Resonant tunneling with a time-dependent voltage

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The effects of an ac component in the voltage across a double-barrier structure are investigated in a simple model. The transmission probability for an incoming electron is calculated as a function of time. When the period of the ac voltage is short compared with the lifetime of an electron in the quantum well, photon-assisted tunneling occurs. In the opposite case, when the ac-voltage period is long, the transmission probability is governed by the instantaneous value of the voltage. In an intermediate regime, interesting interference effects, similar to effects seen in level-crossing problems, show up. We also calculate the time-dependent current through the structure.

I. INTRODUCTION

Resonant tunneling (RT) through quantum-well structures (QWS's) has for several reasons attracted a lot of attention in the last few years. From a theorist's point of view resonant tunneling is nice because it illustrates the principles of quantum mechanics in a simple way. Electrical engineers like resonant tunneling since the effect can be exploited when building devices such as microwave generators. When a QWS is used to generate or detect microwaves the structure will inevitably experience some perturbations that are varying in time with a frequency equal to the microwave frequency. In this paper we study what happens when the voltage across a QWS has an ac component.

The work of Sollner *et al.*[1] is the experimental starting point for studies of the effect of time-dependent perturbations in RT. Their experiment presented the first evidence at room temperature for negative differential resistance (NDR), which is an essential feature if one is going to use RT for generation of microwaves. Moreover, they studied the influence of electromagnetic radiation on the tunneling.

Theoretical work on tunneling devices acted on by external time-dependent fields has quite a long history. Tien and Gordon[2] studied the effect microwave radiation has on superconducting tunneling devices. Several workers[3,4] have investigated the effect time-dependent terms in the potential will have in different tunneling problems.

In this paper we calculate how the transmission probability for electrons in RT is changed due to an ac-voltage component across the structure. We do this by using a model based on a simple tunneling Hamiltonian.[5] Sokolovski[6,7] has studied the interaction between radiation and resonantly tunneling electrons in a thorough manner. His model involves a detailed description of the double-barrier structure and the interaction with the radiation field. Sokolovski in his treatment uses explicit electron wave functions. The present work gives an alternative, and as we believe simpler treatment of the same problem. Given the tunneling Hamiltonian we solve the equations of motion for the electron operators and calculate the transmission probability for the electrons.

The results of the two calculations are quite similar in a number of limiting cases. This should be no surprise since as long as the probability for tunneling through one single barrier is small the detailed form of the wave function in the barrier region is of little importance and all the essential physics is described by the tunneling Hamiltonian. In our calculation we have also tried to put more emphasis on the time dependence of the transmission probability than has been done in earlier work. We investigate in some detail the crossover from high-frequency external fields to low-frequency fields.

Our work was initially inspired by theories on the interaction between a tunneling electron and an *internal* time-dependent perturbation, namely the optical phonons.[8–11] We will indeed see that under appropriate conditions, an ac voltage (photons) will have the same effect on the resonant tunneling as have phonons. There is, however, also a crucial difference between the two cases in that the photon field is coherent and thus has a certain phase while the phonon field is incoherent. The result of this is that an ac voltage will give an ac current. The phonons, on the other hand, just modify the magnitude of the dc current flowing through the QWS.

II. THE MODEL

We use a very simple model for the QWS shown in Fig. 1, with the Hamiltonian H given by

$$\begin{aligned} H &= H_0 + H_T \, , \\ H_0 &= \sum_k [\varepsilon_{kL} + V_0\cos(\omega t)] c_{kL}^\dagger c_{kL} + \varepsilon_0 c^\dagger c \\ &\quad + \sum_p [\varepsilon_{pR} - V_0\cos(\omega t)] c_{pR}^\dagger c_{pR} \, , \qquad (1) \\ H_T &= \sum_k T_{kL}(c^\dagger c_{kL} + \text{H.c.}) + \sum_p T_{pR}(c^\dagger c_{pR} + \text{H.c.}) \, , \end{aligned}$$

where c_{kL}, c_{pR}, $c_{kL}^\dagger$, and $c_{pR}^\dagger$ are destruction and creation operators for electrons in the left- and right-hand side leads of the QWS, respectively. The operators c and $c^\dagger$, respectively, destroy and create electrons in the resonant level in the quantum well.

