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Frequency mixing in de Haas–van Alphen oscillations in heavy-fermion compounds

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By means of a simple model calculation we estimate the amplitudes of nonlinear de Haas–van Alphen oscillations in heavy-fermion compounds, i.e., oscillations that correspond to linear combinations of ordinary closed orbits of the Fermi surface. Two possible many-body mechanisms are discussed: (i) frequency mixing via the nonlinear field dependence of the magnetization (Shoenberg effect), and (ii) the modulation of the antiferromagnetic order parameter. The incipient antiferromagnetic order in many heavy-fermion compounds at low T provides a nonlinear coupling mechanism that may lead to cross-sectional areas corresponding to differences of ordinary extremal closed orbits of the Fermi surface.

I. INTRODUCTION

Heavy-electron metals[1–3] have received a large amount of attention in recent years, in particular because of their unusual low-temperature properties. Characteristic to heavy fermions is a very large electronic specific heat at low temperatures, $C=\gamma T$, where γ corresponds to an effective electron mass of 10^2-10^3 times that of free electrons. These systems have typically a large Pauli susceptibility or order antiferromagnetically at low T. Heavy-fermion behavior occurs in a variety of Ce-, U-, and Yb-based compounds.

The large density of states observed experimentally is attributed to the Kondo resonance arising from the screening of the magnetic moment of the quasilocalized f electrons by the conduction electrons. The Kondo resonances of different rare-earth (actinide) sites superimpose coherently and form a narrow band at the Fermi level of width T_K. The temperature dependence of C and χ can be explained[2,3] in terms of this narrow peak in the density of states, as well as the coherence effects observed in the resistivity at low T.

The observation of de Haas–van Alphen (dHvA) oscillations first in the intermediate valence compound[4] $CeSn_3$ and then in several heavy-fermion systems, e.g., $CeCu_6$ (Ref. 5), UPt_3 (Ref. 6), and CeB_6 (Ref. 7), has unambiguously demonstrated the existence of a Fermi surface at low temperatures and the bandlike properties of the heavy-mass electrons. The experimental frequencies are in general consistent with Fermi surfaces emerging from band-structure calculations. The effective masses determined from the oscillation amplitudes, however, are on one hand much larger than those arising from band calculations, but on the other hand considerably smaller than the thermal effective mass from the specific heat.

Many-body correlations are only partially incorporated in band-structure calculations and consequently the resulting effective masses are not heavy enough. The Fermi surface, on the other hand, is essentially determined by the symmetry of the system and Luttinger's theorem.[8] The fact that the dHvA experiments are performed in high magnetic fields could account for the reduced effective mass as compared to the thermal one (from the γ coefficient). The Kondo resonance, responsible for the enhanced masses, is smeared by large magnetic fields and the effect is reduced.

It is at first surprising that the standard Lifshitz-Kosevich[9] theory for the dHvA amplitude is applicable to systems with strong local many-body effects, but possibly the consequence of only low-energy excitations playing a role at low temperatures. The low-energy excitation spectrum is that of quasiparticles with a strongly enhanced mass. There have been several attempts to incorporate the heavy masses into the band picture. A phenomenological approach, devised by Razafimandimby *et al.*,[10] and Fulde *et al.*[11] builds in the correct effective mass into an appropriate resonant level phase shift of a Korringa-Kohn-Rostoker (KKR) band-structure calculation. Microscopic derivations of the quasiparticle behavior within the framework of the Anderson lattice have been incorporated into the exponential dependence of the dHvA amplitude on the effective mass by various authors.[12–15] Rasul[14] has shown that the effective mass in the dHvA amplitude is the same as the one entering the specific-heat coefficient γ. It is also concluded[14] that the Engelsberg-Simpson expression[16] for the dHvA amplitude remains valid despite the high correlations among the electrons.

The large number of bands crossing the Fermi surface in Ce and U heavy-fermion compounds opens the possibility of nonlinear dHvA oscillations due to a magnetic breakdown of ordinary closed orbits of the Fermi surface. As a consequence of the large magnetic field the electrons have a finite probability to tunnel from one closed orbit into another. In a recent paper Rasul and Schlottmann[17] investigated other possible mechanisms leading to nonlinear dHvA oscillations, i.e., to frequency mixing, in the context of heavy-fermion compounds. They qualitatively discussed the following many-body effects that may give rise to dHvA oscillations with frequencies that correspond to linear combinations of ordinary extremal closed orbits of the Fermi surface: (i) magnetic breakdown (magnetically induced tunneling) across the Kondo hybridization gap, as well as across the Brillouin zone boundary, (ii) the incipient antiferromagnetic order in many heavy-fermion compounds at low T provides a nonlinear coupling mechanism via modulation of the antiferromagnetic order parameter,[18] and (iii) the fact that electrons experience the total magnetic induction rather than the applied field gives rise to frequency mixing (Shoenberg effect[19]).

