Quantum Heisenberg spin glasses: Anisotropy effects and field dependence

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The infinite-range quantum Heisenberg spin glass with uniaxial anisotropy (D) and coaxial external magnetic field (h) is studied by means of the thermo-field-dynamics as a substitute for the n-replica trick. Within the mean-field theory a multiplicity of spin-glass phases has been found for the spin values $S=1$ and $S=\frac{3}{2}$, respectively, including: longitudinal, transverse, and mixed phases. The stability of the mean-field-type solution against the action of fluctuations has been investigated and the existence of a crossover from the longitudinal-to-transverse freezing is demonstrated explicitly in the temperature-field (T-h) phase diagram for various values of the constant D.

I. INTRODUCTION

In recent years a strong anisotropy of the magnetic susceptibility has been found experimentally in a number of hexagonal metallic spin-glass systems.[1-3] The systems studied behave either Ising-like or Heisenberg-like depending on the sign and the size of the energy splitting of the magnetic-moment ground state.

In physical terms the above-mentioned metallic systems are best described by a model in which in addition to the random isotropic Heisenberg exchange interaction a single-spin uniaxial anisotropy energy $-D(S_z)^2$ is added,[4,5] where S_z denotes the z component of a spin operator.

From the theoretical side, anisotropic agencies bring about several new features which have been investigated for classical spin models both with the presence of the magnetic field[6-8] and local uniaxial anisotropy[9-11] and a multiplicity of phases has been found. However, as has been demonstrated,[4,5] the corresponding problem in the quantum limit can behave qualitatively distinct from its classical counterpart. Specifically, for a large negative anisotropy D one expects for integer-valued spins at low temperatures a condensation in the $S_z=0$ state resulting in a nonmagnetic spin state accompanied by the destruction of the spin-glass state.

In this paper we investigate in detail the properties of a Heisenberg model with exchange randomness and both uniaxial anisotropy and applied external magnetic field. We consider the quantum limit and we analyze two distinct cases corresponding to the spin dimensionalities $S=1$ and $S=\frac{3}{2}$. For both cases the classical approximation ($S\rightarrow\infty$) is, of course, a rather poor one.

The quantum spin-glass problem in general is far from being a trivial one due to the noncommutativity of the operators involved and different methods have been developed to handle it.[12-16] Typically, quantum mechanics introduces time-dependent self-interactions and order parameters, which complicates the problem considerably. The method we use to deal with both randomness and quantum features has been presented by one of us[17] and has successfully been applied to the quantum Ising-spin glass in a transverse field.[14-16,18] It is based on the thermo-field-dynamics method[19] (TFD), a real-time finite-temperature quantum field theory.

Apart from intrinsic interest in dynamics this method allows one to avoid the use of the n-replica trick, simultaneously dealing with the physical observables such as response and correlation functions. In this respect the TFD method can be regarded as a quantum counterpart of the Martin-Rose-Siggia[20] formalism, known as the dynamic approach to classical spin-glass problems (see e.g., Ref. 21).

In the following we will present calculations of the phase diagrams for the above-mentioned model, including local uniaxial anisotropy and/or external magnetic field. Phase transition points and lines are found separating longitudinal, transverse and mixed spin-glass phases accompanied by ergodicity breaking as indicated by the corresponding stability conditions. Special attention will be paid to the analysis of the crossover from the Ising-like longitudinal freezing to the X-Y-like freezing of the transverse components both as a function of field and anisotropy.

II. ANISOTROPIC SPIN GLASS WITH FIELD

The Hamiltonian of the model is given by

$$H=-\sum_{i,j=1}^{N} J_{ij}\mathbf{S}_i\cdot\mathbf{S}_j+\sum_{i=1}^{N} H_{0i}\ , \tag{1}$$

where $\mathbf{S}=(S_x,S_y,S_z)$ is the quantum spin operator associated with the local moment S and the $J_{ij}(i\neq j)$ are quenched, independently distributed exchange interactions with the probability distribution

$$P(J_{ij})=(N/2\pi J^2)^{1/2}\exp(-NJ_{ij}^2/2J^2)\ . \tag{2}$$

As usual the scaling of the variance J/N ensures a sensible thermodynamic limit $N\rightarrow\infty$. The second term in Eq. (1) is given by

$$H_{0i}=-D(S_{zi})^2-hS_{zi} \tag{3}$$

and describes an easy (hard) uniaxial energy splitting for $D>0$ ($D<0$) along the z direction and the action of the magnetic field, respectively. We shall consider here only the collinear case where the external magnetic field h is pointing in the z direction.

