Spin-density-wave and charge-density-wave fluctuation and electric conductivity

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Making use of a microscopic model, we study theoretically the fluctuation contribution to the electric conductivity in the vicinity of the spin-density-wave (SDW) or the charge-density-wave (CDW) transition. We find that the vertex corrections associated with the SDW (or the CDW) fluctuation, which are neglected in earlier works, are of prime importance. For example, we find the excess resistance in the vicinity of the SDW (or the CDW) transition diverges like $|\tau|^{-\alpha}$, with $\alpha=\frac{1}{2}(4-D)$ and $\tau=\ln(T/T_c)$, where D is the dimension of the fluctuation in the clean system. This exponent is larger by unity from the value obtained by Horn and Guiddoti [Phys. Rev. B **16**, 491 (1977)]. In dirtier samples in which the vertex renormalization is not so important, we recover the result of Horn and Guiddoti. Further, we find a non-Ohmic term in the fluctuation regime. The non-Ohmic conductivity increases with an external electric field ε. Moreover, the effect of ε is equivalent to the shift in the temperature T by $\Delta T=-7\zeta(3)(ev|E|)(4\pi T_c)^{-1}$, with $\zeta(3)=1.202$ and v the Fermi velocity in the chain direction. These results account nicely for recent experimental results by Richard *et al.*, which found no explanation until now.

I. INTRODUCTION

As is well known, a number of quasi-one-dimensional charge-density-wave (CDW) and spin-density-wave (SDW) systems exhibit sharp rises in the electric resistance at the CDW and the SDW transition, which is commonly interpreted in terms of a model proposed by Horn and Guiddoti.[1] However, when this model is used to analyze the resistivity anomaly[2] at the second CDW transition in $NbSe_3$, it appears to yield an unphysical dimension $D=0$. Further, Richard *et al.*[3] observed a non-Ohmic component of the resistivity above the CDW transitions of both orthorhombic and monoclinic TaS_3 as well as of $NbSe_3$, which decreases with applied electric field.

The object of this paper is to study systematically the fluctuation induced conductivity in both SDW and CDW in the vicinity of the SDW and the CDW transition. As a model we take an anisotropic Hubbard model as used by Yamaji[4] in analyzing the SDW transition in Bechgaard salts, since as long as the fluctuation contribution to the electric conductivity in the vicinity of $T=T_c$ is concerned, there is no difference between the SDW and the CDW. Further, Yamaji's model is slightly simpler than the Fröhlich's model for the CDW. Further recent observations of the Fröhlich conduction in the SDW's of the Bechgaard salts[5,6] support the idea that the similar principle works for both the SDW and the CDW.[7,8]

In order to analyze properly the transport properties, it is crucial to include the dissipation mechanism, which we introduce by means of randomly distributed impurities.[9] Then the fluctuation contribution to the electric conductivity is classified in terms of diagrams as in a related analysis in a superconductor.[10,11] However, unlike in a superconductor one of the contributions (which we call the regular term in analogy to the one in a superconductor) depends on whether the SDW (or the CDW) is pinned or not. Indeed this term gives rise to the non-Ohmic contribution in the fluctuation regime. The anomalous term, on the other hand, is essentially the one considered by Horn and Guiddoti.[1] However, introduction of the vertex renormalization due to the impurity scattering which they neglected modifies strongly their conclusion. Especially in the clean limit ($l/\xi>10^2$, where l is the electron mean free path and $\xi=v/2\pi T_c$ is the coherence distance) the derivative of the resistance in the vicinity of $T=T_c$ diverges like

$$d\rho/dT \propto |\tau|^{-(6-D)/2}\ , \tag{1}$$

where $\tau=\ln(T/T_c)$ and D is the dimension of the fluctuation. This is because the vertex renormalization introduces an extra diffusionlike pole in the total scattering amplitude so that the total scattering is much enhanced from the one without vertex renormalization.[11] Therefore we interpret the exponents 1.5 and 2 observed for the CDW of O-TaS_3 and $NbSe_3$ as associated with $D=3$ and $D=2$ fluctuations, respectively. We have another evidence[12] that $D=2$ is most likely for the first CDW in $NbSe_3$. On the other hand, in dirtier samples (say $l/\xi\leq 10$) we recover the result by Horn and Guiddoti since here the vertex renormalization is not so important.