The electrons in the leads are considered to be approximately free with an effective mass m^*, i.e., ε_{kL} and ε_{pR}

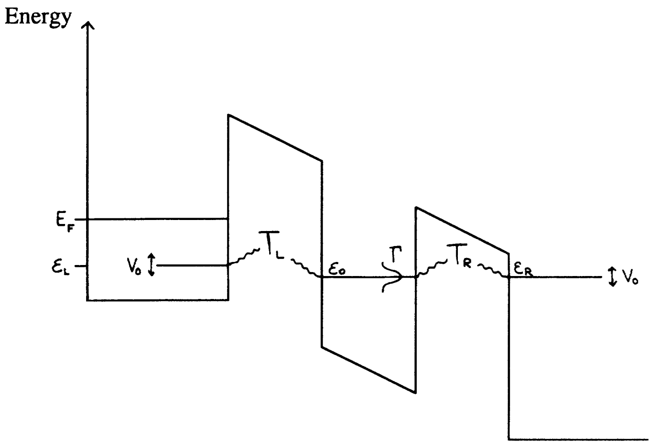

FIG. 1. Schematic picture of the quantum-well structure. The electrons are free in the leads except for the time-dependent shifting of the energy due to the ac voltage. In the well the only allowed energy is ε_0. The matrix elements T_L and T_R provide the coupling between the different regions of the structure.

are given by $\varepsilon_{kL}=k_L^2/2m^*$, etc. Here and in the following calculations units such that $\hbar=1$ has been used.

To find the correct value for the electron energy ε_0 in the quantum well one has to solve the Schrödinger equation in one dimension using the potential which is given by the shape of the well. We have only included the resonant level with the lowest energy in the model. This means that we cannot treat situations where the frequency of the external field is so large that it can excite electrons to higher energies inside the well.

In this context we want to point out that the Hamiltonian we use is in principle one dimensional. All the energies we have defined refer only to the motion perpendicular to the planes of the barriers. Of course there is motion also in the directions parallel to the barriers but since we are not considering any interactions that can change the parallel momentum $\mathbf{k}_\parallel$ of an electron we actually solve a one-dimensional problem for each and every value of $\mathbf{k}_\parallel$.

The effect of the external field is taken care of by the two terms $V_0\cos\omega t$. The effect of the time-dependent voltage in our model is thus to make the electron energies in both leads vary with time. The parameter V_0 is related to the ac-voltage amplitude U_W across the well by $V_0=eU_W/2$.

The possibility for an electron to tunnel through the barriers into and out of the quantum well is modeled by H_T. The matrix elements T_{kL} and T_{pR} can be calculated using the prescription first given by Bardeen.[12]

Our model is thus very simple which has the advantage that only a few parameters are needed to characterize the situation. On the other hand, a simple model of course means that one cannot describe all effects in a correct way. Our model of the external field for instance does not say anything about the details of the electric field in the barriers and the quantum well.

The use of a tunneling Hamiltonian means that we cannot describe any of the structure of the tunneling events. The matrix elements T_{kL} and T_{pR} just tell how large the overlaps between the different states are. This should be no restriction as long as the time it takes for an electron to travel through a barrier is considerably shorter than all the other time scales in the problem. With other relevant time scales we mean the lifetime of an electron inside the quantum well and the period of the external field.

Finally, we have not taken into account any electron-electron interactions and we will concentrate on the properties at zero temperature.

III. CALCULATION OF THE TRANSMISSION PROBABILITY

We begin by calculating the transition probability for an electron from the left lead to the right lead. Since we are interested in time-dependent quantities we want to know how this probability changes with time. Let us define

$$P(t)=A^*(t)A(t)=|\langle pR|U(t,-\infty)|kL\rangle|^2 . \qquad (2)$$

Here $|kL\rangle$ and $|pR\rangle$ represent many-electron wave functions for states with $N+1$ electrons in total. We focus our attention on the last electron in these many-electron states, thus kL means that the extra electron is in the left lead and has a perpendicular momentum k and pR means that it is in the right lead with the perpendicular momentum p. All the other N electrons are supposed to be in the same one-electron states in both $|kL\rangle$ and $|pR\rangle$.