III. TFD HAMILTONIAN AND STATISTICAL AVERAGE

To proceed within the TFD approach we recall the correspondence between the conventional statistical average and the TFD expectation value[19] for a given operator A

$$\langle O(\beta)|A|O(\beta)\rangle = \mathrm{Tr}(e^{-\beta H}A)/\mathrm{Tr}(e^{-\beta H}) . \tag{4}$$

Here the temperature-dependent vacuum is introduced where $\beta=1/k_B T$ with k_B being the Boltzmann constant and T the temperature, while H represents the Hamiltonian of the system. In order to have a consistent operator formalism, one needs to double the operator degrees of freedom. Corresponding to any operator A a tilde operator $\tilde{A}$ is introduced. There is a mapping between A and $\tilde{A}$ called the tilde conjugation rules,[19] being equivalent to the Kubo-Martin-Schwinger condition.[19] By using two equivalent operator sets $\{A\}$ and $\{\tilde{A}\}$ the thermal vacuum is expressed as

$$|O(\beta)\rangle = \sum_n e^{-\beta E_n/2}|n\tilde{n}\rangle Z^{-1/2}(\beta) , \tag{5}$$

where $|n\rangle$ and $|\tilde{n}\rangle$ are the eigenstates (with the eigenenergies E_n) of the Hamiltonians H and $\tilde{H}$, respectively, and $Z(\beta)=\mathrm{Tr}[\exp(-\beta H)]$. Correspondingly, many properties of the usual quantum field theory can be extended to finite temperatures provided that one works with the total thermal Hamiltonian

$$\hat{H}=H-\tilde{H} \tag{6}$$

rather than with H alone. The best merit of the TFD method when applied to the quantum disordered systems lies in the fact that due to the vacuum normalization condition[22] $\langle O(\beta)|O(\beta)\rangle=1$ one avoids the so-called "denominator problem" obstructing the calculation of the quenched average and forcing one to use the n-replica trick.

IV. QUENCHED-AVERAGE AND FUNCTIONAL-INTEGRAL FORMULATION

As usual in the dynamic approach we shall discuss the thermodynamics of the system in terms of the disorder-averaged generating functional for the TFD causal Green's functions

$$\langle Z[\eta]\rangle_J = \int \prod_{i,j} dJ_{ij} P(J_{ij}) Z[\eta,\{J_{ij}\}] , \tag{7}$$

where $Z[\eta,\{J_{ij}\}]$ is the unaveraged generating functional for a fixed realization of random bonds and $\langle\cdots\rangle_J$ represents the subsequent average with respect to the probability distribution (2). Specifically, in the interaction picture with respect to the single-body Hamiltonian (3) one has for the unaveraged generating functional

$$Z[\eta,\{J_{ij}\}]=\langle O,\beta|T\exp\left[i\int_{-\infty}^{+\infty}dt\sum_{i,j}J_{ij}[\mathbf{S}_i(t)\cdot\mathbf{S}_j(t)-\tilde{\mathbf{S}}_i(t)\cdot\tilde{\mathbf{S}}_j(t)]+\Lambda[\eta]\right]O,\beta\rangle , \tag{8}$$

where $|O,\beta\rangle$ refers to the thermal vacuum corresponding to the single-site Hamiltonian (3) while

$$\Lambda[\eta]=\frac{1}{N}\int_{-\infty}^{+\infty}dt\int_{-\infty}^{+\infty}dt'\sum_i\sum_{ab}\sum_{\mu\nu}(\epsilon_a\epsilon_b)^{1/2}\eta^{ab}_{\mu\nu}(t,t')S^a_{\mu i}(t)S^b_{\nu i}(t') \tag{9}$$

represents the source term to account for the nonlocal (in time) expectation values of the composite spin operators. Furthermore, $\epsilon_1=1$, $\epsilon_2=-1$ and the spin operators are defined in the interaction picture in the standard way as

$$S^a_{\mu i}(t)=\exp(i\hat{H}_0 t)S^a_{\mu i}\exp(-i\hat{H}_0 t) , \tag{10}$$

where $\mu=x,y,z$ and the thermo-doublet-notation has been adopted $S^1_\mu=S_\mu$, $S^2_\mu=\tilde{S}_\mu$. The disorder average (7) amounts to a Gaussian integration over J_{ij} variables. Parametrizing *à la* Sherrington and Kirkpatrick[23] we find