In the presence of a large electric field the regular term, which was completely neglected by Horn and Guiddoti, gives rise to the non-Ohmic contribution to the resistance. Indeed the E dependence of the non-Ohmic term is equivalent to the shift in the temperature T by

$$\Delta T=-7\zeta(3)ev|E|(4\pi T)^{-1}\ , \tag{2}$$

where v is the Fermi velocity in the chain direction and $\zeta(3)=1.202\ldots$. The present result describes quite well the observed non-Ohmic conductivity[3] in O-TaS_3 in the region $T > T_c$.

So far we considered only the SDW (or the CDW) fluctuation. In the SDW there are spin-wave fluctuations as well. They contribute to the anomalous term the same as the one due to the SDW fluctuation since they have two more degrees of freedom. On the other hand, there will be no regular term due to spin wave since they do not couple to the electric current. Therefore in an ordinary antiferromagnet we expect the diverging resistance with the same exponent as given above but no non-Ohmic effect.

II. FLUCTUATION PROPAGATOR

We shall consider an anisotropic Hubbard model given by[4]

$$H=\sum_{p\sigma}\xi(p)C^{\dagger}_{p\sigma}C_{p\sigma}+U\sum_{q}n_{\uparrow q}n_{\downarrow -q}\ , \tag{3}$$

where

$$\begin{aligned}\xi(p)&=-2t_a\cos ap_1-2t_b\cos bp_2-2t_c\cos cp_3-\mu\\ &\simeq v(|p_1|-p_F)-2t_b\cos bp_2-\varepsilon_0\cos(2bp_2)\ ,\end{aligned} \tag{4}$$

with typical values for the Bechgaard salts

$$t_a:t_b:t_c\simeq 10:1:0.03\ ,$$

and μ is the chemical potential

$$\varepsilon_0=-\tfrac{1}{4}t_b^2\cos ap_F(t_a\sin^2 ap_F)^{-1}\ , \tag{5}$$

and U is the on-site Coulomb repulsion. In the second line of Eq. (4) we make use of a simplification introduced by Hasegawa and Fukuyama.[13] At low temperatures the system described by the Hamiltonian (3) undergoes the SDW transition with the SDW transition temperature given by[4,13]

$$1=\pi T\bar{U}\sum_{n=0}^{n_0}(\omega_n^2+\varepsilon_0^2)^{-1/2}\ , \tag{6}$$

where $\bar{U}=N_0U$ and ω_n is the Matsubara frequency and N_0 is the density of states at the Fermi surface per spin and the ω_n sum is cut off at $\omega_{n_0}=E_F$ the Fermi energy. In the following we consider only the small-ε_0 limit for simplicity. The electron Green's function in the normal state (i.e., for $T > T_c$) is given by

$$G(\omega_n,\mathbf{p})^{-1}=i\omega_n-\xi(p)\ . \tag{7}$$

So far we have neglected the scattering term. In the presence of impurities the effect of the impurity scattering is incorporated in the Green's function (7) by replacing ω_n by $\tilde{\omega}_n$ where

$$\tilde{\omega}_n=\omega_n(1+\tilde{\Gamma}/|\omega_n|) \tag{8}$$

and $\tilde{\Gamma}=\frac{1}{2}(\Gamma_1+\Gamma_2)$ and Γ_1 and Γ_2 are the forward and the back scattering rate due to the impurities.[9] Further, in order to determine the SDW transition temperature and the fluctuation propagator in the presence of the impurity scattering we have to include the vertex renormalization associated with the SDW order parameter $\Delta(q)$; in the presence of the impurity scattering $\Delta(q)$ is replaced by $\tilde{\Delta}(q)$ where

$$\tilde{\Delta}(\mathbf{q})=\Lambda(\omega_n,\omega_{n+\nu};\mathbf{q})\Delta(\mathbf{q}) \tag{9}$$

and

$$\Lambda(\omega_n,\omega_{n+\nu};\mathbf{q})=\begin{cases}1 & \text{for } \omega_n\omega_{n+\nu}<0\ ,\\ [(\tilde{\omega}_n+\tilde{\omega}_{n+\nu})^2+\zeta^2][|\tilde{\omega}_n+\tilde{\omega}_{n+\nu}|(|\omega_n+\omega_{n+\nu}|+2\Gamma)+\zeta^2]^{-1} & \text{for } \omega_n\omega_{n+\nu}>0\ ,\end{cases} \tag{10}$$

and

$$\begin{aligned}\Gamma&=\Gamma_1+\tfrac{1}{2}\Gamma_2\ ,\\ \zeta^2&=v^2q_1^2+v_2^2q_2^2+v_3^2q_3^2\ ,\\ v&=2t_aa\sin ap_F,\quad v_2=\sqrt{2}t_bb,\quad v_3=\sqrt{3}t_cc\ .\end{aligned} \tag{11}$$

