Thermal fluctuations of vortex lines, pinning, and creep in high-T_c superconductors

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Thermal fluctuations of vortex lines are shown to be capable of strongly reducing the value of the critical current in the mixed state of high-T_c superconductors. The theory of pinning in the presence of thermal fluctuations is developed. The current relaxation law in the regime of single-vortex pinning is obtained.

The magnetic behavior of single-crystal high-T_c superconductors shows several unusual properties, namely, (i) very rapid (presumably exponential) fall of the critical current with temperature T and magnetic field H,[1] (ii) high values of logarithmic creep rates,[2-4] and (iii) nonmonotonic T dependencies of the creep rate.[3] There are several proposals in the literature[4,5] that thermal fluctuations can play an important role in these phenomena due to low pinning energies and high characteristic temperatures of Y-Ba-Cu-O and Bi-Sr-Ca-Cu-O.

There are two different types of thermal fluctuations that can be relevant: phononlike harmonic fluctuations of vortex lines (VL) or vortex line lattice (VLL) reducing the effective pinning strength and critical current, and activated jumps leading to flux creep. In this paper we begin with the study of the first effect, i.e., we incorporate harmonic thermal fluctuations into the Larkin-Ovchinnikov theory of vortex pinning.[6,7] One can note that at any nonzero temperature any current would lead to a relaxation through a flux creep; nevertheless, one can define the critical current j_c as the crossover "point" between linear ($j \gg j_c$) and nonlinear ($j \le j_c$) current-voltage relations. We see below that in extreme type-II superconductors, harmonic thermal fluctuations can strongly reduce the value of j_c. In particular, regions on the (T,B) plane are found where j_c is an exponentially decreasing function of T and/or B, resembling the experimental results of Ref. 1. The second problem that we shall address is the relation between the current that is really measured in the magnetic relaxation experiments and $j_c(T)$. These experiments are usually analyzed[1,4] in terms of Bean's critical state,[8] where the current is supposed to be near critical and its relaxation due to flux creep is very slow. However, the values of flux creep rate observed in high-T_c superconductors appear to be rather large; at relevant time scales $t \sim 10^2 + 10^5$ sec, it can lead to a substantial decrease of current from its initial microscopic-time value $j_c(T)$. To find the temporal dependence of j, one needs some model of VL creep. We shall consider such a model relevant at sufficiently low T and B values, and obtain the asymptotic behavior of $j(t)$ in the form $j(t) \sim (T \ln t)^{-1/\alpha}$, $\alpha < 1$.

The elastic energy of the VLL has the form

$$F = \int d^3r \left[\tfrac{1}{2}(C_{11} - C_{66})(\nabla \cdot \mathbf{u})^2 + \tfrac{1}{2} C_{66} (\nabla_\perp \mathbf{u})^2 + \tfrac{1}{2} C_{44} \left[\frac{\partial \mathbf{u}}{\partial z} \right]^2 + \delta V(\mathbf{r}, \mathbf{u}) \right] , \tag{1}$$

where $\mathbf{u}$ is the lattice distortion, C_{11} and C_{44} are the (nonlocal) elastic moduli for compression and tilt, respectively, C_{66} is the shear modulus, and

$$\delta V(\mathbf{u}) = \sum_{\mathbf{n}} \int dz_{\bar{\mathbf{n}}} \delta\varepsilon(z_n, \mathbf{u}_n(z_n))$$

represents the local fluctuations of VLL energy density due to pins [here $\delta\varepsilon(z_{\bar{\mathbf{n}}}, \bar{u}_{\bar{\mathbf{n}}}(z_n))$ is the fluctuation of the linear density of the $\mathbf{n}$th vortex line, and the sum $\sum_{\mathbf{n}}$ is over all vortices in VLL]. The correlation properties of the disorder potential are characterized by the relation

$$\int d^3r \overline{\delta\varepsilon(0)\delta\varepsilon(\mathbf{r})} e^{i\mathbf{K}\cdot\mathbf{r}} = \gamma F(K\xi) , \tag{2}$$

where $F(0)=1$ and $F(x)$ decreases rapidly at $x > 1$.

