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Skew-scattering effect on the Hall-conductance fluctuation in high-temperature superconductors

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Skew scattering due to spin-orbit interaction is shown to give a new fluctuation effect to the Hall conductance near the superconducting transition point. The magnetic-field dependence of the Hall conductivity is investigated for high-temperature superconductors.

The fluctuation effect of a high-temperature superconductor near the transition point has been clearly observed. Recently, the magnetoresistance has been measured[1] and compared to the theoretical calculations.[2-4] There are two different superconducting fluctuations: one is the Aslamazov-Larkin term and the other is the Maki-Thompson term. The observed magnetoresistance agrees with the theoretical calculations based upon these two contributions. The Hall conductivity of the high-temperature superconductors has been intensively studied and it has been suggested that the superconducting fluctuation of the Hall conductivity has an opposite sign compared to the sign of the normal part.[5-7] Since the usual Aslamazov-Larkin term does not give the fluctuation part for the Hall conductivity to any order, it is of interest to investigate the reason for the existence of superconducting fluctuations. In this paper, we consider the effect of skew scattering due to spin-orbit interaction. This skew scattering gives a nonvanishing superconducting fluctuation to the Hall conductivity. It is known that skew scattering gives an anomalous Hall effect in a normal state. The high-temperature superconductors are known to have an anomalous temperature dependence for the Hall conductivity.[8] Therefore, it may be reasonable to discuss the effect of skew scattering although the origin of the anomalous Hall effect of the high-temperature superconductors in a normal part is not clarified.

The Hall conductivity of a normal metal is easily evaluated in the presence of the skew scattering. It is expressed by

$$\sigma_{xy}=\left(\omega_c\tau+\frac{\tau}{\tau_{sk}}\right)\sigma_{xx}\ , \tag{1}$$

where ω_c is a cyclotron frequency and τ is the impurity scattering lifetime. The skew-scattering lifetime is denoted by τ_{sk}. We consider the following momentum-dependent scattering amplitude,

$$f_{pp'}=a_{pp'}+ib_{pp'}\boldsymbol{\sigma}(\mathbf{p}\times\mathbf{p}')\ , \tag{2}$$

where $\boldsymbol{\sigma}$ is a spin vector. The skew scattering appears due to the presence of the cross term of the first and second term of Eq. (2). The quantity a and b have imaginary parts in this case. If they are real, skew scattering is absent. The skew-scattering lifetime τ_{sk} is given by

$$\frac{1}{\tau_{sk}^{\mu\nu}}=i\epsilon_{\mu\nu\gamma}\left\langle(a_{pp'}b_{p'p}-a_{p'p}b_{pp'})\frac{p_\mu^2}{m}\frac{p_\nu^2}{m}\right\rangle_{\rm imp}\frac{2\pi\nu_1 n_{\rm imp}}{v_F^2/3}\langle\sigma_\gamma\rangle\ , \tag{3}$$

where $\epsilon_{\mu\nu\gamma}$ is an antisymmetric tensor and $\langle\cdots\rangle_{\rm imp}$ means the impurity average. The space coordinates are represented by μ and ν. The single-spin density of state is denoted by ν_1 and $n_{\rm imp}$ is the impurity concentration. $\langle\sigma_\gamma\rangle$ is the average of the spin of the γ component, and it is considered to be proportional to the external magnetic field.

Since the usual Aslamazov-Larkin term with skew scattering does not give a contribution to the Hall conductivity, we consider the diagram of the next order of Fig. 1, which involves the four-point vertex Γ_s with a skew-scattering process. The superconducting fluctuation propagator $K(q,\omega)$ is denoted by the wavy line and the vertex parts have Cooperon corrections $C(q,\omega)$, which are represented by shaded parts in Fig. 1. The skew scattering is denoted by a dotted line with a cross.