$$\langle Z[\eta,\{J_{ij}\}]\rangle_J=\int\prod_{ab}\prod_{\mu\nu}DQ^{ab}_{\mu\nu}\exp(-NL[\mathbf{Q}]+\Omega[\eta]) , \tag{11}$$

where the single-site dynamic effective Lagrangian reads

$$L[\mathbf{Q}]=\mathrm{Tr}\mathbf{Q}^2-\ln\Phi[\mathbf{Q}] \tag{12}$$

with

$$\mathrm{Tr}\mathbf{Q}^2=\int_{-\infty}^{+\infty}dt\int_{-\infty}^{+\infty}dt'\sum_{ab}\sum_{\mu\nu}Q^{ab}_{\mu\nu}(t,t')Q^{ba}_{\nu\mu}(t',t) . \tag{13}$$

Here, $Q^{ab}_{\mu\nu}(t,t')=Q^{ba}_{\nu\mu}(t',t)$ represents a symmetric tensor field which is nonlocal in time. Subsequently, $\Omega[\eta]=\mathrm{Tr}(\mathbf{Q}\eta)/J^2$ and

$$\Phi[\mathbf{Q}]=\langle O,\beta|U_Q(-\infty;+\infty)|O,\beta\rangle , \tag{14}$$

with

$$U_Q(-\infty;+\infty)=T\exp\left[-i\int_{-\infty}^{+\infty}dt\int_{-\infty}^{+\infty}dt' H_Q(t,t')\right] \tag{15}$$

is the time-ordered exponential resulting from the interaction picture. Furthermore, the effective time-dependent single-site thermal Hamiltonian reads

$$\hat{H}_Q(t,t')=-\sum_{ab}\sum_{\mu\nu}(\epsilon_a\epsilon_b)^{1/2}JQ^{ab}_{\mu\nu}(t,t')S^a_\mu(t)S^b_\nu(t') . \tag{16}$$

From Eq. (15) it can be read off that the quantum generalization of the problem results in a time-dependent self-interaction $JQ^{ab}_{\mu\nu}(t,t')$ between spin operators at the same site, which has to be determined self-consistently.

V. SADDLE-POINT APPROXIMATION AND ORDER PARAMETERS

In the limit $N\to\infty$ limit the steepest-descent method can be used, which amounts to finding the stationary point $Q^{ab}_{0,\mu\nu}$ determined by the extremal conditions

$$\delta L[\mathbf{Q}]/\delta Q^{ab}_{\mu\nu}=0 . \tag{17}$$

Thus, one obtains

$$Q^{ab}_{0,\mu\nu}(t,t')=\tfrac{1}{2}(\epsilon_a\epsilon_b)^{1/2}JG^{ab}_{\mu\nu}(t,t') , \tag{18}$$

where

$$G^{ab}_{\mu\nu}(t,t')=-i\frac{\langle O,\beta|TS^a_\mu(t)S^b_\nu(t')U_{Q_0}(-\infty;+\infty)|O,\beta\rangle}{\langle O,\beta|TU_{Q_0}(-\infty;+\infty)|O,\beta\rangle} . \tag{19}$$

Below the freezing temperature in zero field the onset of the glassy phase is marked by a nonzero value of the spin-glass order parameter. Within the context of the dynamic theory the spin-glass order parameter has to be determined via time-persistent quantities. To accomplish it, we factorize the matrix of the causal TFD Green's functions into regular finite-time $(G_{\rm reg})^{ab}_{\mu\nu}$ and singular time-persistent parts $(G_{\rm sing})^{ab}_{\mu\nu}$ as follows:

$$G^{ab}_{\mu\nu}(t,t')=(G_{\rm reg})^{ab}_{\mu\nu}(t,t')+(G_{\rm sing})^{ab}_{\mu\nu} , \tag{20}$$

where

$$(G_{\rm sing})^{ab}_{\mu\nu}=\lim_{|t-t'|\to\infty}G^{ab}_{\mu\nu}(t,t') . \tag{21}$$