Then in the presence of impurities the fluctuation propagator is given by

$$\begin{aligned}D(\mathbf{q},\omega_\nu)&=\langle[\delta\Delta,\delta\Delta]\rangle_0(1-\tfrac{1}{2}U\langle[\delta\Delta,\delta\Delta]\rangle_0)^{-1}\\ &\simeq N_0^{-1}\left[\ln\frac{T}{T_c}+\frac{\pi}{8T}\left[|\omega_\nu|+\frac{7\zeta(3)}{2\pi^3T}\zeta^2\right]\right]^{-1}\ ,\end{aligned} \tag{12}$$

where T_c the transition temperature is given by

$$-\ln\frac{T_c}{T_{c0}}=\psi\left[\frac{1}{2}+\frac{\Gamma}{2\pi T_c}\right]-\psi(\tfrac{1}{2})\simeq\pi\Gamma/4T_c \tag{13}$$

and T_{c0} is the transition temperature in the absence of the impurity scattering and $\psi(z)$ is the di-gamma function. In deriving Eq. (12) we assumed that $\Gamma/2\pi T_c \ll 1$ (the clean limit). In the following we consider only the clean limit. Unlike in a superconductor the dirty limit here is of no interest, since the SDW transition will be completely suppressed.

We note that Eq. (12) is almost the same as that in a superconductor except the numerical coefficient of ζ^2. This difference arises from the quasi one dimensionality of the present system. Derivation of Eqs. (9) and (12) will be sketched in Appendix A. Now we are ready to analyze the fluctuation contribution to the electric conductivity.

III. ELECTRIC CONDUCTIVITY

Within the present model the electric conductivity in the normal state in the most conducting direction is given by[9]

$$\sigma_0=(ev)^2N_0\Gamma_2^{-1} . \tag{14}$$

The lowest-order corrections due to the SDW fluctuations to the electric conductivity are shown in Fig. 1, where the diagrams a, b, and b' give rise to the anomalous term[11] while these c and c' contribute to the regular term.[10] However, unlike in a superconductor the regular term is sensitive to whether the SDW is pinned or not and we can neglect the regular term when the SDW is pinned.[14] The effect of the pinning may be approximated by a negative shift in the transition temperature in the fluctuation propagator, though this shift should not change the actual transition temperature. Then an applied electric field can remove this shift. As we shall see then the regular term gives rise to the non-Ohmic term in the fluctuation regime.

First let us consider the anomalous term, which is essentially studied by Horn and Guiddoti.[1] However, they neglected the vertex renormalization associated with the CDW fluctuation, and their conclusion does not apply to the extremely clean samples (i.e., $\Gamma_i/\pi T_c \leq 10^{-2}$). Following the conventional method we obtain from diagrams a, b, and b'

$$\sigma_{\rm an}=(-i\omega)^{-1}(I_a+2I_b) \tag{15}$$

$$\sigma_{\rm an}=\begin{cases} -\dfrac{\pi}{2}[7\zeta(3)]^{1/2}\dfrac{e^2Tv}{v_2v_3}\left[\dfrac{2\tilde{\Gamma}}{\Gamma+\tilde{\Gamma}}\right](\sqrt{\tau}+\sqrt{\delta})^{-1} & \text{for } D=3 ,\\ -\dfrac{e^2v}{8dv_2}(\tau-\delta)^{-1}\ln(\tau/\delta) & \text{for } D=2 , \end{cases} \tag{16}$$

where $\tau=\ln(T/T_c)$, $\delta=7\zeta(3)\Gamma\tilde{\Gamma}(2\pi T)^{-2}$, and d is the sample thickness.