The time-development operator $U(t,-\infty)$ is to be calculated in the interaction representation of our Hamiltonian so that $P(t)$ expresses the probability for an electron initially starting in the state kL to be in state pR at time t. The perturbation term in the Hamiltonian is H_T. Since the tunneling matrix elements T_{kL} and T_{kR} both are very small in all applications we calculate $U(t,-\infty)$ to lowest possible order in these quantities. Thus

$$U(t,-\infty)=(-i)^2\int_{-\infty}^{t}ds\int_{-\infty}^{s}d\tau\, T_{kL}T_{pR}c_{pR}^\dagger(s)\times c(s)c^\dagger(\tau)c_{kL}(\tau) . \qquad (3)$$

The equations of motion for the operators are $idc/dt=[c,H_0]$ etc. and we easily find the following expression:

$$\langle pR|c_R^\dagger(s)c(s)c^\dagger(\tau)c_L(\tau)|kL\rangle=\exp\{i[\varepsilon_{pR}s-\varepsilon_{kL}\tau-(V_0/\omega)\sin(\omega s)-(V_0/\omega)\sin(\omega\tau)]\}\langle 0|c(s)c^\dagger(\tau)|0\rangle \qquad (4)$$

where $|0\rangle$ stands for the vacuum state of the quantum well. It is possible to factorize out the last expectation value in Eq. (4) because $|kL\rangle$ and $|pR\rangle$ are product wave functions containing one factor from each lead and also one factor from the quantum well. When calculating $\langle 0|c(s)c^{\dagger}(\tau)|0\rangle$ we introduce a broadening[8] of the resonant level due to the finite probability for the electron to tunnel out of the well. We assume that the decay is exponential and thus

$$\langle 0|c(s)c^{\dagger}(\tau)|0\rangle = e^{-i(\varepsilon_0 - i\Gamma/2)(s-\tau)}, \quad \text{for } s>\tau . \qquad (5)$$

The broadening Γ of the resonant level appears to be a constant in Eq. (5). This is of course not true. The broadening depends indeed very strongly on such things as the energy of the resonant level and the bias voltage across the QWS. Moreover the time dependence in our problem also means that Γ varies with time. To be able to simplify the calculations we will neglect these complications, but we write down the basic equations for the energy-dependent broadening. The golden rule gives us

$$\Gamma(\varepsilon)=\Gamma_L(\varepsilon)+\Gamma_R(\varepsilon) = 2\pi\left[\sum_k T_{kL}^2\delta(\varepsilon-\varepsilon_{kL})+\sum_p T_{pR}^2\delta(\varepsilon-\varepsilon_p R)\right] . \qquad (6)$$

Expressed in another way $\Gamma(\varepsilon)$ is the imaginary part of the electron self-energy due to the coupling of the resonant level to the states in the leads. The self-energy of course also has a real part $\Sigma(\varepsilon)$ which enters the calculation as a shift of the resonant level energy. For ε close to the band edges Γ and Σ are strongly energy dependent, and then the expectation value $\langle 0|c(s)c^{\dagger}(\tau)|0\rangle$ will be more complicated than in Eq. (5). In the rest of this paper we will neglect these dispersion effects since we feel that they are irrelevant considering the simplicity of our model. Putting (2)–(5) together we arrive at the following integral for $A(t)$:

$$A(t)=-T_{kL}T_{pR}\int_{-\infty}^{t} ds \exp\{i[\varepsilon_{pR}s-\varepsilon_0 s+i\Gamma s/2-a\sin(\omega s)]\}\int_{-\infty}^{s} d\tau \exp\{i[\varepsilon_0\tau-\varepsilon_{kL}\tau-i\Gamma\tau/2-a\sin(\omega\tau)]\} \qquad (7)$$