For the finite-time part one has restored time-translational invariance in thermal equilibrium $G_{\rm reg}(t,t')=G_{\rm reg}(t-t')$ and the correspondence with measurable quantities is achieved by the following decomposition of the Fourier-transformed causal Green's function in the space of thermo-field-components

$$(G_{\rm reg})^{ab}_{\mu\nu}(\omega)=[U_B(\omega)\tau\bar{G}_{\mu\nu}(\omega)U_B(\omega)]^{ab}=[\tau\bar{G}_{\mu\nu}(\omega)]^{ab}-\frac{2i(C_{\rm reg})_{\mu\nu}(\omega)}{e^{\beta\omega}+1}\begin{bmatrix}1 & e^{\beta\omega/2}\\ e^{\beta\omega/2} & 1\end{bmatrix} , \tag{22}$$

where

$$U_B(\omega)=\begin{bmatrix}\sinh\phi(\omega) & \cosh\phi(\omega)\\ \cosh\phi(\omega) & \sinh\phi(\omega)\end{bmatrix} , \qquad \sinh^2\phi(\omega)=\frac{1}{e^{\beta\omega}-1} \tag{23}$$

is the thermal transformation matrix[19] while

$$\bar{G}^{ab}_{\mu\nu}(\omega)=\begin{bmatrix}G^R_{\mu\nu}(\omega) & 0\\ 0 & G^A_{\mu\nu}(\omega)\end{bmatrix} , \qquad \tau=\begin{bmatrix}1 & 0\\ 0 & -1\end{bmatrix} \tag{24}$$

with $G^{R(A)}_{\mu\nu}(\omega)$ being the matrix of retarded (advanced) Green's functions. Correspondingly, $(C_{\rm reg})_{\mu\nu}(\omega)$ refers to the matrix of the thermodynamic correlation functions in the spin component space being related to $G^R_{\mu\nu}(\omega)$ by means of the usual fluctuation-dissipation theorem. Furthermore, it turns out that the time-persistent part $(G_{\rm sing})^{ab}_{\mu\nu}(\omega)$ has the form

$$(G_{\rm sing})^{ab}_{\mu\nu}(\omega)=-2\pi i q_{\mu\mu}\delta_{\mu\nu}\delta(\omega)\begin{bmatrix}1 & 1\\ 1 & 1\end{bmatrix}^{ab} , \tag{25}$$

where

$$q_{\mu\mu}=q_{\rm LP}\delta_{\mu z}+q_{\rm TP}(1-\delta_{\mu z}) \tag{26}$$

is the Edwards-Anderson (EA) spin-glass order parameter[24] and $q_{\rm LP},q_{\rm TP}$ are the order parameters associated with longitudinal and transverse spin-glass ordering, respectively. In fact, by substituting Eq. (25) into (20) and using (22) one obtains for the total correlation function

$$C_{\mu\nu}(\omega)=(C_{\rm reg})_{\mu\nu}(\omega)+2\pi q_{\mu\nu}\delta(\omega) \tag{27}$$

in accordance with the standard dynamic definition of the EA spin-glass order parameter.

Because of the appearance of the dynamic self-interaction $JQ^{ab}_{\mu\nu}(\omega)$ in the effective thermal Hamiltonian (16), the explicit solution of Eq. (18) is a rather formidable task. For this reason we will focus on the effects of quantum fluctuations on a time scale such that the finite-time part of the dynamic self-interaction can be presented by an instantaneous term

$$(Q_{\rm reg})^{ab}_{\mu\nu}(t-t')=\tfrac{1}{2}(\epsilon_a\epsilon_b)^{1/2}J\chi_{\mu\nu}\delta(t-t')\delta_{ab} , \tag{28}$$

where

$$\chi_{\mu\nu}=\lim_{\omega\to0}G^R_{\mu\nu}(\omega)=\lim_{\omega\to0}G^A_{\mu\nu}(\omega) \tag{29}$$

is the matrix of static susceptibilities which can be decomposed into longitudinal and transverse parts according to

$$\chi_{\mu\nu}=\delta_{\mu\nu}[\chi_{\rm LP}\delta_{\mu z}+\chi_{\rm TP}(1-\delta_{\mu z})] . \tag{30}$$

