends critically on the electron-phonon relaxation rate which provides the main energy relaxation mechanism. At high temperatures this rate is high and the temperature gradient is small. When the temperature decreases the Bloch-Grüneisen regime sets in, the electron-phonon relaxation rate falls off, and so the temperature gradient and thermocurrent increase. This leads to the change of the sign of the total current which is a result of a competition between the current due to the momentum exchange and the thermocurrent due to the energy exchange.

This mechanism of the thermoelectric effect is substantially different from the conventional one. No phonons are involved in the energy transfer, and the lattice temperature gradient, diminished by the phonon thermal conductivity, does not play any substantial role.

The theory developed in this paper gives an explanation of the following characteristics of the effect.

1. Change of the induced current sign when temperature decreases and the decrease of the current value (with one more possible sign change) for a further temperature decrease.
2. The reciprocal effect, i.e., the same behavior of the current induced in the channel when the signal is applied to the gate.
3. The dependence of the effect on the gate voltage, i.e., the 2D EG concentration.
4. The main contribution to the effect from the ends of the sample in the experiments with four contacts on the gate.
5. The nonlinear effect, with signal applied to the channel.

The theory is essentially semiquantitative. The reason for this is partly due to simplifications in the model considered compared to the experiment. The effect strongly depends on details of CMS and the structure and geometry of the contacts. CMS takes place in the gate within a screening length of the gate-insulator interface. The screening radius is only a little greater than the Fermi wavelength so that the detailed structure of the electron wave function depending on the doping near the surface and band bending is very important. Surface scattering, which is not seen in the bulk measurements and, hence, cannot be estimated, plays a very important role in the current induced by the momentum transfer. The amount of energy arriving at the 2D EG due to the Peltier effect strongly depends on the microscopic structure and the geometry of the contacts because of electron thermoconductivity between the 2D EG and the contacts. This may be the reason for a considerable variation of the sign reversed effect from sample to sample.[1] To make the calculations simpler we consider a flatband model and a uniform doping in the gate which is not true in the device used in Ref. 1. Essentially the result of these simplifications is an overestimation of the coupling constant for CMS and underestimation of 3D EG scattering. We also often use simplifications justified for strong inequalities in cases when real inequalities are not very strong. These simplifications allow us to obtain nearly all qualitative features of the effect although we overestimate its magnitude.

In the next section we evaluate the electron-electron collision term describing CMS and calculate the current due to the momentum transfer. In Secs. III and IV the energy balance and the electron temperature distribution in the 3D EG are studied. In the last section the whole picture is considered and the temperature dependency and other traits of the effect are discussed and compared with the experiment.

II. THE MOMENTUM TRANSFER FROM 2D to 3D EG

The authors of Ref. 4 evaluated the resistivity of a 2D EG resulting from momentum transfer to an equilibrium 3D EG. Here we need to take the next step and calculate the current in the 3D EG induced by this transfer. To this end we consider the Boltzmann equation for the 3D EG taking into account its interaction with the 2D EG carrying a current.

For the electron concentration in the gate $n_g \simeq 2\times 10^{18}$ cm^{-3} the inverse screening radius $q_s = [(4e^2 m k_F)/(\pi\kappa\hbar^2)]^{1/2} \simeq 2.3\times 10^6$ cm^{-1} is only a little smaller than the Fermi wave vector $k_F \simeq 3.9\times 10^6$ cm^{-1} (here $m = 0.61\times 10^{-28}$ g is the effective mass and $\kappa \simeq 13$ is the dielectric constant). It means that the Coulomb interaction decreases over a rather short distance and does not lead to a dominance of scattering events with a very small momentum transfer, so that one does not need to use the electron-electron collision operator in the Landau form. On the other hand, a large distance $a \simeq 300$ Å between the gate and the channel makes the transferred momentum and energy small enough to neglect any dynamical effect in screening. Thus we can use an elementary approach where the interaction between 2D EG and 3D EG has to include static screening.

To find the interaction potential we calculate the potential inside the gate caused by a point charge e positioned in the 2D channel at the distance a from the flat gate surface. Coulomb interaction between the electrons of the 3D EG is weak and the screening can be considered in the gas approximation. The potential ϕ satisfies the Poisson equation

$$\Delta\phi - 2q_s^{\text{ch}}\phi\delta(z+a) = -\frac{4\pi e}{\kappa}\delta(z+a)\delta(\mathbf{r}_\perp), \quad z<0\ ; \tag{1a}$$

$$\Delta\phi = -\frac{4\pi e}{\kappa}n(z), \quad z>0\ , \tag{1b}$$

where the z axis is directed perpendicular to the layers, $\mathbf{r}_\perp$ is the radius vector in the x,y plane, and $q_s^{\text{ch}} = 2e^2 m/\kappa\hbar^2 \simeq 2.2\times 10^6$ is the screening parameter of the 2D EG.[5] The difference between the dielectric constants of the substrate, insulating layer, and the gate usually is about 10% and we neglect it. We neglect as well electron tunneling into the insulating layer, so that the electron concentration n is not equal to zero only inside the gate, and assume that

$$n = \int \frac{df_0}{dE_k} e\phi \frac{2d^3k}{(2\pi)^3} = -\frac{mk_F}{\pi^2\hbar^2}e\phi\ , \tag{2}$$

which, strictly speaking, is justified for $q_s \ll k_F$ only. Here f_0 is the Fermi function and E_k is the energy of an electron with the wave vector $\mathbf{k}$. The boundary conditions to Eqs. (1) are the continuity of ϕ and $\partial\phi/\partial z$ at $z=0$. The solution to Eqs. (1) for $z>0$ has the form

$$\phi = \frac{4\pi e}{\kappa}\int \frac{d^2q}{(2\pi)^2}\frac{q}{g(q)}\exp[-(q^2+q_s^2)^{1/2}z + i\mathbf{q}\cdot\mathbf{r}_\perp]\ , \tag{3}$$

where

$$g(q)=(q+q_s^{\mathrm{ch}})[(q^2+q_s^2)^{1/2}+q]e^{qa} -q_s^{\mathrm{ch}}[(q^2+q_s^2)^{1/2}-q]e^{-qa} . \tag{3a}$$