We start from a simple estimate of the mean-squared thermal displacement $\langle u^2 \rangle$ of the position of an unpinned VLL in the field region $H_{c1} \ll B \ll H_{c2}$. Considering the transverse deformation modes one gets

$$\langle u^2 \rangle = T \int \frac{d^3q}{(2\pi)^3} [C_{66} q^2 + C_{44}(q) q_z^2]^{-1} , \tag{3}$$

where the z-axis is in the direction of the magnetic field, the elastic moduli C_{66} and $C_{44}(q)$ are[7,9]

$$C_{66} = \frac{\phi_0 B}{(8\pi\lambda)^2}, \quad C_{44}(q) \simeq \frac{B^2}{4\pi} \frac{1}{(q\lambda)^2 + 1}, \quad q \le \frac{1}{a} \tag{4}$$

where $\Phi_0 = \pi\hbar c/e$, λ is the London screening length, and a is VL lattice constant. The compressional modulus $C_{11}(q)$ is irrelevant since $C_{11}(q) \gg C_{66}$ and, therefore, compressional deformation is small in comparison with shear deformation. The main contribution to the integral (3) comes from the region $q \sim a^{-1}$, where these simple formulas are not valid. However, for the qualitative analysis we shall use them and cut off the integration in (3) at $q=q_{\max}=\frac{1}{2}K_1$, where $K_1=2^{3/2}3^{-1/4}\pi(B/\Phi_0)^{1/2}$ is the length of reciprocal-lattice vector. Then we get

$$\langle u^2\rangle \simeq 4\pi^2\frac{T\lambda^2}{\Phi_0^{3/2}\sqrt{B}} . \tag{5}$$

Let us now assume that pinning is due to some small-scale weak disorder (e.g., fluctuations in oxygen positions). This probably means that our results are most readily applicable for Bi-based compounds which are free from twins. Therefore the pinning energy for an individual VL varies on the scale of the coherence length, and will presumably be reduced by thermal fluctuations if $\langle u^2\rangle \geq \xi^2$, i.e., at

$$T \gtrsim T_L^* = \frac{\Phi_0^{3/2}\sqrt{B}}{(2\pi\kappa)^2}; \quad H_{c1} \ll B \ll H_{c2} , \tag{6}$$

where $\kappa=\lambda/\xi$ is the Ginzburg-Landau parameter, e.g., for $B=10^4$ G and $\kappa=200$ one would obtain $T_L^*\simeq 30$ K. In the following we estimate the critical current j_c at $T\gtrsim T_L^*$ and show that it drops very rapidly with temperature.

To estimate critical current j_c we use the dynamic approach,[6,10] which is slightly more tedious than simple scaling estimates,[7] but is more suitable when dealing with thermal fluctuations. Following Refs. 6 and 10 we derive from (1) the equation of VLL motion in transverse direction:

$$\Gamma^{-1}\frac{\partial \mathbf{u}}{\partial t} - \left[C_{66}\frac{\partial^2}{\partial \mathbf{p}^2}+C_{44}\frac{\partial^2}{\partial z^2}\right]\mathbf{u} = -\frac{\partial}{\partial \mathbf{u}}\delta V(\mathbf{u}) - \frac{1}{c}(\mathbf{j}\cdot\mathbf{B}) + \mathbf{f}(\mathbf{r},t) , \tag{7}$$

where $\Gamma^{-1}=\sigma B^2/c^2$ is the kinetic coefficient (σ is the conductivity of a superconductor in the unpinned mixed state), $\mathbf{f}(\mathbf{r},t)$ is the thermal Langevin force with correlation function

$$\langle f_\alpha(\mathbf{r},t) f_\beta(\mathbf{r}',t')\rangle = 2T\Gamma\delta_{\alpha\beta}\delta_{\mathbf{r}\mathbf{r}'}\delta_{tt'} ,$$

and ρ is a two-dimensional (2D) vector: $\mathbf{r}=(\rho,z)$. At strong current one can solve Eq. (7) perturbatively with respect to disorder:

$$\mathbf{u}(\mathbf{r},t)=\mathbf{v}_0 t+\mathbf{u}_1(\mathbf{r},t)+\mathbf{u}_2(\mathbf{r},t)+\mathbf{u}_{\text{th}}(\mathbf{r},t) , \tag{8}$$

where $\mathbf{v}_0=(c/\sigma B^2)[\mathbf{j}\cdot\mathbf{B}]$, $|\mathbf{u}_1|\propto\delta V$, $|\langle\partial\mathbf{u}_2/\partial t\rangle|\propto(\delta V)^2$, and $\mathbf{u}_{\text{th}}(\bar{r},t)$ describes thermal vibrations of VLL.