The time-persistent contribution to the effective thermal Hamiltonian can be represented by using auxiliary Gaussian integrations having the form of a static Gaussian noise component which acts as a random field to generate time-persistent autocorrelations. Accordingly, the self-consistent equation (19) becomes

$$G^{ab}_{\mu\nu}(t-t')=\langle G^{ab}_{\mu\nu}(t-t'|\mathbf{z})\rangle_z , \tag{31}$$

where, with the use of the vacuum normalization condition

$$G^{ab}_{\mu\nu}(t-t'|\hat{\mathbf{z}}) = -i\langle O(\beta,\mathbf{z})|TS^a_\mu(t)S^b_\nu(t')U_{Q_0}(-\infty;+\infty|\mathbf{z})|O(\beta,\mathbf{z})\rangle \tag{32}$$

with

$$U_{Q_0}(-\infty;+\infty|\mathbf{z}) = T\exp\left[-i\int_{-\infty}^{+\infty} dt\,\hat{H}(t|\mathbf{z})\right], \tag{33}$$

and the Gaussian average over the static noise $\mathbf{z}=(z_x,z_y,z_z)$ is given by

$$\langle\cdots\rangle_z = \int_{-\infty}^{+\infty}\frac{d^3\mathbf{z}}{(2\pi)^{3/2}}e^{-|\mathbf{z}|^2/2}. \tag{34}$$

Here, $|O(\beta,\mathbf{z})\rangle$ is the thermal vacuum associated with the single-site effective Hamiltonian

$$H_0'(\mathbf{z}) = H_0 - Jq_{\mathrm{TP}}^{1/2}(z_xS_x+z_yS_y) - Jq_{\mathrm{LP}}^{1/2}z_zS_z \tag{35}$$

which contains the static noise, while the time-ordered exponential contains only the finite-time part of the dynamic self-interaction. The corresponding total single-site effective Hamiltonian can then be established via the correspondence (6) with its thermal counterpart and reads

$$H(\mathbf{z}) = -\tfrac{1}{2}J^2\{\chi_{\mathrm{LP}}(S_z)^2+\chi_{TP}[(S_x)^2+(S_y)^2]\}+H_0'(\mathbf{z}). \tag{36}$$

From Eq. (36) it is seen that various susceptibilities couple to the squares of the spin operators and thus χ_{LP} and χ_{TP} can be considered as kinds of quadrupolar order parameters which should be determined self-consistently together with q_L and q_T. The corresponding self-consistency equations which follow from Eqs. (31) and (32) are then

$$q_\Theta = \langle m_\Theta^2(\mathbf{z})\rangle_z,\quad \chi_\Theta = \langle\chi_\Theta(\mathbf{z})\rangle_z, \tag{37}$$

where Θ=LP, TP while $m_\Theta(\mathbf{z})$ and $\chi_\Theta(\mathbf{z})$ are the magnetization induced by the static random fields and the unaveraged susceptibility, respectively. Furthermore,

$$m_\Theta(\mathbf{z}) = -\sum_{l=0}^{2S}[\partial_\Theta\lambda_l(\mathbf{z})]\rho_l(\mathbf{z}), \tag{38}$$

while

$$\chi_\Theta(\mathbf{z}) = -\sum_{l=0}^{2S}[\partial_\Theta^2\lambda_l(\mathbf{z})]\rho_l(\mathbf{z}) + \beta\left[\sum_{l=0}^{2S}[\partial_\Theta\lambda(\mathbf{z})]^2\rho_l(\mathbf{z}) - m_\Theta^2(\mathbf{z})\right] \tag{39}$$

and

$$\rho_l(\mathbf{z}) = \frac{\exp(-\beta\lambda_l(\mathbf{z})}{\sum_{k=0}^{2S}\exp(-\beta\lambda_k(\mathbf{z})]}. \tag{40}$$

Here, $\lambda_l(\mathbf{z})$ $(l=0,\ldots,2S)$ represents one of the $2S+1$ eigenvalues of the effective single-site quantum spin Hamiltonian (36), while

$$\partial_\Theta\lambda_l(\mathbf{z}) \equiv \left[\frac{\partial\lambda_l(\mathbf{z})}{\partial h_\Theta}\right]_{h_\Theta=0}, \tag{41}$$

where h_θ denotes an infinitesimal longitudinal (transverse) applied magnetic field.