We denote the contribution from Fig. 1 by $\Delta\sigma_{xy}^{sk}$. In a weak magnetic field, the magnetic-field dependences of the vertex parts and superconducting propagators may be neglected since $1/\tau$ already has a linear dependence on the magnetic field. $\Delta\sigma_{xy}^{sk}$ is given by

$$\Delta\sigma_{xy}^{sk}=-2T^2\sum_{\Omega}\sum_{\Omega_1}\int d\mathbf{q}d\mathbf{q}_1\Gamma_x(q)\Gamma_s(q,q_1)\Gamma_y(q_1)K_\Omega(q)\times K_{\Omega-\omega}(q)K_{\Omega_1}(q_1)K_{\Omega_1-\omega}(q_1)\ . \tag{4}$$

The vertex part Γ_x becomes eCq_x/m and $\Gamma_s(q,q_1)$ is $3q_xq_{1y}C^2/\pi\nu_1k_F^2\tau_{sk}^{xy}T$. The constant C is calculated for two dimensions as $C=\sqrt{2}m\nu_1\eta$, where $\eta=D\pi/8T$ (D is a diffusion constant). The fluctuation term $\Delta\sigma_{xy}^{sk}$ is evaluated by a similar calculation as the Aslamazov-Larkin

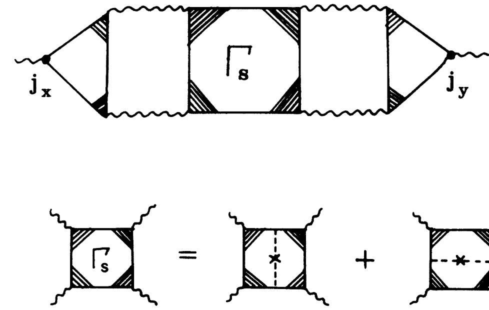

FIG. 1. The nonvanishing Hall-conductivity fluctuation diagram with a skew-scattering vertex Γ_s.

(AL) term for σ_{xx},[9]

$$\Delta\sigma_{xy}^{sk} = -\frac{\Delta\sigma_{xx}^{\rm AL}}{\sigma_0}\frac{\tau}{\tau_{sk}^{xy}}B^{yy}, \quad (5)$$

$$B^{yy} = \frac{e^2C^2}{m^2v_{\rm f}^2}\frac{8}{\pi}\int d^d\mathbf{q}\frac{q_y^2}{\epsilon_q^2}, \quad (6)$$

$$\epsilon_q = \epsilon + \eta q^2, \quad (7)$$

where σ_0 is the normal conductivity and $\epsilon = (T - T_c)/T_c$. The usual Aslamazov-Larkin term for σ_{xx} is denoted by $\Delta\sigma_{xx}^{\rm AL}$. In two dimensions, B^{yy} becomes $(2e^2/\pi^2 d) \times \ln(\pi^2 e/4\epsilon)$, and the Hall conductance Δg_{xy}^{sk} has the following expression:

$$\Delta g_{xy}^{sk} = -\frac{2e^2}{\pi^2\hbar}\frac{\tau}{\tau_{sk}}\left[\frac{\Delta\sigma_{xx}^{\rm AL}}{\sigma_0}\right]\ln\frac{\pi^2 e}{4\epsilon}, \quad (8)$$

where $\Delta\sigma_{xx}^{\rm AL} = e^2/16\hbar d\epsilon$ and d is the thickness of the two-dimensional film. In three dimensions, B^{yy} is a finite constant instead of a logarithmically divergent term, and it is necessary to calculate the integration with Cooperon vertex corrections, as

$$B^{yy} = \frac{8e^2C^2}{m^2v_{\rm f}^2\pi}\int\frac{d^3\mathbf{q}}{(2\pi)^2}\frac{q_y^2}{(\epsilon+\eta q^2)^2[1+(Dq^2/2\pi T)]^4} = \frac{5e^2}{12\pi^2}\left[\frac{2\pi T}{D}\right]^{1/2}. \quad (9)$$