where $a=V_0/\omega$. To evaluate the integral we use the Fourier expansion

$$e^{ia\sin\phi}=\sum_{n=-\infty}^{\infty} J_n(a)e^{in\phi} . \qquad (8)$$

In this equation $J_n(a)$ denotes a Bessel function of order n. In order for the s integral to be well defined a convergence factor $e^{\delta s}$ where δ is infinitesimally small and positive is introduced. The result for $A(t)$ reads

$$A(t)=T_{kL}T_{pR}\sum_{n,m}\frac{\exp[i(\varepsilon_{pR}-\varepsilon_{kL}-n\omega-m\omega)t]e^{\delta t}J_n(a)J_m(a)}{(\varepsilon_{pR}-\varepsilon_{kL}-n\omega-m\omega-i\delta)(\varepsilon_0-\varepsilon_{kL}-m\omega-i\Gamma/2)} . \qquad (9)$$

The time derivative of the transition probability is

$$\begin{aligned}\frac{dP}{dt}&=\frac{dA^*}{dt}A+A^*\frac{dA}{dt}\\ &=2\mathrm{Re}\left[A^*\frac{dA}{dt}\right]\\ &=2\,\mathrm{Re}\left[iT_{kL}^2T_{pR}^2\sum_{p,q,n,m}\frac{e^{i(p+q-n-m)\omega t}}{(\varepsilon_{pR}-\varepsilon_{kL}-p\omega-q\omega+i\delta)(\varepsilon_0-\varepsilon_{kL}-q\omega+i\Gamma/2)}\right.\\ &\qquad\left.\times\frac{1}{\varepsilon_0-\varepsilon_{kL}-m\omega-i\Gamma/2}J_n(a)J_m(a)J_p(a)J_q(a)\right] . \end{aligned} \qquad (10)$$

This is the rate at which electrons go into one specific state in the right lead after having started in another specific state in the left lead.

However, we do not need such detailed information. We just want to know the rate $w_1(\varepsilon_L,t)$ at which an electron with a certain initial energy ε_L in the left lead tunnels through the QWS into any state in the right lead. Summing the expression in Eq. (10) over final states we get the following appealingly simple result:

$$w_1(\varepsilon_L,t)=T_L^2\Gamma_R\sum_{m,q}\frac{e^{i(q-m)\omega t}J_m(a)J_q(a)}{(\varepsilon_0-\varepsilon_L-q\omega+i\Gamma/2)(\varepsilon_0-\varepsilon_L-m\omega-i\Gamma/2)} . \qquad (11)$$

This simplicity relies heavily on the assumption that $\Gamma_R(\varepsilon)$ defined in Eq. (6) is energy independent. Furthermore we have neglected the real-valued principal part $\sum(\varepsilon_{pR}-\varepsilon_{kL}-p\omega-q\omega)^{-1}$ of the sum over final states. The latter approximation is justified physically by the fact that at high bias voltages there are a lot of empty states in the right lead both below and above the energy where overall energy conservation is satisfied, leading to cancellations in the sum.

The total number of electrons with initial energy near ε_L tunneling through the QWS per unit time is

$$w(\varepsilon_L,t)d\varepsilon_L=\frac{\Gamma_L\Gamma_R}{2\pi}\sum_{m,q}\frac{e^{i(q-m)\omega t}J_m(a)J_q(a)}{(\varepsilon_0-\varepsilon_L-q\omega+i\Gamma/2)(\varepsilon_0-\varepsilon_L-m\omega-i\Gamma/2)}d\varepsilon_L\ . \tag{12}$$

To calculate the current through the structure one has to integrate $w(\varepsilon_L,t)$ over ε_L and sum over parallel momentum. This will be done in Sec. V.