Making use of the iterative procedure,[6,10] one can find the first disorder correction $\delta\mathbf{v}^{(1)}$ to the VLL velocity, $\mathbf{v}=\partial\mathbf{u}/\partial t$ in the form

$$-\frac{\delta\mathbf{v}^{(1)}}{\mathbf{v}_0}=\frac{1}{S_0^2}\sum_{\mathbf{K}}\int\frac{d^3q}{(2\pi)^3}\frac{\gamma F(K\xi)K^2K_\parallel^2\exp(-K^2\langle u^2\rangle)}{[C_{66}q^2+C_{44}(q)q_z^2]^2+(jBK_\parallel/c)^2} , \tag{9}$$

where $S_0=\Phi_0/B$ is the area for one VL, the sum is over reciprocal-lattice vectors K whose component $K_\parallel$ lies in the direction of VLL motion, and the Debye-Waller factor on the right-hand side (rhs) describes the effect of thermal fluctuations. It was supposed in the derivation of Eq. (9) that pinning is of 3D collective nature, so that an influence of disorder on VLL thermal fluctuations can be neglected. The integral over q in Eq. (9) can easily be estimated in two limits corresponding to large or small values of $(q\lambda)$. The sum over K can be converted to an integral as long as $\langle u^2\rangle \ll (a/2\pi)^2$ [this means in fact that we restrict ourselves to the temperatures lower than the melting temperature $T_M(B)$]. We also choose $F(K\xi)=\exp(-K^2\xi^2)$ for the sake of simplicity. Then the value of the critical current j_c can be estimated as the one that corresponds to $|\delta\mathbf{v}^{(1)}/\mathbf{v}_0|$ of the order of unity. We start from the case of large dispersion, $q\lambda\gg 1$, so $C_{44}\simeq C_4\,(q_\perp\lambda)^{-2}$ (we shall see just below that essential values of q_z are much smaller than those of $q_\perp$) and $C_4=C_{44}$ (0). Introducing variables $\tilde{q}_z=(\sqrt{C_4C_{66}}/\lambda)q_z$ and $\tilde{q}_\perp^2=C_{66}q_\perp^2$ and performing integration over $\tilde{q}_z$ one gets

$$-\frac{\delta v}{v}=\frac{\gamma\lambda}{4\sqrt{2}S_0C_{66}^{3/2}C_4^{1/2}}\frac{c}{jB}\int\frac{d^2K}{(2\pi)^2}K^2|K_\parallel|\exp(-K^2\xi_T^2)\int_{q_1^2}^{q_2^2}\frac{d\tilde{q}_\perp^2}{2\pi}\frac{[\sqrt{\tilde{q}_\perp^4+(jBK_\parallel/c)^2}-\tilde{q}_\perp^2]^{1/2}}{\sqrt{\tilde{q}_\perp^4+(jBK_\parallel/c)^2}}\tilde{q}_\perp ,$$

where $\xi_T^2=\xi^2+\langle u^2\rangle_T=\xi^2(1+T/T_L^*)$, $q_1^2=jB/\xi_T c$, and $q_2^2=C_{66}/a^2$. The limits of integration over $q_\perp^2$ stem from the fact that the main contribution to the integral over $\mathbf{K}$ comes from $|\mathbf{K}|\approx\xi_T^{-1}$, and that to provide the large dispersion essential $q_\perp^2$ should satisfy condition $q_\perp^2 \gg jB/c\xi_T$. Then one can expand the square root in the integrand and obtain

$$-\frac{\delta v}{v}=\frac{1}{\pi\sqrt{2}}\frac{\gamma}{\gamma_m}\left[\frac{H_{c2}}{B}\right]^{3/2}\left[\frac{T}{T_L^*}+1\right]^{-3}\ln\left[\frac{3\sqrt{3}}{32\pi}\frac{j_0}{j}\frac{B}{H_{c2}}\left[1+\frac{T}{T_L^*}\right]^{1/2}\right] ,$$

and finally

$$j_c=\frac{3\sqrt{3}}{32\pi}j_0\frac{B}{K_{c2}}\left[1+\frac{T}{T_T^*}\right]^{1/2}\exp\left[-\pi\sqrt{2}\left[\frac{B}{H_{c2}}\right]^{3/2}\frac{\gamma_m}{\gamma}\left[\frac{T}{T_T^*}+1\right]^3\right] \tag{10}$$

if the condition

$$\frac{j_0}{\kappa^2}\ll j_c\ll\frac{B}{H_{c2}}\left[\frac{T}{T_T^*}+1\right]^{1/2}j_0 \tag{11}$$

is satisfied. Here $j_0=c\Phi_0(12\pi^2\sqrt{3}\lambda^2\xi)^{-1}$ is the depairing current and $\gamma_{\max}=(H_c^2\xi^3)^2\xi$ is the value of the parameter γ corresponding to extremely strong short-scale disorder.