Here I_a and I_b are evaluated as

$$I_a=(ev)^2N_0\Big[7\zeta(3)[(2\pi)^2T]^{-1}\sum_q D(q) + \frac{\pi}{4}i\omega(-i\omega+2\tilde{\Gamma})(-i\omega+\Gamma+\tilde{\Gamma})^{-1}\sum_q[(-i\omega+2\tilde{\Gamma})(-i\omega+2\Gamma)+\zeta^2]^{-1}D(q)\Big] , \tag{17}$$

$$I_b=-\tfrac{1}{2}(ev)^2N_0[7\zeta(3)][(2\pi)^2T]^{-1}\sum_q D(q) , \tag{18}$$

where

$$D(q)=D(q,0) . \tag{19}$$

A derivation of Eqs. (16)–(18) is given in Appendix B. The electric conductivity of the pinned SDW (or CDW) for $T>T_c$ is then given by

$$\sigma=\sigma_0+\sigma_{\rm an} . \tag{20}$$

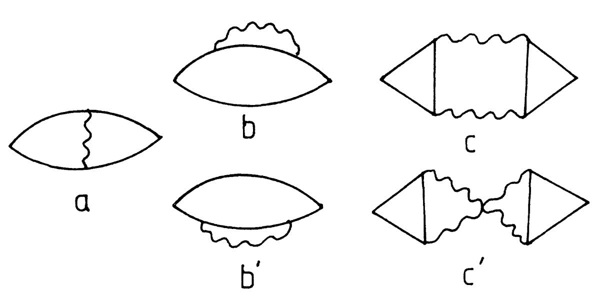

FIG. 1. The lowest-order fluctuation contributions to the electric conductivity are shown. We call the diagram a, b, and b' the anomalous term, while the diagram c and c' is the regular term. Here solid lines are the electron Green's function and wavy lines are fluctuation propagators. Vertex renormalizations are omitted from the figure for clarity.

The power law stated in the Introduction follows from Eq. (16) when $\tau \gg \delta \sim 10^{-4}$ (i.e., in the clean samples). For $|\tau|\sim\delta$, on the other hand, Eq. (16) gives the same exponent as given by Horn and Guiddoti.

In the light of the present result we interpret the power laws found in the electric resistance of the CDW's in O-TaS_3 and $NbSe_3$ as due to the $D=3$ and the $D=2$ fluctuations, respectively. As already mentioned this $D=2$ behavior has been deduced from the temperature dependence of the threshold field[12] in the first CDW of $NbSe_3$.

Second, the regular term is given by

$$\sigma_r=(ev)^2N_0[7\zeta(3)]^2(\pi^5T^2)^{-1}\sum_q D(q) , \tag{21}$$

$$\sigma_r=\begin{cases} \dfrac{16}{\pi^3}[7\zeta(3)]^{1/2}(e^2Tv/v_2v_3)(c_1-\sqrt{\tau}) & \text{for } D=3 ,\\ \dfrac{28\zeta(3)}{\pi^4}(e^2v/dv_2)(-\ln\tau) & \text{for } D=2 , \end{cases} \tag{22}$$

where C_1 is the cutoff constant of the order of unity. Here we made use of the fact that j_x, the electric current associated with the SDW order parameter, is given by

$$j_x=[7\zeta(3)/(2\pi T)^2]evN_0(2i)^{-1}[(\partial_t\Delta^*)\Delta-\Delta^*\partial_t\Delta] , \tag{23}$$

which is consistent with the well known formula[8] for $T<T_c$

$$j_x = evN_0 f \frac{\partial \phi}{\partial t} , \tag{24}$$

with $f \simeq [7\zeta(3)/(2\pi T)^2]\Delta^2$ for $T \simeq T_c$ and ϕ is the phase of the order parameter.

We note that σ_r gives the positive contribution to the electric conductivity. Further the divergence of σ_r is weaker than that of σ_{an}. Therefore σ_r cannot cancel out σ_{an} even when the SDW fluctuation is completely depinned. So far we neglect the pinning effect as well as the E dependence of σ_r where E is an electric field applied in the sample. The non-Ohmic effect is easily incorporated in Eq. (22), so that we have the electric conductivity for $T > T_c$ as

$$\sigma = \sigma_0 + \sigma_{an} + \sigma_r(E) , \tag{25}$$

where the E dependence comes from the τ dependence of σ_r, which is now written $\tau + \varepsilon_c - \varepsilon$ where ε_c is the threshold field for $T > T_c$ due to the pinning and

$$\varepsilon = 7\zeta(3)(ev|E|)(4\pi T_c^2)^{-1} . \tag{26}$$

This follows from the fact that the effect of the electric field is incorporated into $D(q)$ by shifting ζ^2 to

$$\zeta^2 - 4ev\langle \phi \rangle E , \tag{27}$$

which follows from the microscopic phase Hamiltonian[8] and $\langle \phi \rangle$ is the average of ϕ, which we took $\langle \phi \rangle = \pi$. The effect of the electric field is identical to the shift in the temperature T to lower temperatures. Indeed this scaling behavior has been already observed in the non-Ohmic conductivity of TaS_3 above $T = T_c$.