The most important quantity in a tunneling problem is, however, the transmission probability for an electron impinging on the barrier structure with a certain energy. If we let D denote the normalization length of the left lead the following relation holds between w_1 and the transmission probability T:

$$w_1(\varepsilon_L,t)=\frac{v(\varepsilon_L)}{2D}T(\varepsilon_L,t)\ . \tag{13}$$

Here $v(\varepsilon_L)$ is the velocity of the electron in the left lead and Eq. (13) just states that the tunneling rate is equal to the attempt frequency times the transmission probability. From Eq. (6) we get

$$\Gamma_L(\varepsilon)=2\pi\sum_k T_{kL}^2\delta(\varepsilon-\varepsilon_L)$$
$$=2\pi\int d\varepsilon_L\frac{D}{\pi}\frac{T_L^2}{v(\varepsilon_L)}\delta(\varepsilon-\varepsilon_L)=\left.\frac{2DT_L^2}{v(\varepsilon_L)}\right|_{\varepsilon_L=\varepsilon}\ . \tag{14}$$

In Eq. (14) D/π is the density of states in perpendicular momentum space when the basis is built up by standing wave functions. The factor $1/v(\varepsilon_L)$ comes from changing the integration variable from k_L to ε_L. From Eq. (14) and the assumption that Γ_L is not depending on energy we get the following time-dependent transmission probability:

$$T(\varepsilon_L,t)=\Gamma_L\Gamma_R\sum_{m,q}\frac{e^{i(q-m)\omega t}J_m(a)J_q(a)}{(\varepsilon_0-\varepsilon_L-q\omega+i\Gamma/2)(\varepsilon_0-\varepsilon_L-m\omega-i\Gamma/2)}\ . \tag{15}$$

We see that $T(\varepsilon_L,t)$ as well as $w_1(\varepsilon_L,t)$ and $w(\varepsilon_L,t)$ are absolute squares and we can write

$$T(\varepsilon_L,t)=f^*(t)f(t)=|f(t)|^2\ ,$$
$$f(t)=(\Gamma_L\Gamma_R)^{1/2}\sum_m\frac{e^{-im\omega t}J_m(a)}{(\varepsilon_0-\varepsilon_L-m\omega-i\Gamma/2)}\ . \tag{16}$$

Sokolovski[7] obtained the same result in a limit in his calculation.

For numerical calculations of the transmission probability Eq. (16) is the best starting point but the limiting behavior of $f(t)$ in some cases becomes more apparent if the sum is converted into an integral. Using the inverse of Eq. (8),

$$J_m(a)=\frac{1}{2\pi}\int_0^{2\pi}d\phi\, e^{i(a\sin\phi-m\phi)} \tag{17}$$

and then interpreting the sum in the expression for $f(t)$ as a sum of residues, $f(t)$ can be brought into the form

$$f(t)=\frac{i(\Gamma_L\Gamma_R)^{1/2}}{\omega}\times\int_0^{2\pi}d\beta\frac{\exp[ia\sin(\beta-\omega t)-i\beta\Delta-\beta\gamma]}{1-e^{-2\pi\gamma}e^{-2\pi i\Delta}}\ , \tag{18}$$

where $\gamma=\Gamma/2\hbar\omega$ and $\Delta=(\varepsilon_0-\varepsilon_L)/\hbar\omega$.

The technique we have used in order to perform this transformation is identical to the standard methods[13] for summation over Matsubara frequencies.

IV. RESULTS FOR THE TRANSMISSION PROBABILITY

The behavior of the transmission probability $T(\varepsilon_L,t)$ is mainly determined by the two parameters a and γ. To vary $\gamma=\Gamma/2\hbar\omega$ one has to change the frequency ω, since Γ is in principle fixed after the device has been built. The parameter $a=V_0/\hbar\omega$ can be changed either by varying the ac-voltage amplitude V_0 or the frequency ω. We find that the limiting behaviors split into two subgroups (low intensity and high intensity) depending on the value of a.

(i) Low intensity. When $V_0/\hbar\omega$ is small ($a<1$) only the Bessel functions with index close to zero will give any considerable contributions to $f(t)$ in Eq. (16). Then if the broadening of the resonant level is small compared to the ac frequency ($\Gamma/2\hbar\omega\ll1$) the transmission probability will show peaks at integer values of $(\varepsilon_0-\varepsilon_L)/\hbar\omega$. The physical meaning of this is that only electrons whose unperturbed energy is either "correct" from the beginning or is off resonance by an integer number of photon energies can tunnel through the QWS. When the level width is larger than the photon energy $\hbar\omega$ the effects of this photon-assisted tunneling cannot be resolved anymore.