In a strong magnetic field $4DeH/\epsilon > 1$, the superconducting fluctuation propagator K is quantized and the integral is replaced by a summation as in the $\Delta\sigma_{xx}^{\rm AL}$ case,

$$\Delta\sigma_{xy}^{sk} = -\frac{e^2h^4}{\sigma_0}\left[\frac{\tau}{\tau_{sk}}\right]\sum_{n=0}^{n=\infty}\frac{n+1}{\Gamma_n\Gamma_{n+1}(\Gamma_n+\Gamma_{n+1})} \times\frac{4e^2}{\pi^2\hbar}\sum_{n=0}^{n_D}\frac{n+1}{\Gamma_n\Gamma_{n+1}}, \quad (10)$$

where n_D is a cutoff number and is to the order of $c\hbar\pi T/2eHD$. In Eq. (10), Γ_n is $\epsilon+(2n+1)h$ and $h = \ln[T_c(0)/T_c(H)] = 2e\xi_{ab}^2(0)H/\hbar c$.[2] Two summations in Eq. (10) denoted by $A(H)$ and $B(H)$, respectively, are expressed by the digamma function ψ as

$$A(H) = \frac{\epsilon}{8h^2}\left[\frac{h}{\epsilon}+\psi\left[\frac{1}{2}+\frac{\epsilon}{2h}\right]-\psi\left[1+\frac{\epsilon}{2h}\right]\right],$$
$$B(H) = \frac{1}{4}\left[-1-\psi\left[\frac{1}{2}+\frac{\epsilon}{2h}\right]+\psi\left[\frac{3}{2}+n_D+\frac{\epsilon}{2h}\right] + \frac{1}{2}\frac{1+\epsilon/h}{\epsilon/2h+\frac{3}{2}+n_D}\right]. \quad (11)$$

As discussed before,[2] for the weakly coupled anisotropic system, we replace the constant ϵ in Eq. (11) by $\epsilon[1+\alpha(1-\cos k_{\parallel}d)]$ and integrate about $k_{\parallel}$ where $\alpha = 2\xi_{\parallel}^2(0)/d^2\epsilon$ and d is the distance between the layers. The explicit formula for the small magnetic-field case is given in Ref. 2 for $A(H)$. The magnetic-field dependence of $B(H)$ for the quasi-two-dimensional case is also calculated as Ref. 2.

From the above equations, we find that the singularity of $\Delta\sigma_{xy}^{sk}$ is the same as $\Delta\sigma_{xx}^{\rm AL}$ with an opposite sign. In three dimensions, it is proportional to $1/\sqrt{\epsilon}$ for $T > T_c(0)$. However, near $T_c(H)$ $[T < T_c(0)]$, the $n = 0$ term in the summations of Eq. (10) becomes important for a strong magnetic field. It can be seen that $\Delta\sigma_{xy}^{sk}$ has a singularity proportional to $1/\epsilon_H^2$ for the two-dimensional case and it has $1/\epsilon_H$ singularity for the three-dimensional case. Here, ϵ_H is defined by $T - T_c(H)/T_c(H)$. Therefore, near $T_c(H)$, the singularity of $\Delta\sigma_{xy}^{sk}$ becomes stronger than $\Delta\sigma_{xx}^{\rm AL}$. This behavior may be relevant to the observation of Iye, Nakamura, and Tamegai.[5]

There is also the Maki-Thompson (MT) term which gives the contribution to the Hall conductivity. In addition to the usual contribution of the Maki-Thompson term to the Hall conductivity, a skew scattering in the Maki-Thompson diagram gives nonvanishing contribution to the Hall conductivity. We denote the contribution from the Maki-Thompson term including the skew scattering by $\Delta\sigma_{xy}^{\rm MT}$ and it is evaluated by four diagrams as

$$\Delta\sigma_{xy}^{\rm MT} = \left[2\omega_c\tau+\frac{\tau}{\tau_{sk}}\right]\Delta\sigma_{xx}^{\rm MT}, \quad (12)$$

where $\Delta\sigma_{xx}^{\rm MT}$ is a diagonal Maki-Thompson contribution which has been discussed before.[2]