So far we limit ourselves to $T > T_c$. The fluctuation contribution to the conductivity for $T < T_c$ is obtained as follows. First in the absence of the fluctuation and when the SDW is pinned σ_0 in Eq. (14) is replaced by[9,15]

$$\sigma_1 = \sigma_0 \left[1 - \frac{\pi}{4T} \frac{\Delta^2}{(\Gamma_2^2 + \Delta^2)^{1/2}} \right] . \tag{28}$$

Second, the fluctuation contribution associated with the anomalous term is obtained by replacing τ in Eq. (16) by $2|\tau| = 2\ln(T_c/T)$. This follows from the fact the SDW fluctuations below $T = T_c$ split into the phase and the amplitude fluctuation and that $D(q)$ in Eqs. (17) and (18) are replaced by the amplitude fluctuation $D_A(q)$. Here

$$D_\phi(q,\omega_\nu) = N_0^{-1}(8T/\pi) \left[|\omega_\nu| + \frac{7\zeta(3)}{2\pi^3 T}\zeta^2 \right]^{-1} \tag{29}$$

and

$$D_A(q,\omega_\nu) = N_0^{-1} \left[2|\ln(T/T_c)| + \frac{\pi}{8T} \left[|\omega_\nu| + \frac{7\zeta(3)}{2\pi^3 T}\zeta^2 \right] \right]^{-1} . \tag{30}$$

Therefore for a pinned SDW for $T < T_c$ we obtain

$$\sigma = \sigma_1 + \sigma_{an} , \tag{31}$$

where τ in σ_{an} is replaced by $2|\tau|$. When the SDW is completely unpinned, we obtain instead of σ_1

$$\sigma_{unp} = \sigma_0 [1 + 7\zeta(3)\Delta^2 / 2\pi^2 T^2] . \tag{32}$$

Further, in the presence of the fluctuation Δ^2 is replaced by $\langle \Delta^2 \rangle$ with

$$\langle \Delta^2 \rangle = \Delta^2 - T \sum_q D_A(q) , \tag{33}$$

$$\langle \Delta^2 \rangle = \Delta^2 - \begin{cases} [7\zeta(3)]^{-3/2}(2\pi T)^3 T(t_b t_c)^{-1}(c_1 - \sqrt{2}|\tau|) & \text{for } D=3 , \\ [7\zeta(3)]^{-1} 2\sqrt{2}(\pi T)^2 T t_b^{-1}(c/d)[-\ln(2|\tau|)] & \text{for } D=2 . \end{cases} \tag{34}$$

Second, we have to include σ_{an}, which is more singular than the correction term given above. Finally we have to add σ_r below $T = T_c$. Though the general expression of σ_r below $T = T_c$ is different from the one given in Eqs. (21) and (22) it takes the identical form in the low frequency limit if we replace τ by $|\tau|$. Therefore when the SDW is unpinned the electric conductivity is given by

$$\sigma = \sigma_{unp}(\Delta^2 \to \langle \Delta^2 \rangle) + \sigma_{an} + \sigma_r \simeq \sigma_{unp} + \sigma_{an} . \tag{35}$$

Derivations of Eq. (33) and σ_r are given in Appendix C.