In Fig. 2(a) we have plotted the transmission probability T as a function of $(\varepsilon_0-\varepsilon_L)\hbar\omega$ at two different times. The parameter values are $a=1$ and $\gamma=0.2$. One can see that the time dependence of the transmission probability is quite small in this case.

(ii) High intensity. When a is much larger than 1 we find a number of interesting results depending on γ.

We start by considering the case $\gamma \gg 1$ while at the same time $\gamma^2 \gg a$. Physically this means that the lifetime of an electron inside the quantum well is much shorter than the period of the ac voltage, while the amplitude of the ac voltage is not too high. We expand the sine function in the exponent in Eq. (18) in powers of β,

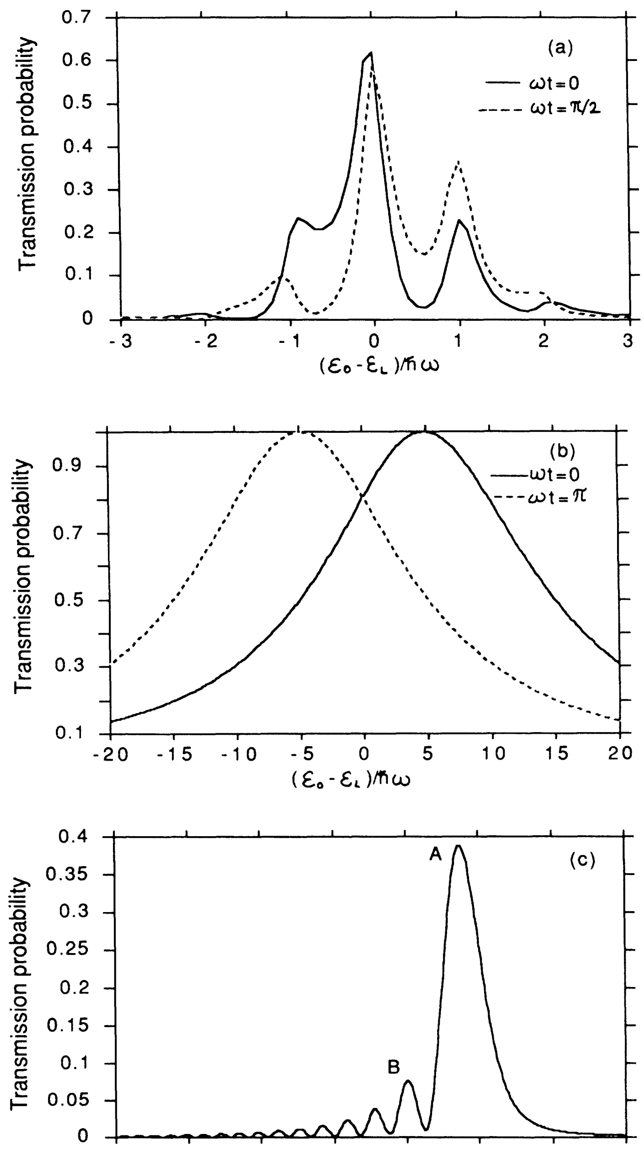

FIG. 2. The transmission probability T in units of $4\Gamma_L\Gamma_R/\Gamma^2$ plotted as a function of $\Delta=(\varepsilon_0-\varepsilon_L)/\hbar\omega$. *Thus different values on Δ refer to different energy levels in the left lead. Each curve corresponds to a particular time t.* (a) The transmission probability with $V_0/\hbar\omega=1$ and $\Gamma/2\hbar\omega=0.2$ plotted at two different times. (b) The parameter values are $V_0/\hbar\omega=5$ and $\Gamma/2\hbar\omega=10$, and we have the adiabatic limit. (c) In this plot $V_0/\hbar\omega=100$, $\Gamma/2\hbar\omega=2$, and $\omega t=0$. Peak A corresponds to energy levels in the left lead which at the moment have approximately the same energy as the resonant level.