The interaction potential falls off inside the metal on a scale of q_s^{-1} which is typically much smaller than the electron mean free path. It means that transitions in 3D EG due to this potential can take place when an electron wave packet is reflected from the metal surface. The transition probability is evaluated making use of wave functions

$$\psi_{\mathbf{k}}=\left[\frac{2}{SL_z}\right]^{1/2} e^{i\mathbf{k}_\perp\cdot\mathbf{r}_\perp}\sin(k_z z),\quad k_z>0 , \tag{4}$$

where S and L_z are normalization surface and length. The integral with respect to z in the interaction matrix element converges on a finite z which leads to an extra factor L_z^{-1} in the collision operator. The normalization length L_z is chosen much greater than k_F^{-1} and q_s^{-1} but much smaller than the electron mean free path, and that extra factor can be replaced by $\delta(z)$. As a result we obtain the following expression for this operator:

$$\hat{I}_{ee}f_{\mathbf{k}}=\delta(z)\int_0^\infty\frac{dk_z'}{\pi}\int\frac{d^2q}{(2\pi)^2}[R(\mathbf{k},\mathbf{k}')(1-f_{\mathbf{k}})f_{\mathbf{k}'} -R(\mathbf{k}',\mathbf{k})(1-f_{\mathbf{k}'})f_{\mathbf{k}}] , \tag{5}$$

where

$$R(\mathbf{k},\mathbf{k}')=\int\frac{2\,d^2k_1}{(2\pi)^2}Q(q,k_z,k_z')f^{\mathrm{ch}}_{\mathbf{k}_1+\mathbf{q}}(1-f^{\mathrm{ch}}_{\mathbf{k}_1}) \times\delta(E_k-E_{k'}+E_{k_1}-E_{|\mathbf{k}_1+\mathbf{q}|}) , \tag{5a}$$

$$Q(q,k_z,k_z')=\frac{2\pi}{\hbar}\left[\frac{4\pi e^2}{\kappa}\right]^2\frac{q^2}{g(q)^2}M^2(k_z,k_z') , \tag{5b}$$

$$M(k_z,k_z')=\frac{(q^2+q_s^2)^{1/2}}{(k_z-k_z')^2+q^2+q_s^2}-\frac{(q^2+q_s^2)^{1/2}}{(k_z+k_z')^2+q^2+q_s^2} , \tag{5c}$$

$E_k=\hbar^2k^2/(2m)$, and $\mathbf{q}=\mathbf{k}_\perp-\mathbf{k}_\perp'$ is the transferred momentum. Quantities related to the channel are marked with the corresponding superscript or subscript and no appellations are made for gate quantities if this does not cause confusion. We consider the interaction between 3D EG and 2D EG as a perturbation for 3D EG, so that the distribution function of 3D EG in Eq. (5) can be replaced by the Fermi function. For 2D EG

$$f^{\mathrm{ch}}_{\mathbf{k}_1}=f_0(E_{k_1})-eFv_{1x}\tau_{\mathrm{ch}}\frac{df_0}{dE_{\mathbf{k}_1}} , \tag{6}$$

where F is the electric field, $\mathbf{v}=\hbar\mathbf{k}/m$ is the electron velocity, and τ_{ch} is the electron elastic relaxation time in the channel. The main scattering mechanisms in GaAs layers at low temperatures are the scattering by deformation potential of acoustical phonons and impurity scattering, and in both cases τ_{ch} does not depend on the electron energy.[6] Apparently, if the electron temperature of 2D EG and 3D EG is the same, which we assume in this section, the first term in the right hand side of Eq. (6) does not contribute to the collision operator.

For a typical electron concentration in the channel $n_{\mathrm{ch}}\approx 5\times10^{11}$ cm^{-2} the Fermi wave vector there $k_F^{\mathrm{ch}}\approx 1.8\times10^6$ cm^{-1}, and we can make use of the inequality

$$k_F^{\mathrm{ch}}a, k_Fa \gg 1 . \tag{7}$$

The momentum transferred in one scattering event is of the order of $\hbar q \lesssim \hbar/2a$. The resulting transferred energy is of the order of $\hbar^2k_F^{\mathrm{ch}}/2ma \sim 5\times10^{-15}$ erg which is less than k_BT for temperature $T\simeq 35$ K. Thus collisions between 2D and 3D electrons can be considered to be quasielastic. In evaluation of the momentum transfer, where a small energy transfer is unimportant, we can use the elastic approximation neglecting the energy transfer in the arguments of the distribution functions. This allows us to reduce the collision operator to

$$\hat{I}_{ee}f_{\mathbf{k}}=\delta(z)\left[\frac{4\pi e^2}{\kappa}\right]^2\frac{\pi}{\hbar^2q_s^2(q_s^{\mathrm{ch}})^2}eF\tau_{\mathrm{ch}}k_BT\frac{df_0}{dE_k} \times\int_0^\infty\frac{dk_z'}{\pi}\int\frac{d^2q}{(2\pi)^2}\frac{2\,d^2k_1}{(2\pi)^2}q_x\frac{df_0}{dE_{k_1}}M^2(k_z,k_z')\left[\frac{q}{\sinh qa}\right]^2\delta[k_z^2-k_z'^2+2\mathbf{q}(\mathbf{k}-\mathbf{k}_1)] . \tag{8}$$

Here we neglected also q compared to q_s and q_s^{ch}.