IV. CONCLUDING REMARKS

We have studied the fluctuation contribution to the electric conductivity in the vicinity of the SDW transition within a microscopic model. Although we limit our analysis to the SDW transition, the present result applies for the CDW in the quasi-one-dimensional systems. We find that the fluctuation contribution is quite different depending on whether the SDW (or CDW) is pinned or not. When the SDW (or the CDW) is pinned the electric conductivity for $T > T_c$ is given by Eq. (20). Further, in the clean limit σ_{an} is negative and diverges like $|\tau|^{-(1/2)(4-D)}$ for not too small τ. The present exponent is larger by unity from the one obtained by Horn and Guiddoti. For samples with intermediate cleanness, on the other hand, we recover the early result of Horn and Guiddoti. The present result shows that the CDW fluctuation in O-TaS_3 is three dimensional and not one dimensional as assumed until now. Indeed we have accumulating evidences that the CDW fluctuation in most of quasi-one-dimensional CDW systems is three dimensional. The only exception we know is that in $NbSe_3$, which appears to be two dimensional.[12]

In the presence of an electric field the conductivity is now given by Eq. (25). In particular the regular term depends on E. The dependence of E scales with the shift in T. The present results describe quite well a number of experimental results,[2,3] which are unexplained until now. We have extended the similar analysis to the sound propagation,[16] which will be published elsewhere. The theory predicts a dip in the elastic constant at $T=T_c$ when the SDW (or the CDW) is pinned. When the SDW (or the CDW) is unpinned, the dip will disappear completely. The theory also predicts a sharp peak in the sound attenuation at $T=T_c$, when the SDW (or the CDW) is pinned. When the SDW (or the CDW) is unpinned, the magnitude of the attenuation almost doubles.

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APPENDIX A: VERTEX RENORMALIZATION AND FLUCTUATION PROPAGATOR

Making use of the Green's function (7) with ω_n replaced by $\tilde{\omega}_n$, the equation for Λ is given by

$$\begin{aligned}\Lambda(\omega_n,\omega_{n+\nu},\mathbf{q})&=1+n_i|V_1^2|\int\frac{d^2p}{(2\pi)^3}(i\tilde{\omega}_n-\xi)^{-1}\Lambda(\omega_n,\omega_{n+\nu},n\mathbf{q})(i\tilde{\omega}_{n+\nu}+\xi+\mathbf{v}\cdot\mathbf{q})^{-1}\\&=1+\Theta(\omega_n\omega_{n+\nu})\Gamma_1|\tilde{\omega}_n+\tilde{\omega}_{n+\nu}|(|\tilde{\omega}_n+\tilde{\omega}_{n+\nu}|^2+\zeta^2)^{-1}\Lambda\ ,\end{aligned}\tag{A1}$$

where $\Theta(z)$ is the step function and

$$\zeta^2=v^2q^2+v_2^2q_2^2+v_3^2q_3^2\tag{A2}$$

and V_1 is the Fourier component of the impurity potential with the zero momentum transfer.[9] Then (A1) is solved as Eq. (10) in the text.

Now the fluctuation propagator is obtained from the first line of Eq. (12), where

$$\begin{aligned}\langle[\delta\Delta,\delta\Delta]\rangle_0&=T\sum_n\int\frac{d^3p}{(2\pi)^3}(i\tilde{\omega}_n-\xi)^{-1}(i\tilde{\omega}_{n+\nu}+\xi+vq)^{-1}\Lambda\\&=2\pi TN_0\sum_n\frac{\Theta(|\tilde{\omega}_n+\tilde{\omega}_{n+\nu}|)}{(\tilde{\omega}_n+\tilde{\omega}_{n+\nu})^2+\zeta^2}\Lambda\\&=2\pi TN_0\sum_n\Theta\left[\frac{|\tilde{\omega}_n+\tilde{\omega}_{n+\nu}|}{|\tilde{\omega}_n+\tilde{\omega}_{n+\nu}|(|\omega_n|+|\omega_{n+\nu}|+2\Gamma)+\zeta^2}\right]\\&\simeq2\pi TN_0\sum_n\Theta\left[\frac{1}{|\omega_n|+|\omega_{n+\nu}|+2\Gamma}-\frac{\zeta^2}{|\tilde{\omega}_n+\tilde{\omega}_{n+\nu}|(|\omega_n|+|\omega_{n+\nu}|+2\Gamma)^2}\right]\\&=N_0\left[\psi\left[\frac{\varepsilon_F}{2\pi T}\right]-\psi\left[\frac{1}{2}+\frac{\Gamma}{2\pi T}+\frac{|\omega_\nu|}{4\pi T}\right]-\frac{7\zeta(3)}{4(2\pi T)^2}\zeta^2\right]\\&=N_0\left[2(\bar{U})^{-1}+\ln\frac{T_c}{T}-\frac{\pi}{8T}|\omega_\nu|-\frac{7\zeta(3)\zeta^2}{4(2\pi T)^2}\right]\ ,\end{aligned}\tag{A3}$$

where we neglected the higher-order terms in $\Gamma/2\pi T_c$ and $\tilde{\Gamma}/2\pi T_c$. Inserting this into the first line of Eq. (12) we find the fluctuation propagator.