$$a\sin(\beta-\omega t)=-a\sin\omega t+\beta a\cos\omega t +(\beta^2 a/2)\sin\omega t+O(\beta^3)\ . \qquad (19)$$

Because of the exponential factor $e^{-\beta\gamma}$ the integral in Eq. (18) will get its major contributions from small β $(0<\beta<1/\gamma)$. In this interval $\beta^2 a/2<a/2\gamma^2\ll 1$ and we can neglect the β^2 term in Eq. (19) when calculating the integral. We get for $T(t)$

$$T(t)=|f(t)|^2\approx\Gamma_L\Gamma_R\frac{1}{|1-e^{-2\pi\gamma}e^{-2\pi i\Delta}|^2}\times\frac{1}{[\varepsilon_0-\varepsilon_L-V_0\cos(\omega t)]^2+\Gamma^2/4}\ . \qquad (20)$$

From this result we see that in this so-called adiabatic limit[7] the transmission probability is determined by the instantaneous positions of the energy levels. See Fig. 2(b).

When the condition on the amplitude $(a\ll\gamma^2)$ for the adiabatic limit is no longer satisfied the transmission probability shows a more dramatic behavior. In Fig. 2(c) we have plotted T with the parameter values $a=V_0/\hbar\omega=100$ and $\gamma=\Gamma/2\hbar\omega=2$. One can see that the transmission probability has a peak (peak A) for electrons with an energy in the left lead that for the moment is close to the resonant level energy.

However, the curve also shows oscillations in the transmission probability. These oscillations are the result of quantum interference phenomena. The peak marked B in Fig. 2(c) is due to electrons whose energy in the left lead coincided with the resonant energy before the time corresponding to the curve. When the level in question passed the resonant level, part of the electron wave function leaked over into the quantum well. Another part of the electron wave function, however, stayed in the left lead a little longer before going into the quantum well. As a result of this the two parts of the wave function will differ in phase. In Fig. 2(c) electrons with $(\varepsilon_0-\varepsilon_L)/\hbar\omega$ at peak B have a phase difference of $\approx 2\pi$ between these two parts of the wave function. The result of this is constructive interference.

When the ac voltage is so large that what is happening from the point of view of an energy level in the left lead is a quick passing of the resonant level twice every period, we are in principle facing a level-crossing problem. In fact the curve in Fig. 2(c) shows a striking qualitative agreement with results from calculations on a clean-cut level-crossing problem.[14]

If we had plotted the curve in Fig. 2(c) using $\gamma\approx 10$ we would have reached the limit where the lifetime of the electron in the well is so short that the interference effect can no longer be seen.

Choosing a small γ $(\gamma\ll 1)$ while still having a large amplitude for the ac voltage $(a\gg 1)$ will give very complicated results for the transmission probability. The peaks at integer $(\varepsilon_0-\varepsilon_L)/\hbar\omega$ typical for photon-assisted tunneling will appear again, but the time dependence of the curve will be pronounced, i.e., the largest peaks will be seen for electrons whose energy at the moment is close to ε_0.

In all the cases with $a\gg 1$ the average shape of the transmission probability (for $\Delta<a$) can be calculated by

applying the stationary phase approximation to the integral in Eq. (18). This is done by Sokolovski.[7]

V. CALCULATION OF THE CURRENT

To get the current we start from the rate $w(t)$, sum over parallel momenta, and finally integrate over the perpendicular energy of the electron in the left lead. We write

$$j(t)=e\int_0^{E_F} d\varepsilon_L \sum_{\mathbf{k}_\parallel} w(t)=e\int_0^{E_F} d\varepsilon_L \frac{A_c}{2\pi^2}\pi k_{\parallel\max}^2 w(t)\ . \tag{21}$$

In this equation E_F is he Fermi energy of the material in the left lead, e is the elementary charge, and A_c is the cross-section area of the QWS. The density of states in parallel momentum space is, including both spin directions, $A_c/2\pi^2$.