A further simplification is possible if we take into account that the main contribution to the current comes from small k_z, i.e., from electrons grazing the surface. Assuming that $k_z,k_z' \ll q_s$ (the real inequality is not very strong), we have $M(k_z,k_z')\simeq 4k_zk_z'/q_s^3$. At the same time q is so small that we can consider $kq \ll k_z^2$. We use also the inequality $k_F^{\mathrm{ch}} \ll k_F$. As a result

$$\int_0^\infty\frac{dk_z'}{\pi}\delta[k_z^2-k_z'^2+2\mathbf{q}\cdot(\mathbf{k}-\mathbf{k}_1)]M^2(k_z,k_z') \simeq\frac{8}{\pi}\frac{k_z^3}{q_s^6}\left[1+\frac{\mathbf{k}\cdot\mathbf{q}}{k_z^2}\right] . \tag{9}$$

All other integrations are carried out without any difficulty and we have

$$\hat{I}_{ee} f_{\mathbf{k}} = \delta(z) \tilde{I}_{ee} , \tag{10}$$

where

$$\tilde{I}_{ee} = - \left[\frac{4\pi e^2}{\kappa} \right]^2 \frac{\pi C_1}{2} \frac{eF\tau_{\rm ch} m k_x k_z}{\hbar^4 q_s^4 (q_s a)^4 (q_s^{\rm ch} a)^2} k_B T \frac{df_0}{dE_k} , \tag{11}$$

$$C_1 = \frac{5}{4\pi^3} \int_0^\infty \frac{x^4 dx}{e^x - 1} \simeq 1 . \tag{12}$$

Now we are in a position to evaluate the electric current in the gate induced by the collision operator [Eqs. (10), (11)] which is considered as a perturbation in the Boltzmann equation. Let the electron distribution function in the gate be $f_{\mathbf{k}} = f_0(E_k) + f_{\mathbf{k}}^{(1)}$. The perturbation affects only at the boundary and the main relaxation mechanism is impurity scattering, so that $f_{\mathbf{k}}^{(1)}$ depends on z and satisfies the equation

$$\mathbf{v}_z \frac{\partial f_{\mathbf{k}}^{(1)}}{\partial z} = - \frac{f_{\mathbf{k}}^{(1)}}{\tau_g} + \hat{I}_{ee} f_{\mathbf{k}} , \tag{13}$$

where τ_g is the electron elastic relaxation time in the gate. Equation (13) can be replaced by the homogeneous equation with the boundary condition

$$f_{\mathbf{k}}^{(1)} = \frac{\tilde{I}_{ee}}{\mathbf{v}_z}, \quad z = 0 . \tag{14}$$

Electrons coming to the boundary have the distribution function f_0, and $f^{(1)}$ is not equal to zero for reflected electrons only. Thus the induced current

$$\begin{aligned} J &= L_y \int_0^\infty dz \int_{\mathbf{v}_z > 0} \frac{2 d^3 k}{(2\pi)^3} e \mathbf{v}_x f_{\mathbf{k}}^{(1)} \\ &= L_y \int_{\mathbf{v}_z > 0} \frac{2 d^3 k}{(2\pi)^3} e \mathbf{v}_x \tau_g \tilde{I}_{ee} , \end{aligned} \tag{15}$$

where L_y is the sample size in the y direction. Carrying out the last integration one can see that more than half of the contribution comes from $k_z / k_F < 0.55$ which justifies the approximations made above. The final result is

$$J = L_y \frac{\pi^3 C_1}{32} \frac{\tau_g k_F^2 k_B T}{\hbar (q_s a)^4 (q_s^{\rm ch} a)^2} \frac{e^2 F \tau_{\rm ch}}{m} . \tag{16}$$

Equation (16) explains the main features of the experiment[1] at high temperature. The temperature dependence of the current is determined by the product $T\tau_{\rm ch}$. At low temperature the main contribution to $\tau_{\rm ch}$ comes from impurity scattering, which does not depend on the temperature, and the current increases with the temperature. For a higher temperature the main mechanism of electron scattering is scattering by the deformation potential of acoustical phonons, $\tau_{\rm ch} \propto 1/T$ (see, e.g., Ref. 6), so that the current does not depend on temperature. The saturation of the current with temperature takes place at $T \simeq 80$ K. At this temperature the mobility of 2D EG in pure GaAs, related to the phonon scattering only, is about 1.5×10^5 cm^2/V s (see Ref. 7) which is quite close to the mobility of 2D EG in the experiment.[1]

The current Eq. (16) does not depend on the electron concentration in the channel. This results from the fact that not all electrons in the channel contribute to the current but only those which are in the energy layer with the width equal to the drift velocity $eF\tau_{\rm ch}/m$ times x component of the momentum transferred, $\hbar q_x$. For $T \gtrsim 80$ K the main scattering mechanism is acoustical phonons, and $\tau_{\rm ch}$ and hence this portion do not depend on the Fermi energy E_F.

Comparing the magnitude of the current to the experiment we encounter the problems mentioned above. We have evaluated the interaction between 2D EG and 3D EG making use of an ideal flatband model and uniform doping in the gate. Actually, in the samples used in the experiment[1] the gate has a spacer, an undoped layer of about 20 Å thickness near the insulating Al$_x$Ga$_{1-x}$As layer. There is also possible accumulation of negative charge on the interface between the gate and the Al$_x$GA$_{1-x}$As layer.[8] Both the spacer and the accumulated charge give rise to a potential barrier preventing electrons with a small k_z, which give the main contribution to the current, from coming close to the insulating layer. On the other hand, those electrons on the Fermi sphere which have a large enough k_z overcome the barrier and screen the coupling between the 2D EG and 3D EG. This screening can substantially decrease the estimate of the current. If we neglect this screening and use Eq. (16) then the mobilities in the gate 2.5×10^3 cm^2/V s and in the channel 10^5 cm^2/V s lead correspondingly to $\tau_g \simeq 0.9 \times 10^{-13}$ and $\tau_{\rm ch} \simeq 0.4 \times 10^{-11}$ s so that for $F = 0.5$ V/cm and $L_y = 40$ μm we have $J \simeq 4 \times 10^{-9}$ Å which is well above the observed value of the current.