To make an estimate in the case $\lambda q\ll 1$, let us define

$$x=(c'C_{44}/jB|K_{\parallel}|)^{1/2}q_z,\quad y=(cC_{66}/jB|K_{\parallel}|)q_{\perp}^2\ ,$$

and rewrite (9) in the form

$$-\frac{\delta v}{v}=\frac{\gamma}{2S_0C_{66}\sqrt{C_{44}}}\left[\frac{c}{jB}\right]^{1/2}\int\frac{d^2K}{(2\pi)^4}K^2|K_{\parallel}|^{3/2}\exp(-K^2\xi_T^2)\int dx\int\frac{dy}{(x^2+y)^2+1}\ .$$

Noticing once again that the main contribution to the integral over $\mathbf{K}$ comes from $|K|\approx\xi_T^{-1}$, and taking into account that essential x and y are of order of unity, one can find that the condition of the small dispersion is satisfied as long as $j_c\ll j_0/\kappa^2$. Performing a trivial integration one gets

$$j_c\simeq 10\frac{j_0}{\kappa^2}\left[\frac{\gamma}{\gamma_m}\right]^2\left[\frac{H_{c2}}{B}\right]^3\left[\frac{T_L^*}{T_L^*+T}\right]^{11/2} \tag{12}$$

at $j_c\ll j_0/\kappa^2$.

Before analyzing the results [Eqs. (10)–(12)], we shall obtain formulas for the critical current for another regime, where each VL is pinned independently. We proceed analogously to Ref. 10 (the only difference is that VL is situated in 3D space rather than in 2D as it was in Ref. 10). The equation of motion for VL is

$$\frac{1}{\Gamma_1}\frac{\partial\mathbf{u}}{\partial t}=C_1\nabla^2\mathbf{u}+\frac{\Phi_0}{c}[\mathbf{j}\cdot\mathbf{n}]-\frac{\partial\delta\varepsilon(z,\mathbf{u})}{\partial\mathbf{u}}+f(z,t)\ , \tag{13}$$

where $\Gamma_1^{-1}=\sigma B\Phi_0$ is the kinetic coefficient, $\mathbf{n}$ is the unit tangent vector along the vortex line, and $C_1=(\Phi_0/4\pi\lambda)^2[\ln(1/q\xi)+1]$ is VL tension. Looking for the solution in the form

$$\mathbf{u}(z,t)=\mathbf{v}t+\delta\mathbf{u}_{\rm dis}(z,t)+\delta\mathbf{u}_T(z,t)\ , \tag{14}$$

where the second term on the right-hand side (rhs) is the correction to the displacement of VL due to the disorder, and the last term accounts for the thermal fluctuations of VL. One can obtain the first correction to the VL velocity analogously to (9):

$$\delta\mathbf{v}^{(1)}=-\gamma\int dt\int\frac{dq\,d^2K}{(2\pi)^3}F(q,\mathbf{K})\exp\left[i\mathbf{K}\cdot\mathbf{v}t-\frac{K^2}{2}\langle[u(0)-u(t)]^2\rangle\right]iK^2K_{\parallel}G_0(q,t) \tag{15}$$

with $G_0(q,t)=\Theta(t)\exp(-C_1q^2t)$. Performing the integration, one gets the critical current from the condition $|\delta\mathbf{v}^{(1)}/\mathbf{v}_0|\sim 1$ [$\Theta(t)$ is the step function]:

$$j_c(T\ll T_v^*)\simeq\frac{3^{1/3}j_0}{2\pi^{2/3}[\ln(\gamma_{\max}/\gamma)]^{1/3}}\left[\frac{\gamma}{\gamma_{\max}}\right]^{2/3}, \tag{16}$$

$$j_c(T\gg T_v^*)\simeq j_c(T\ll T_v^*)\left[\frac{T}{T_v^*}\right]^2\exp\left[-\left[\frac{T}{T_v^*}\right]^3\right], \tag{17}$$

where the crossover temperature T_v^* is given by

$$T_v^*=\left[\frac{\gamma C_1}{6}\right]^{1/3}. \tag{18}$$

The picture of single VL pinning is valid at sufficiently strong critical currents, i.e., when the inequality opposite to the rhs of Eq. (11) holds.