In this paper we present a more quantitative estimate of the dHvA amplitudes corresponding to frequency mixing in heavy-fermion systems. We perform a simple model calculation for the mechanisms (ii) and (iii), i.e., the modulation of the antiferromagnetic order parameter (Sec. II) and the Shoenberg effect (Sec. III), respectively. Concluding remarks follow as Sec. IV.

Since the quasiparticle picture appears to be valid, we consider quasiparticles with very large mass. It is then implicitly assumed that only low-energy excitations in the Fermi liquid are relevant to the dHvA oscillations.

II. MODULATION OF THE ANTIFERROMAGNETIC ORDER PARAMETER

Incipient antiferromagnetic order with a very small ordered magnetic moment has been found via neutron scattering and muon spin resonance in some heavy-fermion compounds, e.g., UPt_3 (Ref. 20), $CeCu_6$ (Ref. 21), and $CeAl_3$ (Ref. 22). We argue that the motion of electrons along extremal closed orbits in an applied field affects the antiferromagnetic order parameter giving rise to frequency mixing. This mechanism is expected to be effective if the same portion of the Fermi surface is responsible for both the nesting (giving rise to antiferromagnetism) and the extremal closed orbits. This mechanism has been proposed[18] to explain observed small area oscillations in transition metal chalcogenide-layer compounds showing charge density waves. Here we extend these arguments to a three-dimensional situation involving heavy electrons and a low T_N.

Long-range order in general modifies the properties of the Fermi surface. The electronic spectrum is determined self-consistently by the many-body collective properties, including the antiferromagnetic correlations via the energy gap or order parameter Δ, which satisfies a nonlinear integral equation (the gap equation or Ginzburg-Landau functional). The kernel of this integral equation is modified by the presence of a magnetic field **H**. As a consequence the order parameter Δ has an oscillatory contribution which affects the one-electron spectrum and hence the dHvA oscillation spectrum with the appearance of new frequencies, which actually do not correspond to cross-sectional areas of the Fermi surface.

The order parameter is determined by minimizing the free energy functional $F_{\mathrm{SDW}}(\Delta)$. The equilibrium value Δ_0 in the absence of a magnetic field satisfies[23]

$$(dF_{\mathrm{SDW}}/d\Delta)_{\Delta=\Delta_0}=0\ ,$$
$$(d^2F_{\mathrm{SDW}}/d\Delta^2)_{\Delta=\Delta_0}=\alpha\rho>0\ , \tag{1}$$

where ρ is the electronic density of states and α is a dimensionless parameter which in the mean-field approximation and close to the transition can be parametrized by $\alpha=a(1-T/T_N)$, $a>0$. We choose the mean-field order parameter as given by (BCS-like)

$$\Delta_0(T)=3.2k_BT_N(1-T/T_N)^{1/2}\ , \tag{2}$$

with T_N being the Néel temperature.

The applied magnetic field gives rise to several effects: (i) the spin-density wave gap or order parameter is reduced, (ii) it induces Landau diamagnetism, and (iii) the free energy acquires an oscillatory contribution, which gives rise to dHvA oscillations. We incorporate (i) into an effective T_N and $\Delta_0(T)$, we disregard (ii) and the oscillatory part of the free energy is given by[24]

$$F_{\mathrm{osc}}(\Delta,H)=2k_BT\sum_{i,n}(-)^n\left[\frac{eH}{2\pi nc\hbar}\right]^{3/2}\times\frac{|A_i''|^{-1/2}}{\sinh(2\pi^2nk_BTm_i/\hbar\omega_c)}\times\cos\left[\frac{\hbar cn}{eH}A_i+\theta_{in}\right]\ , \tag{3}$$

where i labels the extremal closed cross-sectional areas $A_i(\Delta)$ of the Fermi surface with effective cyclotron mass $m_i(\Delta)$ measured in units of the free-electron mass, n indicates the harmonic order, and θ_{in} is an arbitrary phase. Here $\omega_c\equiv eH/mc$ is the cyclotron frequency and A_i'' is the second derivative of the cross-sectional area in the direction parallel to the field.