III. HEATING OF 2D EG

In this section we evaluate the heating of the 2D EG due to the Peltier effect. We use the electron temperature approximation which will be shown to be justified in this case. The distribution of the 2D EG temperature $T_{\rm ch}$ along the channel can be found from the thermal conductivity equation where the heat is transferred along the channel electronically and from the channel to the substrate by means of acoustic phonons

$$\lambda_{\rm ch} \frac{d^2 T_{\rm ch}}{dx^2} - \gamma_{\rm ch} (T_{\rm ch} - T) = 0 . \tag{17}$$

Here the energy dissipation term due to an interaction with phonons is linearized with respect to the difference between the electron and lattice temperatures ($\lambda_{\rm ch}$ is the electron thermoconductivity). The first problem is to estimate the magnitude of $T_{\rm ch} - T$ and the length on which $T_{\rm ch}$ decays to the lattice temperature T. To this end it is enough to set the source of the Peltier heat near the contact giving the boundary condition

$$-\lambda_{\rm ch} \frac{dT_{\rm ch}}{dx} = r \sigma_{\rm ch} \alpha T F, \quad x = 0 , \tag{18}$$

where $\sigma_{\rm ch}$ and α are conductivity and thermoelectric power, and the coefficient r determines the portion of the

Peltier heat absorbed by the 2D EG. Assuming that $T_{\rm ch}=T$ for $x\to\infty$ we have

$$T_{\rm ch}-T=(\Delta T)_{\rm ch}e^{-x/l_T} , \tag{19}$$

where

$$(\Delta T)_{\rm ch}=r\frac{\sigma_{\rm ch}\alpha T}{(\lambda_{\rm ch}\gamma_{\rm ch})^{1/2}}F,\quad l_T=(\lambda_{\rm ch}/\gamma_{\rm ch})^{1/2} . \tag{20}$$

For an evaluation of $\gamma_{\rm ch}$ we make use of the results of Refs. 9–12 which lead to the following expression for the energy transferred from the 2D EG to phonons per unit area in the case of $T_{\rm ch}-T\ll T$:

$$\left[\frac{dE}{dT}\right]_{eph}^{\rm ch}=-\frac{T_{\rm ch}-T}{k_BT^2}\int\frac{2d^2k}{(2\pi)^2}\frac{d^3q}{(2\pi)^3}W_{\rm ch}(q)(\hbar\omega_q)^2 \times N_{\mathbf{q}}f_0(E_{|\mathbf{k}+\mathbf{q}_\perp|})[1-f_0(E_k)] \times\delta(E_{|\mathbf{k}+\mathbf{q}_\perp|}-E_k+\hbar\omega_q) . \tag{21}$$

Here ω_q and N_q are the frequency and the number of phonons with wave vector **q**. An elastic anisotropy in GaAs and $\mathrm{Al}_x\mathrm{Ga}_{1-x}\mathrm{As}$ is about 10% and we neglect it. In the isotropic approximation electrons interact with longitudinal photons only. We will neglect also the screening of the electron-phonon interaction. The screening adds the factor $q_\perp/[q_\perp+PH(q_\perp)]$ to the interaction matrix element, where $H(q_\perp)$ is an integral containing the electron wave functions along the z direction.[13] Electrons interact with the phonons which wave vector $q_\perp\lesssim 2k_F^{\rm ch}\approx 3.6\times10^6$ cm^{-1}. On the other hand, $H(q_\perp)<1$ and in GaAs $P\leq 2me^2/\kappa\hbar^2\simeq 2\times10^6$ cm^{-1}. Thus the neglect of screening, corresponding to the case $q_\perp\gg PH(q_\perp)$, is a more or less good approximation.

The interaction of 2D EG with phonons is strongly anisotropic and depends on the channel thickness $d_{\rm ch}$. In the calculation of the interaction matrix element we used the simplest approximation for the 2D electron wave function, $(1/2d_{\rm ch}^3)^{1/2}z\exp(-z/2d_{\rm ch})$, where $d_{\rm ch}=[\kappa\hbar^2/(48\pi e^2n_{\rm ch})]^{1/3}\simeq 25$ Å (see Ref. 5). Thus

$$W_{\rm ch}(q)=\frac{\pi\Xi^2q}{\rho c}[1+(q_zd_{\rm ch})^2]^{-3} , \tag{22}$$

where Ξ is the deformation potential, ρ is the material density, and c is the longitudinal sound velocity.

Using the value $c\approx 5\times10^5$ cm/s we see that the inequalities $2\hbar ck_F^{\rm ch}<k_BT$ and $\hbar c/d_{\rm ch}<k_BT$ are satisfied above 15 K, so that we can use the equipartition and quasielastic approximation. As a result we have

$$\gamma_{\rm ch}=\frac{\Xi^2m^2k_B}{96\pi\rho\hbar^3d_{\rm ch}^3}(1+6k_F^{\rm ch2}d_{\rm ch}^2) . \tag{23}$$

In the calculation of $\gamma_{\rm ch}$ the assumption was made that the electron distribution function is the Fermi function with some effective temperature. This assumption can be justified by comparison of the inelastic relaxation time due to electron-phonon scattering $\tau_{\rm ch}^{\rm in}=n_{\rm ch}k_B/\gamma_{\rm ch}$ with the electron-electron relaxation time. Equation (23) gives $\tau_{\rm ch}^{\rm in}\simeq 3\times10^{-9}$ s. For the electron-electron relaxation time we have an estimate[14] $\hbar E_F^{\rm ch}/(k_FT)^2\simeq 1.6\times10^{-12}$ s at $T\sim 30$ K. This means that any possible distortion of the distribution function at the contact dissipates very fast and can be neglected.

The thermoconductivity of 2D EG,

$$\lambda_{\rm ch}=\frac{\pi^2}{3}\frac{\tau_{\rm ch}n_{\rm ch}}{m}k_B^2T , \tag{24}$$

so that we obtain

$$l_T^2=32\pi^3\frac{\rho\hbar^3n_{\rm ch}\tau_{\rm ch}d_{\rm ch}^3}{\Xi^2m^3(1+6k_F^{\rm ch2}d_{\rm ch}^2)}k_BT . \tag{25}$$

For $\Xi\simeq 13.5$ eV (Ref. 7) and $T\simeq 30$ K we have $l_T\simeq 25$ μm. This estimate shows that in a sample of 70 μm length the most prominent temperature gradient is near the ends while the middle is at the lattice temperature.