APPENDIX B: EVALUATION OF THE ANOMALOUS TERMS

First let us consider I_a, which is given by

$$I_a=-2\langle ev\rangle^2T^2\sum_n\sum_q\int\frac{d^3p}{(2\pi)^3}(i\tilde{\omega}_n-\xi)^{-1}(i\tilde{\omega}_n+\xi+\mathbf{v}\cdot\mathbf{q})^{-1}\Lambda_1(i\tilde{\omega}_{n+\nu}-\xi)^{-1}(i\tilde{\omega}_{n+\nu}+\xi+\mathbf{v}\cdot\mathbf{q})^{-1}\Lambda_2D(q)\ ,\tag{B1}$$

where

$$\Lambda_1=\Lambda(\omega_n,\omega_n;\mathbf{q})\quad\text{and}\quad\Lambda_2=\Lambda(\omega_{n+\nu},\omega_{n+\nu};\mathbf{q})\ .\tag{B2}$$

Here we neglected the frequency dependence of $D(q)$, which is not important. The momentum integration is easily done and we obtain

$$I_a=-2\pi(ev)^2N_0T^2\sum_q\left\{(\omega_\nu+2\tilde{\Gamma})\sum_a|\tilde{\omega}_n||\tilde{\omega}_{n+\nu}|[|\tilde{\omega}_n|(|\omega_n|+\Gamma)+\tfrac{1}{4}\zeta^2]^{-1}[|\tilde{\omega}_{n+\nu}|(|\omega_{n+\nu}|+\Gamma)+\tfrac{1}{4}\zeta^2]^{-1}\right.$$
$$\left.+4\sum_r\frac{1}{[2|\omega_n|+2\tilde{\Gamma}-(i/2)vq]^3}\Lambda_1\Lambda_2\right\}D(q)\ , \tag{B3}$$

where $\sum_a$ and $\sum_r$ means the frequency sum has to be done for $\omega_n\omega_{n+\nu}<0$ (the anomalous region) and for $\omega_n\omega_{n+\upsilon}>0$ (the regular region), respectively. The sum over the regular region is easily done, since we can neglect the higher-order terms in Γ and $\tilde{\Gamma}$ and we find

$$2\pi T\sum_r\left[2|\omega_n|+2\tilde{\Gamma}-\frac{i}{2}vq\right]^{-3}\Lambda_1\Lambda_2\simeq 7\zeta(3)(4\pi T)^{-2}\ . \tag{B4}$$

On the other hand the sum over the anomalous term gives

$$2\pi T\sum_a|\tilde{\omega}_n||\tilde{\omega}_{n+\nu}|[|\tilde{\omega}_n|(|\omega_n|+\Gamma)+\tfrac{1}{4}\zeta^2]^{-1}[|\tilde{\omega}_{n+\nu}|(|\omega_{n+\nu}|+\Gamma)+\tfrac{1}{4}\zeta^2]^{-1}$$
$$\simeq\frac{\omega_\nu}{4\pi T}\psi^{(1)}(\tfrac{1}{2})\{(1-y)^2[\omega_\nu+2\tilde{\Gamma}+\tfrac{1}{2}\Gamma_1(1-y^{-1})]^{-1}$$
$$+(1+y)^2[\omega_\nu+2\tilde{\Gamma}+\tfrac{1}{2}\Gamma_1(1+y^{-1})]^{-1}+2(1-y^2)(\omega_\nu+2\tilde{\Gamma}+\tfrac{1}{2}\Gamma_1)^{-1}\}$$
$$=\frac{\pi\omega_\nu}{8T}\{2(\omega_\nu+2\tilde{\Gamma})^2(\omega_\nu+\Gamma+\tilde{\Gamma})^{-1}[(\omega_\nu+2\tilde{\Gamma})(\omega_\nu+2\Gamma)+\zeta^2]^{-1}\}\ , \tag{B5}$$

where

$$y=\Gamma_1(\Gamma_1^2-4\zeta^2)^{-1/2}\ .$$

Putting these together we obtain Eq. (17) in the text.