The maximum value of the parallel momentum that we can have for a specific ε_L is given by $k_{\parallel\max}^2=2m^*(E_F-\varepsilon_L)$, where m^* is the effective electron mass. Putting this into Eq. (24) we get the following integral to solve for $j(t)$:

$$j(t)=\frac{em^*A_c}{\pi}\int_0^{E_F} d\varepsilon_L(E_F-\varepsilon_L)w(t)\ . \tag{22}$$

We have also investigated the different frequency components of the current through a Fourier analysis:

$$j(t)=j_0+\sum\left[j_k'\cos(k\theta)+j_k''\sin(k\theta)\right]\ ,$$
$$\theta=\omega t,\quad k=1,2,3,\ldots\ ;$$
$$j_0=\frac{1}{2\pi}\int_0^{2\pi}d\theta\, j(\theta)\ ; \tag{23}$$
$$j_k'=\frac{1}{\pi}\int_0^{2\pi}d\theta\, j(\theta)\cos(k\theta)\ ;$$
$$j_k''=\frac{1}{\pi}\int_0^{2\pi}d\theta\, j(\theta)\sin(k\theta)\ .$$

In our numerical calculation of the tunneling current we have estimated our parameter values from recent experimental data.[15] In this experiment a microwave power of 50 mW is concentrated by a wave guide onto a circular area of 4 μm diameter. The magnitude of the electric field in the QWS is in this case $\approx 10^6$ V/m. Since 200 Å is a quite typical value for the length of the well and both the barriers together, we find that 10 meV is a reasonable value to use for V_0.

The microwave frequencies used are about 250 GHz corresponding to $\hbar\omega \approx 1$ meV and the Fermi energy of the material in the leads is ≈ 50 meV. Finally the typical level width Γ is often around 8 meV. Thus we arrive at the following reasonably realistic parameter values: $a=V_0/\hbar\omega=10$, $\gamma=\Gamma/2\hbar\omega=4$.

The result of the current calculation is presented in Fig. 3. The current as a function of bias voltage with no ac voltage is also in the diagram as a comparison. We see that the microwave radiation indeed is so intense that the I-V characteristics of the QWS have been changed. The large ac components in the current clearly show that the time dependence in the problem is important. However, the alternating current is practically impossible to measure so just as in the phonon case the experimental manifestation of the interaction between the tunneling electrons and the ac voltage is best seen in the modification of the dc current.

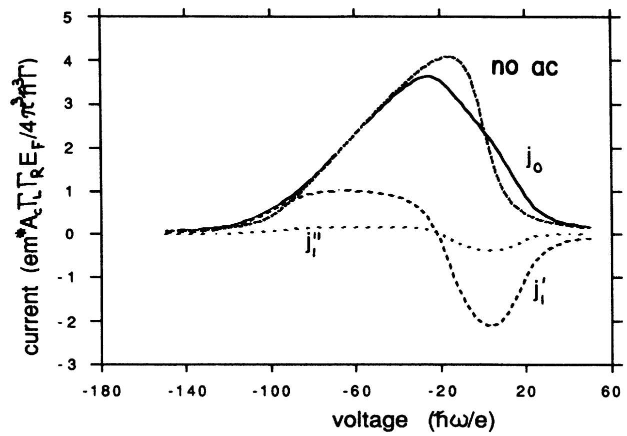

FIG. 3. The Fourier components of the tunneling current through the QWS as a function of the bias voltage. The parameter values used are $V_0/\hbar\omega=10$ and $\Gamma/2\hbar\omega=4$. For comparison we have also plotted the dc current through the same structure with no ac voltage present. Note that the voltage scale zero does not correspond to true zero bias but to the situation when the resonant level coincides in energy with the bottom of the free-electron band in the left lead.

VI. CONCLUSIONS

Starting from a simple tunneling Hamiltonian we have calculated the time-dependent transmission probability for electrons through a double-barrier structure when acted on by an ac voltage. Our solution gives the crossover from photon-assisted resonant tunneling at high frequencies to other limiting behaviors at low frequency.

We find it especially interesting that, for high intensities and ratios of level width to frequency in an intermediate regime, the transmission probability shows clear signatures characterizing Zener tunneling.

Our calculation of the tunneling current clearly shows that the microwave intensities used in recent experiments are high enough to alter the current-voltage characteristics of the QWS.

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