In the case of impurity scattering we can use the expression

$$\alpha=\frac{\pi^2}{6}\frac{k_B^2T}{eE_F^{\rm ch}} \tag{26}$$

for the thermoelectric power of the 2D EG to estimate the magnitude of the temperature difference, and then

$$(\Delta T)_{\rm ch}=r\frac{\pi}{6}\frac{\tau_{\rm ch}(k_BT)^2}{\hbar^2(\lambda_{\rm ch}\gamma_{\rm ch})^{1/2}}eF . \tag{27}$$

The most important result here is the temperature and concentration dependence. The latter comes from $\lambda_{\rm ch}$, $\gamma_{\rm ch}$, and $\tau_{\rm ch}=l_{\rm ch}/v_F$, where the electron mean free path $l_{\rm ch}=$const for impurity scattering. The result is $(\Delta T)_{\rm ch}\propto T^{3/2}n_{\rm ch}^{-3/4}$ if we neglect the second term in the brackets in Eq. (23), and $(\Delta T)_{\rm ch}\propto T^{3/2}n_{\rm ch}^{-5/4}$ for high concentration when this term is greater than the first one. The magnitude of $(\Delta T)_{\rm ch}$ is rather indefinite because of the coefficient r. If we consider the electron gas in a contact as a 2D EG then we obtain the estimate

$$r=\frac{(\lambda_{\rm ch}\gamma_{\rm ch})^{1/2}}{(\lambda_{\rm ch}\gamma_{\rm ch})^{1/2}+(\lambda_1\gamma_1)^{1/2}} , \tag{28}$$

where λ_1 and γ_1 are the coefficients of thermoconductivity and dissipation in the contact. Usually the conductivities of the channel and the contact are of the same order and we can assume the same for the thermoconductivities. But the Fermi energy in the contact is much greater than in the channel which leads to a greater value of the phonon dissipation because more phonons can participate in scattering processes. If we take $\lambda_{\rm ch}\simeq\lambda_1$ and $\gamma_{\rm ch}/\gamma_1\simeq 10$ then $r\simeq 0.25$, and Eq. (27) gives $(\Delta T)_{\rm ch}\simeq 0.5$ K.

IV. ENERGY BALANCE IN 3D EG

The 3D EG receives energy from the 2D EG and transfers it to the lattice. Due to this process a temperature gradient is set up there. First of all one can see that it is a 2D temperature gradient. To check this fact it is enough to estimate the length over which the temperature gradient extends with the help of the expression similar to Eq. (20). The evaluation of the 3D EG energy dissipation to phonons is more complicated than the evaluation for the 2D EG. The problem is that one cannot use the equipartition and quasielastic approximation because of the greater magnitude of k_F compared to k_F^{ch}. In fact, the energy of a typical phonon corresponds to $2\hbar c k_F/k_B \approx 29$ K which is within the studied temperature range. We use here a method similar to that of Ref. 15 and begin with the general expression for the electron energy transferred to phonons in unit volume per unit time,

$$\left[\frac{dE}{dt}\right]_{eph} = -\int \frac{2\,d^3k}{(2\pi)^3}\frac{d^3q}{(2\pi)^3}\hbar\omega_q \frac{\pi\Xi^2}{\rho c} \times q[f_{\mathbf{k}}(1-f_{\mathbf{k+q}})(N_{\mathbf{q}}+1) - f_{\mathbf{k+q}}(1-f_{\mathbf{k}})N_{\mathbf{q}}] \times \delta(E_{|\mathbf{k+q}|}-E_k+\hbar\omega_q)\ . \tag{29}$$

Assuming that $f_{\mathbf{k}}$ is the Fermi function, in the linear approximation with respect to the difference between the electron temperature T_g and the lattice temperature

$$\left[\frac{dE}{dt}\right]_{eph} = -\gamma(T_g - T)\ , \tag{30}$$

where

$$\gamma = \frac{\pi\Xi^2\hbar^2}{\rho c k_B T^2}\int \frac{2\,d^3k}{(2\pi)^3}\frac{d^3q}{(2\pi)^3} f_0(E_k-\hbar\omega_q)[1-f_0(E_k)] \times \delta(E_{|\mathbf{k+q}|}-E_k+\hbar\omega_q)\ . \tag{31}$$

$\hbar\omega_q \sim \min(k_B T, 2\hbar c k_F)$ and it can be neglected in the argument of the δ function. After this the integrations with respect to angles and k can be carried out and

$$\gamma = \frac{\Xi^2 m^2 c}{4\pi^3\rho\hbar^2 T} \times \int_0^\infty N_q(1+N_q) \times \ln\left[\frac{\exp\left[\dfrac{E_{q/2}-E_F}{k_B T}\right]+\exp\left[\dfrac{\hbar\omega_q}{k_B T}\right]}{\exp\left[\dfrac{E_{q/2}-E_F}{k_B T}\right]+1}\right] q^4\,dq\ . \tag{32}$$

The logarithm in the integrand equals $\hbar\omega_q/k_B T$ if $E_{q/2}-E_F<0$ and is exponentially small in the opposite case. The width of the transition region $\Delta q \sim 2mk_B T/(\hbar^2 k_F)$ is so small that $\hbar c\Delta q/k_B T \sim 2mc/(\hbar k_F) \simeq 1.5\times 10^{-2}$ and N_q nearly does not change there. As a result we have

$$\gamma = \frac{\Xi^2 m^2 c}{4\pi^3\rho\hbar^2 T}\int_0^{2k_F} N_q(1+N_q)\frac{\hbar\omega_q}{k_B T} q^4\,dq = \begin{cases} \dfrac{\Xi^2 m^2 k_F^4 k_B}{\pi^3\rho\hbar^3}, & 2\hbar c k_F \ll k_B T \\ C_1\dfrac{\Xi^2 m^2 k_B^5 T^4}{\rho\hbar^7 c^4}, & 2\hbar c k_F \gg k_B T\ . \end{cases} \tag{33}$$

The low-temperature asymptotic form in Eq. (33) should be used very carefully. It gives an error less than 10% only for $2\hbar c k_F/k_B T > 10$.