The total free energy is then $F=F_{\mathrm{SDW}}(\Delta)+F_{\mathrm{osc}}(\Delta,H)$ and the equilibrium value $\Delta(H)$ is obtained by minimizing the free energy with respect to Δ. Writing $\Delta(H)=\Delta_0+\delta\Delta(H)$, where the second term contains the oscillations, we expand the free energy in powers of $\delta\Delta(H)$ [using Eq. (1)]

$$F=F_{\mathrm{SDW}}(\Delta_0)+F_{\mathrm{osc}}(\Delta_0,H)+(dF_{\mathrm{osc}}/d\Delta)_{\Delta=\Delta_0}\delta\Delta(H)+\tfrac{1}{2}\alpha\rho[\delta\Delta(H)]^2\ , \tag{4}$$

so that to first order

$$\delta\Delta(H)=-(dF_{\mathrm{osc}}/d\Delta)_{\Delta=\Delta_0}/\alpha\rho \tag{5}$$

and

$$F=F_{\mathrm{SDW}}(\Delta_0)+F_{\mathrm{osc}}(\Delta_0,H)-\tfrac{1}{2}[(dF_{\mathrm{osc}}/d\Delta)_{\Delta=\Delta_0}]^2/\alpha\rho\ . \tag{6}$$

The last term gives rise to oscillatory terms[18] that vary periodically with inverse magnetic field with frequencies that are characteristic of areas $(n_iA_i\pm n_jA_j)$. In all cases the areas are for $\Delta=\Delta_0$, and n_i and n_j are integers. Hence, antiferromagnetic order can modify the harmonic content of the oscillations, but the main effect is the appearance of new frequencies. These frequencies do not correspond to actual cross-sectional areas of the Fermi surface. Note that the parameter α can be small, but it should remain large enough to ensure the validity of the Taylor expansion.

Denoting with $f(n_iA_i\pm n_jA_j)$ the amplitude factors in front of the cosine functions in the free energy, we obtain

$$f(A_1)=2k_BT\left[\frac{eH}{2\pi c\hbar}\right]^{3/2}\frac{|A_1''|^{-1/2}}{\sinh(2\pi^2k_BTm_1/\hbar\omega_c)}\ , \tag{7}$$

$$f(2A_1)=2k_BT\left[\frac{eH}{4\pi c\hbar}\right]^{3/2}\frac{|A_1''|^{-1/2}}{\sinh(4\pi^2k_BTm_1/\hbar\omega_c)}\left|1+\frac{|A_1''|^{-1/2}}{\alpha\rho}\frac{k_BT}{(\hbar\omega_c)^2}\left[\frac{eH}{\pi c\hbar}\right]^{3/2}(a_1^2+b_1^2)\cos\chi_{2A_1}\right|\ , \tag{8}$$

$$f(A_1\pm A_2)=\frac{1}{\alpha\rho}\left[\frac{2k_BT}{\hbar\omega_c}\right]^2\left[\frac{eH}{2\pi c\hbar}\right]^3\frac{|A_1''A_2''|^{-1/2}}{\sinh[(2\pi^2k_BT/\hbar\omega_c)(m_1+m_2)]}(a_1^2+b_1^2)^{1/2}(a_2^2+b_2^2)^{1/2}\ , \tag{9}$$

where

$$a_i=\frac{\hbar^2}{m}(dA_i/d\Delta)_{\Delta=\Delta_0}\ , \tag{10a}$$

$$b_i=2\pi^2k_BT(dm_i/d\Delta)_{\Delta=\Delta_0}+\frac{\hbar\omega_c}{2|A_i''|}(d|A_i''|/d\Delta)_{\Delta=\Delta_0}\ . \tag{10b}$$

Frequencies corresponding to higher-order harmonics or mixings are expected to have smaller amplitudes in view of the exponential dependence on the heavy masses. We have assumed that $2\pi^2k_BTm_i \gg \hbar\omega_c \gg k_BT$ and χ_{2A_1} is a phase that depends on the phases θ_{in} in Eq. (3).

In order to estimate the relative strength of the amplitudes, we choose a simple BCS-type model for the spin-density waves. This model has been discussed in detail by Rice.[25] The model band structure has one electron pocket, one hole pocket, and a nonmagnetic background. We consider elliptically shaped pockets of unequal radii, so that the nesting is not perfect. The two pockets are coupled via the Coulomb interaction, which drives the system into the antiferromagnetic order and gives rise to the equilibrium order parameter Δ_0. The order parameter hybridizes the two pockets leading to a dispersion relation of the type (we linearize the momentum around the Fermi momentum k_F)

$$E(\mathbf{k})=\left[\left[\frac{\hbar^2k_F}{m_\perp^*}\right]^2(k_\perp-k_0)^2+\Delta^2\right]^{1/2}+\frac{\hbar^2k_\parallel^2}{2m_\parallel^*}-E_0\ , \tag{11}$$

where $m_\perp^*$ and $m_\parallel^*$ are two effective masses (assumed to be heavy). As a function of the parameters E_0 and k_0 the Fermi surface is obtained by equating $E(\mathbf{k})=0$, i.e.,

$$k_\perp^\pm(k_\parallel)=k_0\pm(m_\perp^*/\hbar^2k_F)[(E_0-\hbar^2k_\parallel^2/2m_\parallel^*)^2-\Delta^2]^{1/2}\ . \tag{12}$$