Now we are in a position to discuss the whole picture of the $j_c(T,B)$ dependence on the region $H_{c1}\ll B\ll H_{c2}$. The "phase diagram" on the (T,B) plane is shown in Fig. 1. The numbers in each area show which formula for j_c should be used. At low temperatures the crossover from single-vortex pinning to collective pinning occurs at $B\simeq B_1\simeq 0.17(\gamma/\gamma_{\max})^{2/3}H_{c2}$. The second crossover between different regions of collective pinning occurs at $B_2\simeq B_1\ln^{2/3}(B_1/H_{c1})$. Thermal fluctuations are substantial at $T>\max(T_v^*,T_1^*(B))$.

The critical current value given by Eq. (17) decreases very fast at $T>T_v^*$, so its domain of applicability is practically absent when B is not very close to H_{c1}. With in-

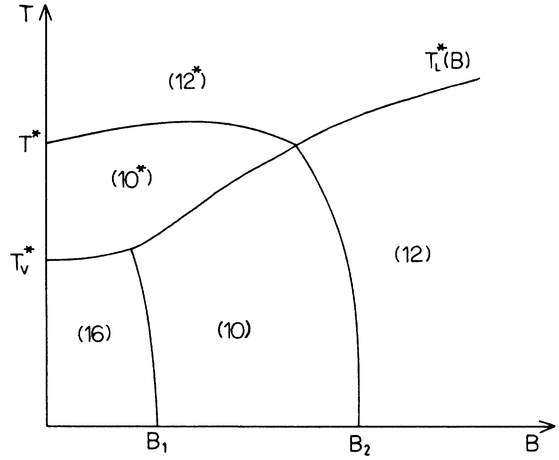

FIG. 1. Phase diagram of the (T,B) plane. The numbers in each area represent which formula for j_c should be used (asterisks denote the high-temperature limits of corresponding formulas).

creasing T the dependence (16) crosses over to (10^*) and then to (12^*). It is interesting that in region (10^*) the exponent is B independent [and coincides up to numerical factors with that of Eq. (17)], so the critical current grows as $B^{3/4}$ in that region. At larger B, crossover $[(10^*)\rightarrow(10)]$ takes place and j_c decreases rapidly with B. Such a nonmonotonic behavior at not too low temperatures resembles the experimental observations of Ref. 1. Moreover, if one considers thermal correction to j_c in region (10) at $T \ll T_L^*$, one obtains

$$-\ln\frac{j_c(T,B)}{j_c(0,B)} \simeq 6\pi^2\frac{\gamma_{\max}}{\gamma}\frac{T\lambda}{\phi_0^2}\frac{B}{H_c} \propto TB \ , \tag{19}$$

which type of behavior was also observed in Ref. 1. In the preceding discussion it was implicitly supposed that the characteristic temperatures $T_v^*, T_L^*(B)$ are much lower than that of T_c, otherwise the temperature dependencies of all the parameters entering Eqs. (10)–(18) should be taken into account.

We now proceed to the discussion of our second subject, that is, current relaxation due to flux creep. Here we restrict our consideration by the single-vortex pinning regime [i.e., region (16) in Fig. 1]. Let us consider single VL placed in 3D unbiased random potential. A similar problem for a domain wall (DW) in 2D space was considered in Ref. 11 where it was shown that a random potential leads to a wandering of the DW so that its transverse displacement u grows with a distance L along the DW:

$$\overline{[u(0)-u(L)]^2} = AL^{2\zeta} \ , \tag{20}$$

where A is some constant. At present, an exact treatment of the similar 3D problem is absent, but numerical simulations[12] point at the validity of Eq. (20) in the 3D case as well. Now our goal is to estimate the energy barrier that should be overcome by a segment of VL when it jumps into some new position under the action of the external current j that produces a transverse force $c^{-1}\phi_0 j = f$ per a unit length of VL (see also Ref. 13 where more detailed discussion of the similar problem is given). Let $L(j)$ be the size of VL line segment that jumps under the action of force density f. The jump length $u(j)$ is related to $L(j)$ by Eq. (20). Then the energy gain due to a jump is $E_g(j) = fu(j)L(j)$, whereas the energy cost is of the order of the depth of energy valley for the same segment, that is, $E_p(j) \sim C_1 u^2(j)/L(j)$. These energies should be of the same order of magnitude, so one gets

$$E_g(j) \sim E_p(j) \sim E_c(j_c/j)^\alpha, \quad \alpha = \frac{2\zeta-1}{2-\zeta} \ , \tag{21}$$