Equation (33) gives for electron-phonon relaxation time $n_g k_B/\gamma \simeq 1.5\times 10^{-10}$ s at $T\simeq 30$ K. Under the same conditions electron-electron relaxation time $\hbar E_F/(k_B T)^2 \simeq 10^{-11}$ s which justifies the electron temperature approximation.

Now making use of the expression for electron thermal conductivity $\lambda = k_B^2 T\tau_g k_F^3/9m$ we obtain the following estimate for length scale of the temperature change at high temperature:

$$l^2 = \frac{\pi^3}{9}\frac{\rho\hbar^3 k_B T\tau_g}{\Xi^2 m^3 k_F}, \quad 2\hbar c k_F \ll k_B T\ . \tag{34}$$

For 30 K Eq. (34) gives $l \simeq 1.7\ \mu$m which is well above the thickness of the gate $d_g \simeq 1000$ Å. For lower temperature this inequality becomes even stronger. It means that while diffusing across the gate in the z direction the 3D electrons do not transfer much of their energy to phonons, which allows us to consider the thermoconductivity in the 3D EG as a 2D problem. It shows also that l is much shorter than the scale of the temperature distribution in the channel, l_T. Hence, this last scale determines the temperature distribution in the gate too, and we can study the energy exchange between the gate and the channel locally, as if the temperature along the channel were constant.

The inequality $l \ll d_g$ allows us to calculate the energy transferred from 2D EG to 3D EG averaged over the thickness of the gate

$$\left[\frac{dE}{dt}\right]_{ee} = \frac{1}{d_g}\int_0^{d_g} dz \int \frac{2\,d^3k}{(2\pi)^3} E_k \hat{I}_{ee} f_{\mathbf{k}}\ . \tag{35}$$

The integrand here is an even function of k_z and the integral with respect to k_z can be reduced to that from zero to infinity. Then we obtain a symmetrical integral with respect to $\mathbf{k}$ and $\mathbf{k}'$, and in the linear approximation with respect to T_{ch}-T_g after standard transformations come up with the expression

$$\left[\frac{dE}{dt}\right]_{ee} = \gamma_{ee}(T_{\mathrm{ch}} - T_g)\ , \tag{36}$$

$$\gamma_{ee} = \frac{2}{d_g k_B T^2}\int_0^\infty \frac{dk_z dk_z'}{\pi^2}\int \frac{d^2k_\perp d^2q\, d^2k_1}{(2\pi)^6} Q(q,k_z,k_z')(E_{|\mathbf{k}_1+\mathbf{q}|}-E_{k_1})^2 \times[1-f_0(E_k)]f_0(E_{k'})[1-f_0(E_{k_1})]f_0(E_{k_1'})\delta(E_k - E_{k'} + E_{k_1} - E_{|\mathbf{k}_1+\mathbf{q}|})\ . \tag{37}$$

This expression can be simplified with the help of Eq. (7), $k_F^{\mathrm{ch}} \ll k_F$, and $q_s a \gg 1$. After substituting the transferred energy $\hbar^2 \mathbf{q}\cdot\mathbf{k}_1/m$ into $f_0(E_{k'})$ it can be neglected in the argument of δ function. Then this argument is reduced to $k_z^2 - k_z'^2 + 2\mathbf{q}\cdot\mathbf{k}_1$. The last term here is much less than the two first terms, which means that $k_z - k_z' = \mathbf{q}\cdot\mathbf{k}_1/k_z \ll k_z$. For $k_z - k_z'$ so small we can put $M(k_z,k_z') \simeq 1/q_s$ without a large error and integrating with respect to k_z' obtain a logarithmic integral with respect to k_z. Its lower limit is determined by the condition $k_z \sim q k_1/k_z \sim (k_F/a)^{1/2}$ and the upper one is k_F. The representation of this integration as one with respect to angles of **k** makes it clear that it is separable. After this it is possible to carry out the integration with respect to $k_\perp$ and k_1, and with logarithmic accuracy

$$\gamma_{ee} = \frac{\hbar^5 \ln(k_F a)}{128\pi d_g m^3 k_F^2 (q_s^{\mathrm{ch}})^2 k_B T^2} \times \int d^2q \left[\frac{q}{\sinh(qa)} \right]^2 \frac{(k_F^{\mathrm{ch}} q_x)^4}{\sinh^2 \left[\dfrac{\hbar^2 k_F^{\mathrm{ch}} q_x}{2mk_B T} \right]} . \tag{38}$$

At high temperature, when $\hbar^2 k_F^{\mathrm{ch}}/2ma \ll k_B T$ sinh in Eq. (38) can be replaced by its argument (quasielastic approximation), and

$$\gamma_{ee} = \frac{\pi^3 C_1}{128} \frac{\hbar E_F^{\mathrm{ch}} k_B}{d_g m E_F a^4 (q_s^{\mathrm{ch}} a)^2} \ln(k_F a) , \tag{39}$$

where C_1 is defined in Eq. (12). In the opposite case the main contribution to the integral comes from the regions where $q_x \ll q_y$. In this region q_x can be neglected in the argument of the exponent and once again the integral is evaluated without any difficulty,

$$\gamma_{ee} = \frac{\pi^5}{360} \frac{m k_B^4 T^3}{d_g \hbar^3 E_F a k_F^{\mathrm{ch}} (q_s^{\mathrm{ch}} a)^2} \ln(k_F a) . \tag{40}$$

It is important to note that in this case the integrand has a maximum at $q_x \approx 3.8 m k_B T/(\hbar^2 k_F^{\mathrm{ch}})$ and the condition $q_x \ll q_y$ is satisfied for $T < 20$ K which gives the upper limit for the asymptotics Eq. (40). The physical meaning of the decrease of γ_{ee} with temperature is that at low temperature the phase space of 3D electrons which can participate in CMS is very limited because not all scattering processes satisfy the energy conservation law.