The analysis of I_b and $I_{b'}$ ($=I_b$) is done similarly. However, there is no contribution from the anomalous region and we obtain Eq. (18) in text. The regular contribution from I_a, I_b, and $I_{b'}$ cancels out exactly.

APPENDIX C: EVALUATION OF THE REGULAR TERM

The diagram C is calculated as

$$I_c=T[7\zeta(3)(8\pi^2T^2)^{-1}(evN_0)]^2\sum_\mu(2\omega_\mu+\omega_\nu)^2\sum_q D_1(q)D_2(q)$$
$$=\frac{\omega_\nu}{2}[7\zeta(3)(8\pi^2T^2)^{-1}(ev)]^2N_0\sum_q D(q)\ , \tag{C1}$$

where

$$D_1(q)=D(q,\omega_{\mu+\nu}),\quad D_2(q)=D(q,\omega_\mu)\ . \tag{C2}$$

Here we made use of the relation

$$T\sum_\mu(\omega_\mu+\omega_{\mu+\nu})^2(|\omega_\mu|+A)^{-1}(|\omega_{\mu+\nu}|+A)^{-1}=4T\omega_\nu A^{-1}\ . \tag{C3}$$

For $T<T_c$,

$$\sum_\mu(\omega_\mu+\omega_{\mu+\nu})^2D_1(q)D_2(q)$$

has to be replaced by

$$\sum_\mu(\omega_\mu+\omega_{\mu+\nu})^2D_{A1}(q)D_{\phi2}(q)\ .$$

Then this will be transformed as

$$T\sum_\mu(\omega_\mu+\omega_{\mu+\nu})^2D_{A1}(q)D_{\phi2}(q)=\left[N_0\frac{\pi}{8T}\right]^{-2}\left[\frac{4T\omega_\nu}{\omega_\nu+B+C}\left[2+\frac{\omega_\nu^2}{4}[B^{-1}(B+\omega_\nu)^{-1}+C^{-1}(C+\omega_\nu)^{-1}]\right]\right]\ , \tag{C4}$$

where

$$B=7\zeta(3)(2\pi^3T)^{-1}\zeta^2, \quad C=B+\frac{16T}{\pi}\ln(T_c/T)\ . \tag{C5}$$

Therefore in the limit the external frequency tends to zero, σ_r below $T=T_c$ is obtained from the one for $T>T_c$ by replacing τ by $|\tau|$.

APPENDIX D: $\langle\Delta^2\rangle$ IN THE REGION $T<T_c$

Perhaps it will be simplest to treat this term within the free energy functional in the Ginzburg-Landau region

$$F=a|\Delta|^2+\frac{b}{2}|\Delta|^4+C\sum_i v_i^2|\partial_i\Delta|^2\ , \tag{D1}$$

where a, b, and c are identified as

$$a=N_0\ln(T/T_c),\quad b=N_0[7\zeta(3)/8\pi^2T^2],\quad \text{and } c=N_0[7\zeta(3)/16\pi^2T^2]\ . \tag{D2}$$

In the presence of fluctuations we replace F by $\langle F\rangle$, where loop corrections[17] are included

$$\langle F\rangle=a(|\Delta|^2+D_1)+\frac{b}{2}(|\Delta|^4+4|\Delta|^2D_1+2|\Delta|^2D_2+D_1^2)\ , \tag{D3}$$

where

$$D_1=\langle|\delta\Delta|^2\rangle=\tfrac{1}{2}(D_\phi+D_A)\ ,$$

$$D_2=\langle(\delta\Delta)^2\rangle=\langle(\delta\Delta^*)^2\rangle=\tfrac{1}{2}(D_A-D_\phi)\ ,$$

and

$$D_\phi=T\sum_q D_\phi(q)\ ,$$

$$D_A=T\sum_q D_A(q)\ . \tag{D4}$$

Then by minimizing F in terms of $|\Delta|^2$, we obtain

$$|\Delta|^2=-\frac{a}{b}-2D_1-D_2\ . \tag{D5}$$

On the other hand, we have

$$\langle|\Delta|^2\rangle=|\Delta|^2+D_1=-\frac{a}{b}-D_1-D_2=8\pi^2T^2[7\zeta(3)]^{-1}\ln(T_c/T)-D_A\ , \tag{D6}$$

which is essentially Eq. (33) in the text. The present result is fully consistent with the early analysis by Scalapino *et al.*[18]

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