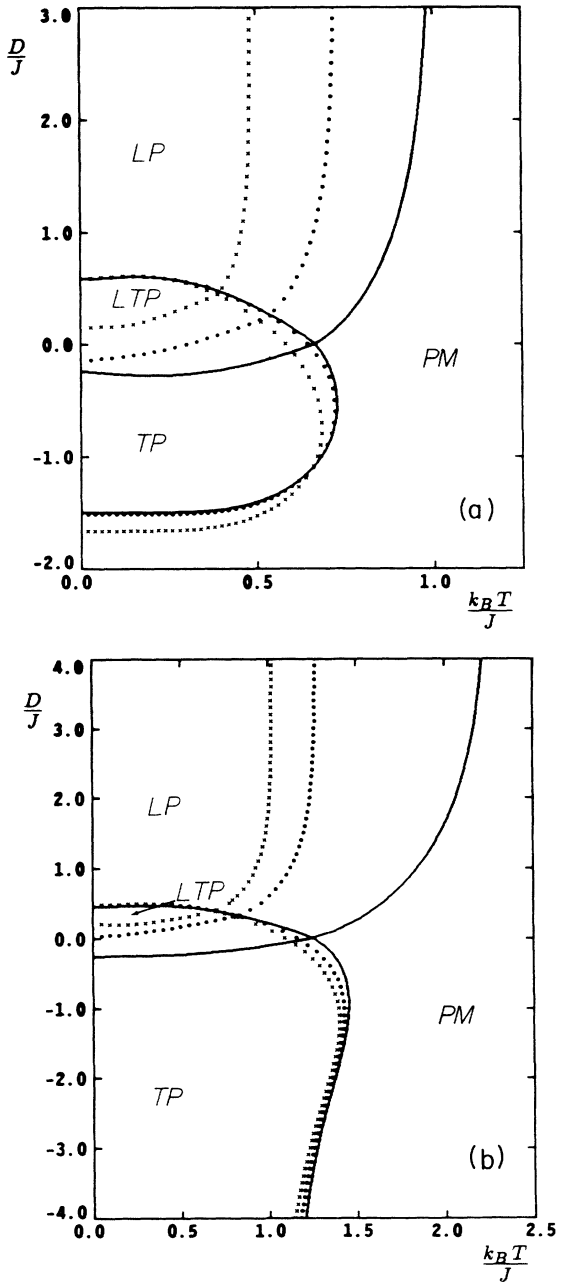


FIG. 1. (a) The anisotropy-temperature phase diagram for $S=1$; zero-field case (solid curve), $h/J=0.2$ (●), 0.6 (×). The temperature for the asymptotic behavior is $k_BT/J=1$. (b) The anisotropy-temperature phase diagram for $S=\frac{3}{2}$; zero-field case (solid curve), $h/J=\frac{2}{3}$ (●), 1.0 (×). The temperatures for the asymptotic behavior are $k_BT/J=1$ and $k_BT/J=2.25$.

VI. PHASE DIAGRAMS AND STABILITY CONDITIONS

Uniaxial anisotropy leads to a strong modification of the phase diagram of a vector spin glass. Specifically, in the quantum case the results are qualitatively different from the behavior found in the classical limit $S \to \infty$. If $D > 0$, ordering is preferred in the longitudinal direction whereas transverse ordering is favored for $D < 0$. The set of complicated self-consistency equations (37)–(41) can be solved only numerically and the resulting phase diagrams are shown in Figs. 1(a) and 1(b). In the zero-field case $h=0$, besides the paramagnetic (PM) phase there are a transverse phase (TP) with $q_{\rm LP}=0$, $q_{\rm TP}\neq 0$, a longitudinal phase (LP) with $q_{\rm LP}\neq 0$, $q_{\rm TP}=0$, and a mixed phase (LTP) in which both orderings occur.

A richer behavior is expected when there is an applied external magnetic field in the z direction in addition to the uniaxial single-site anisotropy. One still expects a transition to a state where the glassy features are manifested in a nonergodic behavior and the occurrence of remanence effects. However, in this case the longitudinal spin-glass order parameter $q_{\rm LP}$ is not appropriate to locate the longitudinal spin-glass phase boundary since $q_{\rm LP}\neq 0$ everywhere due to the applied field. Therefore, one is forced to look for the stability conditions determining the validity of the mean-field approach. Within the present technique it amounts to studying the effect of fluctuations δQ around the saddle-point solution (18) working out the terms which are second order in δQ and calculating the eigenvalues of the matrix associated with the corresponding quadratic form. Since the calculations run along the lines already presented while analyzing the