where E_0 is the energy barrier at $j \sim j_c$. To estimate E_c one would need a value of coefficient A in Eq. (20), but it is much simpler to note that E_c should be approximately equal to the crossover temperature T_v^* [see Eq. (18)] that leads to VL depinning. With all this in mind we can estimate the rate of thermally activated VL creep under the current density j:

$$\frac{dj}{dt} \propto \mathbf{v} \propto \exp\left[-\frac{T_v^*}{T}\left[\frac{j_c}{j}\right]^\alpha\right] \ , \tag{22}$$

where it is assumed that $T \ll T_v^*$, $j \ll j_c$. In the considered case, Eq. (22) substitutes usual flux creep relation[14] with the rhs proportional to exp(const. j). Finally, one obtains

$$j(t) \simeq j_c(T_v^*/T\ln t)^{1/\alpha}, \quad T\ln t \gtrsim T_v^* \ . \tag{23}$$

The value of ζ was found to lie numerically in the interval $0.6+0.65$.[12,15] Using the results of the renormalization-group analysis by Halpin-Healy[16] and Nattermann,[17] we can conclude that $\zeta = 0.6$ and $\alpha = \frac{1}{7}$.

The appropriate interpolation between our result (23) and the Anderson formula is

$$j(t) \simeq j_c\left[1+\frac{T}{7T_v^*}\ln(t/t_0)\right]^{-7} \ ,$$
$$T_v^* \ll T\ln(t/t_0) \ll 7T_v^* \ . \tag{24}$$

This result holds at $T \ll T_v^*$, $B \ll B_1$, i.e., in region (16) of Fig. 1. Moreover, there is an additional restriction whose existence was recognized by Geshkenbein:[18] as j decreases the mean jump length $U(j)$ grows [cf. Eq. (21)] as well as the interaction energy $E_{\rm int}$ between jumping VL and neighboring VL. We estimate

$$E_{\rm int} \simeq (\phi_0/4\pi\lambda)^2 u^2(j)L(j)/a^2$$

and obtain [with Eqs. (20) and (21)] the condition $E_{\rm int} < E_p$ in the form

$$j(t) > j_c(B/B_1)^{2/3} \ . \tag{25}$$

At low values of current density [that do not obey Eq. (25)] the interaction between different VL should be taken into account. An analog of Eq. (23) for that region is found in Ref. 18.

In conclusion, we have shown that thermal fluctuations of vortex lines can considerably reduce the pinning strength in high-T_c superconductors. This can supply us with an alternative explanation of the experimental findings, which were viewed there to be a consequence of VL lattice melting. In fact, it is easy to see that the temperature of the melting transition $T_M(B)$ [for details of calculation see Ref. 22] is lower than our depinning temperature T_L^* only at sufficiently strong magnetic fields $B > B_M = (2C_L)^2 H_{c2}$ ($C_L \approx 0.1$ is the Lindemann number). Indeed, $T_M(B)$ is determined by the condition $\langle u^2 \rangle_T = (C_L a_0)^2$ (a_0 is a VL lattice constant), whereas $T_L^*(B)$ corresponds to the condition $\langle u^2 \rangle_T = r_p^2 = (1.4\xi)^2$. Thus, in the field range $B < B_M$, VLL should be depinned at first at $T = T_L^*(B)$ by thermal fluctuations and would melt at higher temperature $T_M(B)$. Note here that the calculation of the exact value of C_L for VLL would be, therefore, of great importance for the drawing of a complete phase diagram of high-T_c superconductors in a strong magnetic field. Critical current dependence on temperature and magnetic field is obtained and shown to be in qualitative agreement with the experimental results.[1] Vortex lines that creep under the action of weak ($j \ll j_c$) current is considered, and the resulting relaxation law $j(t)$ is obtained.

After this work was completed we became aware of work[19,20] where the concept of thermally assisted flux flow (TAFF) was introduced. This concept has much in common with our description of low-j creep phenomena. However, an important difference exists: it is assumed in Refs. 19 and 20 that $E_g \ll T \ll E_p$ ($E_g \equiv \Delta W$ and $E_p \equiv U$ in Ref. 19), whereas in the regime we have studied $E_p \simeq E_g \gg T$ so that the $V(j)$ dependence is strongly nonlinear even at $j \ll j_c$. It looks very probable that we have considered some moderate current regime [cf. Eq. (24)], whereas the asymptotic regime at very low j is described by the TAFF theory. Moreover, we would like to note that TAFF is a rather probable explanation for the unusual resistivity behavior observed in Ref. 21.

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