Equations (30) and (36) allows us to find the electron temperature from the electron energy balance in the gate

$$T_g = \frac{\gamma T + \gamma_{ee} T_{\mathrm{ch}}}{\gamma + \gamma_{ee}} . \tag{41}$$

At a high temperature Eqs. (33) and (39) give $\gamma/\gamma_{ee} \simeq 14$ so that $T_g - T \simeq (\Delta T)_{\mathrm{ch}}/15$. But in this case, as in Sec. II, an attempt to estimate the thermocurrent according to

$$J = -\frac{2e\tau_g k_F}{3\hbar^2} k_B^2 T \, \Delta T_g \frac{d_g L_y}{L_x} \tag{42}$$

for $L_x = 70$ μm, $L_y = 40$ μm, $d_g = 500$ Å, $T = 30$ K, and the temperature difference between the ends of the channel $(\Delta T)_{\mathrm{ch}} = 0.5$ K leads to the $J \simeq 2 \times 10^{-9}$ Å which is about an order of magnitude greater than the observed value.

V. DISCUSSION

The calculations made in the preceding sections show that the current observed in the experiment[1] is a result of two competing effects resulting from CMS. The first of them is the direct momentum transfer from 2D to 3D EG inducing the current in the gate in the same direction as that in the channel. The second effect is the energy transfer from 2D to 3D EG. This effect is important because the Peltier effect at the contacts to the channel produces an electron temperature gradient along it. The energy transfer results in the electron temperature gradient in the gate which induces a thermocurrent in the direction opposite to that of the channel current. The competition of the two currents explains the major features of the total effect.

1. A nontrivial temperature dependence of the total current is related to different temperature dependencies of the currents comprising it. The current induced by the momentum transfer is proportional to the number of 3D electrons which can interact with nonequilibrium 2D electrons, i.e., $\propto T$, Eq. (16). When the main mechanism for the momentum relaxation both in the channel and in the gates is impurity scattering and the corresponding relaxation times are temperature independent, it is the only source of the current temperature dependence. At higher temperature a phonon contribution to the momentum scattering in the channel becomes more pronounced. It does not happen in the gate because of a heavy doping there. The momentum relaxation time due to the phonon scattering $\propto 1/T$ and the current saturates when the temperature increases. The observed saturation temperature is about 80 K which more or less corresponds to the temperature where phonon scattering in the channel becomes of the order of impurity scattering.

The temperature dependence of the current induced by the energy transfer is more complicated. The temperature gradient which is set up in the channel is the result of a balance between the heating or cooling of electrons at the contacts and an energy relaxation due to phonon scattering. Electrons interact only with the phonons whose wave vectors are smaller than $2k_F^{\mathrm{ch}}$ which is much smaller than the wave vector of the thermal phonons. As a result, the energy relaxation time is constant, and the temperature dependence of the temperature gradient is caused by that of thermoelectric power and thermoconductivity of the 2D EG, so that $(\Delta T)_{\mathrm{ch}} \propto T^{3/2}$, Eq. (27). The energy balance in the gate is determined by the energy coming from the 2D EG and by phonon relaxation. At high temperature, when the energy relaxation time is constant, as in the channel, the incoming energy is transferred to the lattice and the temperature gradient is smaller than that in the channel. The thermocurrent is smaller than the current induced by the momentum transfer. When the temperature decreases, the wave vector of the thermal phonons becomes smaller than $2k_F$, the number of phonons which can participate in the energy relaxation decreases rapidly, and the energy relaxation time increases, Eq. (33). This does not happen in the channel because of $k_F^{\mathrm{ch}} \ll k_F$. The increase of the energy relaxation time leads to an increase of the temperature gradient in 3D EG, which approaches that in the channel. As a result the thermocurrent increases and the total current changes its sign. The change of the electron energy dissipation in the gate happens at the temperature $T \simeq 2\hbar c k_F/k_B \simeq 30$ K which is close to the temperature where the effect changes its sign in the experiment.[1] A further decrease of the temperature eventually leads to a decrease of the thermocurrent (and possible change of the sign of the total current) which can be related to a few reasons. First of all, the Peltier effect and the kinetic coefficient determining thermocurrent decrease with temperature. Also, at low temperature CMS becomes highly inelastic which diminishes the energy exchange between the 2D EG and 3D EG, Eq. (40).

The temperature dependence of the current described above (Fig. 2) qualitatively corresponds to the experimental one. Quantitative estimates of the current magnitude give, however, values greater than the experimental ones approximately by an order of magnitude, both for low and for high temperatures. The physical reason for this is an overestimate of the coupling constant between the 2D EG and 3D EG and the unknown portion of the Peltier heat flowing into the contacts. The large uncertainty concerning these two components of the total current also prevents one from obtaining a good estimate for the temperature where the total current changes its sign. There can be also other reasons for lack of quantitative agreement, such as an uncertainty or nonuniformity of the doping. Nevertheless, we have confidence as to the suggested explanation of the experiment, because the theory explains not only the temperature dependence but also most of the other characteristic features of the effect.

2. The same physical arguments explain the reciprocal effect, where the external signal is applied to the gate and the effect is measured in the channel. Here the temperature gradient due to the Peltier effect arises in the gate and the energy balance there controls the amount of energy transferred to the channel. The total current through the channel is the sum of the thermocurrent and the current due to the direct momentum transfer.

3. At high temperature the theoretical current does not depend on the electron concentration in the channel Eq. (16). This explains the observed saturation of the current when the gate voltage increases. Both the channel current and the current in the gate decrease at low values of the gate bias because the electron concentration has decreased to the point where a continuous 2D EG does not exist in the channel.

At low temperature an increase of the current with the gate voltage eventually changes to a decrease due to a decrease of the temperature difference in the channel, Eq. (27).