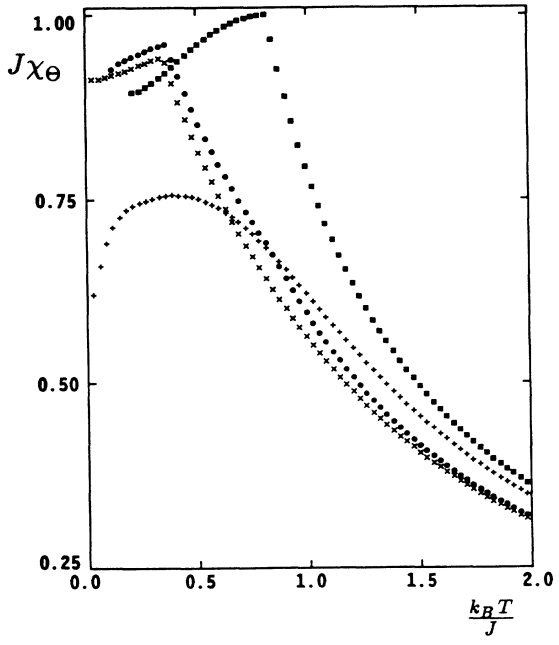
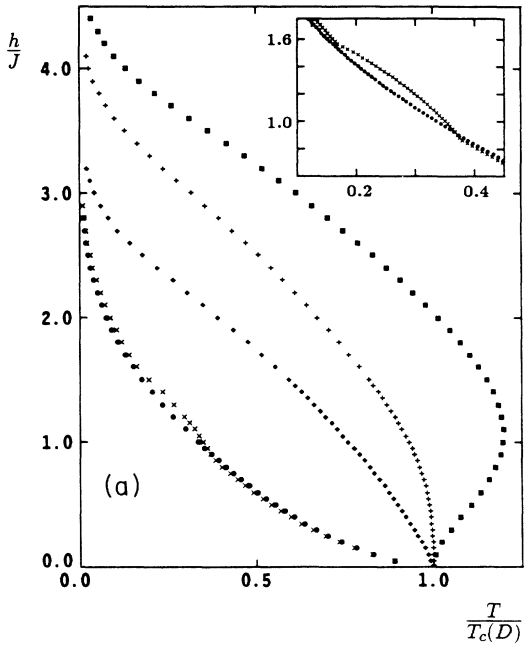


FIG. 2. The local static longitudinal (Θ=LP) and transverse (Θ=TP) susceptibilities as functions of the temperature for $S=1$ (the case of $S=\frac{3}{2}$ is qualitatively similar). (i) Zero-field case: Θ=TP (■) and Θ=LP (●). (ii) Non-zero-field case: $h/J=0.5$, Θ=TP (×), Θ=LP (+). The anisotropy parameter is fixed at $D/J=0.5$.



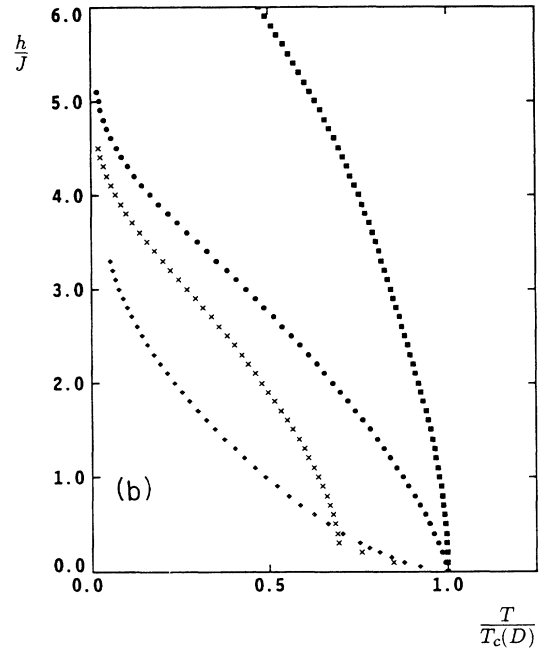


FIG. 3. (a) The field-temperature instability curves for $S=1$ and various anisotropies including: $D/J=-1.4$ (■), -1.0 (+), 0.0 (◆), 0.5 (×), and 2.0 (●). The inset shows the behavior of the instability curve (×) in a region corresponding to the LTP phase. (b) The field-temperature instability curves for $S=\frac{3}{2}$ and various anisotropies including: $D/J=-2.0$ (■), 0.0 (●), 0.2 (×), and 2.0 (◆). The behavior of (×) curve at low fields corresponds to the crossover from the LP to the TP.