Thus the fluctuation parts for the Hall conductivity consist of two terms of Eqs. (4) and (12). Since the signs of (4) and (12) are different for the skew-scattering contribution, the crossover point from the Maki-Thompson to the Aslamazov-Larkin region may appear at a different point from the case of the diagonal conductivity. It is of interest to compare the derived theoretical estimate to the experimental value. It should be noticed that the normal part of the Hall conductivity is anomalous for the high-temperature superconductivity, and therefore it may be difficult to subtract the normal part from the data. In this respect, the magneto-Hall conductivity measurement may be useful since the magnetic-field dependence becomes nonlinear near the critical point.

The skew scattering due to the spin-orbit interaction has been subject to many cases, including normal-metal[10] and heavy-Fermion systems.[11,12] In our calculation, we have not assigned the spin of the spin-orbit interaction. The spin may become the spin of the conduction carrier or of the localized impurity. For the high-temperature superconductors, the role of the hole of oxygen and copper may be different. Therefore, if the susceptibilities of different sites can be measured, one may find the dominant contribution of $\langle\sigma_z\rangle$. For the heavy-Fermion case, the skew-scattering contribution is proportional to the susceptibility.[12]

We have investigated the skew-scattering contribution to the fluctuation part of the Hall conductivity. Usually, the vertex correction of the Ginzburg-Landau Hamiltonian is small. In our case, the skew-scattering vertex part of Fig. 1 is not small. It has been pointed out before[13] that if there is an imaginary part for the superconducting fluctuation propagator, the nonvanishing Aslamazov-Larkin term appears for the Hall conductivity. However, in this case the sign of the fluctuation part of the Hall conductivity depends upon the details of the materials. In the skew-scattering case, as we have shown, the fluctuating part $\Delta\sigma_{xy}^{sk}$ has a minus sign due to the next order of the Ginzburg-Landau Hamiltonian. Therefore, it predicts that the similar opposite sign also exists in electronlike high-temperature superconductors as $Nd_{2-x}Ce_xCuO_4$, which was recently measured and indeed it had an opposite sign.[7]

A detailed discussion of the Hall-conductivity calculation will be given elsewhere.

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[1]Y. Matsuda, T. Hirai, S. Komiyama, T. Terashima, Y. Bando, K. Iijima, K. Yamamoto, and K. Hirata, Phys. Rev. B **40**, 5176 (1989).

[2]S. Hikami and A. I. Larkin, Mod. Phys. Lett. B **2**, 693 (1988).

[3]A. G. Aronov, S. Hikami, and A. I. Larkin, Phys. Rev. Lett. **62**, 965 (1989).

[4]K. Maki and R. S. Thompson, Phys. Rev. B **39**, 2767 (1989).

[5]Y. Iye, S. Nakamura, and T. Tamegai, Physica C **159**, 616 (1989).

[6]T. Hiraoka, Jpn. J. Appl. Phys. Pt. 2 **28**, 1135 (1989).

[7]Y. Matsuda (private communication).

[8]J. Clayhold, N. P. Ong, Z. Z. Wang, J. M. Tarascon, and P. Barboux, Phys. Rev. B **39**, 7324 (1989).

[9]L. G. Aslamazov and A. I. Larkin, Fiz. Tverd. Tela. (Leningrad) **10**, 1104 (1968) [Sov. Phys. Solid State **10**, 875 (1968)].

[10]L. E. Ballentine and M. Huberman, J. Phys. C **10**, 4991 (1977).

[11]A. Fert and O. Jaoul, Phys. Rev. Lett. **28**, 303 (1972).

[12]P. Coleman, P. W. Anderson, and T. V. Ramakrishnan, Phys. Rev. Lett. **55**, 414 (1985).

[13]H. Fukuyama, H. Ebisawa, and T. Tsuzuki, Prog. Theor. Phys. **46**, 1028 (1971).