4. Solomon *et al.*[1] also made measurements on the devices with four contacts on the gate. That is, instead of two contacts G_1 and G_2 (Fig. 1) four contacts were made with the distance between adjacent contacts of about 10 μm (Fig. 3). In these devices the main contribution to the low-temperature effect came from the ends of the sample. That is, the effect was observed between the contacts comprising pairs (G_1 and G_2 or G_3 and G_4) but in linear regime no current was detected between the contacts G_2 and G_3 in Fig. 3. This result is easily understandable because the temperature profile in the gate follows that in the channel, and there the temperature gradient disappears in the middle.

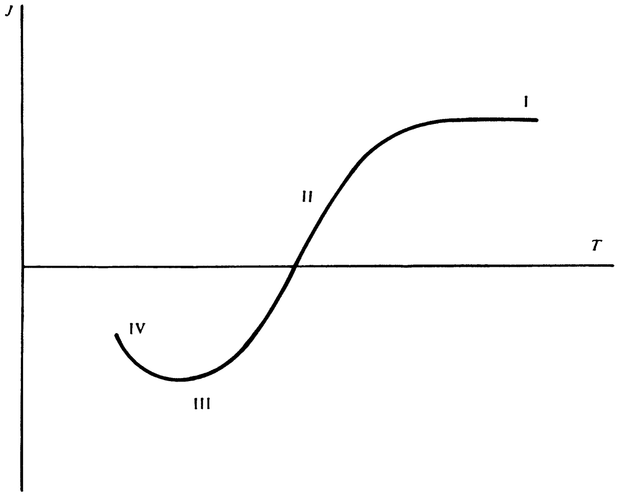

FIG. 2. Qualitative picture of current temperature dependence. I. The current due to direct momentum transfer dominates. II. The region of transition from equipartition regime to Bloch-Grüneisen regime. III. The thermal current dominates. IV. The thermal current decreases with temperature.

5. A nonlinearity of the effect is explained by electric field heating of the 2D EG. The increase of the 2D EG temperature due to the electric field is determined by the equation

$$\frac{T_{\mathrm{ch}}-T}{T}=\frac{(eF)^2\tau_{\mathrm{ch}}\tau_{\mathrm{ch}}^{\mathrm{in}}}{mk_BT} . \tag{43}$$

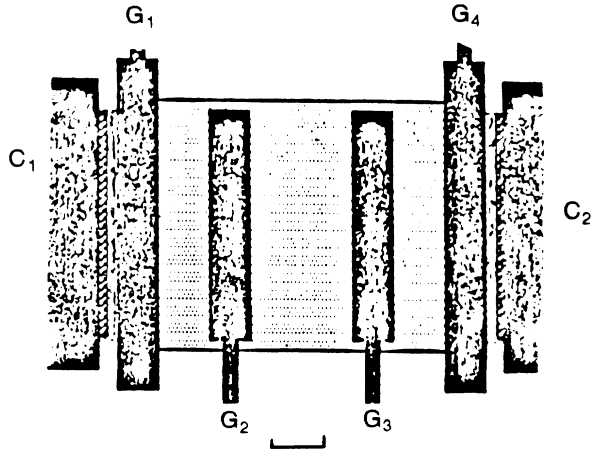

FIG. 3. Four gate-contact device.

At $T=30$ K for voltage of 10 mV on a sample of 70 μm length Eq. (43) gives $(T_{\rm ch}-T)/T \simeq 0.25$. If the voltage increases to 25 mV then $(T_{\rm ch}-T)/T \simeq 1.6$. Thus the voltage corresponding to a pronounced nonlinearity really gives rise to a substantial heating of the 2D EG. The increase of the electron temperature leads to an increase of the Peltier effect and other kinetic coefficients which increases the thermocurrent. It is important to note that the heating of the 2D EG by the field and a heating of the 3D EG, resulting from the energy exchange, do not lead to a lattice heating which could decrease the thermocurrent.

The field heating of the 2D EG increases l_T, Eq. (25), i.e., the part of the sample producing the thermocurrent. It explains the fact that the nonlinear effect was also observed in the middle part of the sample in the devices with four contacts on the gate. In short samples l_T can be of the order or longer than the sample length, which leads to an increase of the nonlinear threshold field due to the electron thermoconductivity. This increase of the nonlinear threshold was also observed in the experiment. The aim of these arguments is only to make an estimate for the threshold and to show possible results of nonlinear effects. The developed theory is essentially linear and it does not consider, for instance, such an interesting question as the effect of the nonuniform Joule heating of the 2D EG which also can contribute to the thermocurrent near the contacts.

In conclusion, a theory is suggested explaining the temperature dependence of the mutual drag of 2D EG and 3D EG in GaAs-Al$_x$Ga$_{1-x}$As heterostructures and the change of the sign of the effect. It explains the dependence of the effect on the gate voltage, nonlinear effect, and the reciprocal effect too. Our results show that the Coulomb mutual scattering is the most important mechanism of energy and momentum exchange between the layers in small heterostructure devices if there is no spatial transfer. Both energy and momentum exchange can induce a current in one of the layers when a current is driven in the other one. The currents induced by these two mechanisms can have opposite directions and the result of the competition between them strongly depends on the delicate details of the electron energy relaxation. A few problems are left unsolved. The nonlinearity of the reciprocal effect which is nearly the same as that of the direct effect is not explained by the current theory. An additional study of the 3D EG properties near the surface is necessary to obtain a better estimate for the magnitude of the effect.

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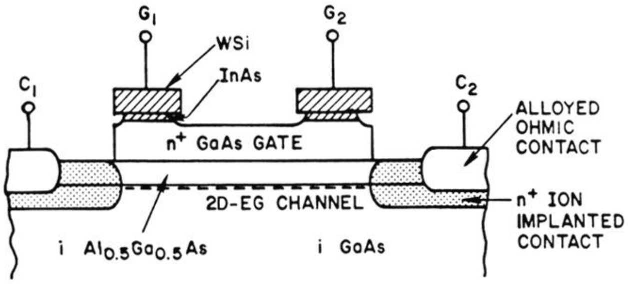

FIG. 1. Device structure.