transverse spin-glass model[18] we quote here only the final results. The condition for stable solution of Eq. (18) reads

$$1-J^2\langle\chi_\Theta^2(\mathbf{z})\rangle_z \geq 0 \ , \tag{42}$$

where Θ=LP,TP and the local susceptibility χ_Θ is given by Eq. (39). The behavior of χ_Θ as a function of temperature for anisotropies corresponding to the region (LTP) at various fields h is depicted in Fig. 2. In the case of transverse freezing ($\Theta=T$), the condition (42) reduces simply to $1-J\chi_{\rm TP}\geq 0$, since $q_{\rm TP}=0$ along the transverse freezing line.[6] Because for $h\neq 0$ one has $q_{\rm LP}\neq 0$ everywhere, Eq. (42) represents the generalization of the de Almeida–Thouless (AT) line[25] to the present problem. To conclude, in order to single out the instability region of the (k_BT,h,D) parameter space one has to look for the lines determined by the following condition:

$$\max(J^2\langle\chi_{\rm LP}^2(\mathbf{z})\rangle_z, J\langle\chi_{\rm TP}(\mathbf{z})\rangle_z)=1 \ . \tag{43}$$

If one deals with uniaxial anisotropy two qualitatively distinct cases have to be distinguished according to the sign of the energy splitting D, as emphasized earlier. For large positive D the system is of the Ising-type while in the opposite case the spins prefer to lie in the basal plane, but no direction is then preferred because of the rotational X-Y symmetry. The continuous crossover behavior is best seen if one looks at the field-temperature phase diagram of the system for different values of the constant D [Figs. 3(a) and 3(b)]. For large positive D the system exhibits a typical AT behavior characteristic for the freezing of the longitudinal spin components. For negative D, in turn, the system is of the X-Y-type and for small magnetic fields one observes the field-temperature dependence corresponding to the Gabay-Toulouse[6] (GT) line indicating the freezing of the transverse spin components. At large fields, however, the transverse spin components order and the system crosses over to the longitudinal behavior determined by the AT-like line. It is interesting to note that, as emphasized earlier, the cases of $S=1$ and $S=\frac{3}{2}$ behave quite distinctly. For example, for $S=1$ and anisotropies $-1>D\geq -1.5$ one sees in the temperature-field phase diagram a temperature region with a reentrance behavior. In this case, as the field is lowered, the system passes from paramagnetic-ergodic phase to the spin-glass one and, by further lowering the field, the system reenters an ergodic region. This occurrence of glass ordering and the nonergodic behavior would deserve a special theoretical treatment. Since, e.g., LP-LTP and TP-LTP boundaries lie in the spin-glass phase, i.e., in an instability region, a precise calculation of these lines presumably would require a quantum analog of the replica-symmetry-breaking scheme, which has not been done yet.

The remarkable difference between classical and quantum spins takes place for $S=1$, while for large negative anisotropy D a condensation in the $S_z=0$ nonmagnetic spin state results, accompanied by the destruction of the spin-glass order as indicated by the finite critical value D_c as the temperature goes to zero. Moreover, the quantum mechanics brings about new features which are ultimately connected with the dynamics of the system. For example, the critical value $D_c(T=0)=-1.5J$ found here differs from the one found by means of the so-called static approximation[4,5] within the imaginary-time Matsubara technique $D_c(T=0)=-4J$. The reason for this discrepancy lies in the fact that for the quantum spin-glass problem an exact calculation of the transition lines requires precise knowledge of the time dependence of the dynamic spin self-interaction involved in the problem. Even in the static limit the local susceptibilities χ_{LP} and χ_{TP} will depend on the detailed time dependence of $JQ_{\mu\nu}^{ab}(t)$. The complexity of the problem prevents an analytically tractable approach which goes beyond the ansatz(28). However, a calculation of the exact critical line $T_c(D)$ is presently under study using extensive Monte Carlo simulations.[26] As in the case of the transverse Ising spin glass with field,[18] one expects that the exact critical value $D_c(T=0)$ will lie between the values determined by the present method and imaginary-time approach within the static approximation.

To summarize, the calculated phase diagrams and curves show a remarkable similarity to those found experimentally.[1–3] Especially the occurrence of the two successive transitions LP-LTP in the Ising-like ($D\rightarrow\infty$) also found experimentally is worth mentioning. Concerning the Ising and X-Y limits, the experiments seem to be in accordance with the theoretical results found here. Moreover the reentrant behavior found here seems to correlate with the experimental data.[3]

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