Hence, if the magnetic field is parallel to the $k_\parallel$ direction we obtain two circular orbits of radius $k_\perp^\pm(k_\parallel)$ for each value of $k_\parallel$. These orbits are extremal if $k_\parallel=0$ with cross-sectional areas

$$A_+=\pi[k_\perp^+(0)]^2 \text{ and } A_-=\pi[k_\perp^-(0)]^2\ .$$

In order to obtain the dHvA amplitudes we need expressions for $dA_\pm/d\Delta$, $A_\pm''(\Delta)$, etc., which can be derived from Eq. (11):

$$\frac{dA_\pm}{d\Delta}=\mp\frac{2\pi m_\perp^*}{\hbar^2k_F}\frac{\Delta}{(E_0^2-\Delta^2)^{1/2}}k_\perp^\pm(0)\ , \tag{13}$$

$$A_\pm''(\Delta)=\mp2\pi\left[\frac{m_\perp^*}{m_\parallel^*}\right]\left[\frac{k_\perp^\pm(0)}{k_F}\right]\frac{E_0}{(E_0^2-\Delta^2)^{1/2}}\ , \tag{14}$$

$$\frac{dA_\pm''(\Delta)}{d\Delta}=\mp2\pi\left[\frac{m_\perp^*}{m_\parallel^*}\right]\left[\frac{k_\perp^\pm(0)}{k_F}\right]\frac{E_0\Delta}{(E_0^2-\Delta^2)^{3/2}}\ . \tag{15}$$

The effective mass entering the amplitudes is renormalized by the spin-density wave. This renormalized effective mass can be defined as $(m_\pm^*)^{-1}=|\partial E/\partial k_\perp|/\hbar^2k_F$, so that

$$m_\pm^*=m_\perp^*/[1-(\Delta/E_0)^2]^{1/2}\ , \tag{16}$$

$$\frac{dm_\pm^*}{d\Delta}=\frac{m_\perp^*\Delta}{E_0^2}[1-(\Delta/E_0)^2]^{-3/2}\ . \tag{17}$$

In Fig. 1 we show the dHvA oscillation amplitudes $f(A_+)$, $f(2A_+)$, and $f(A_+\pm A_-)$ as a function of field at a temperature of 100 mK. We have chosen the following parameters $m_\perp^*=m_\parallel^*=40m$ (m being the free electron mass), $T_N=200$ mK, $k_F=0.5$ Å^{-1}, $k_0=0.2$ Å^{-1}, $\chi_{2A_1}=0$, $a=1$, $E_0=k_BT_0$ with $T_0=1.2$ K, i.e., some-

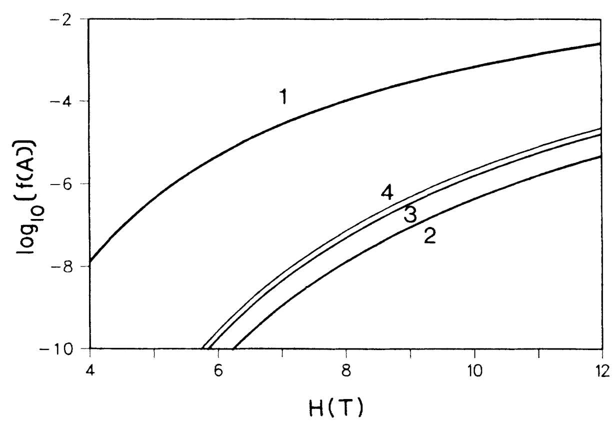

FIG. 1. The amplitudes of the oscillatory free energy for the example given in the text. The field is given in teslas, the ordinates are in arbitrary units, and the temperature is 100 mK. Curve 1 corresponds to the fundamental area A_+, curves 3 and 2 to the second harmonic with and without mixing due to antiferromagnetic order (i.e., $\cos\chi_{2A_1}=1$ and $\cos\chi_{2A_1}=0$), respectively, and curve 4 is the amplitude of the frequency mixing, $A_+\pm A_-$.

what smaller than a typical Kondo temperature, and ρ is the density of the states participating in the formation of the spin-density wave, which can be estimated from the dispersion relation (11) by assuming cylindrical symmetry around the Fermi surface

$$\rho=\frac{1}{4\pi}\frac{k_0}{k_F}\frac{m_\pm^* \Delta k_\parallel}{\pi\hbar^2} , \tag{18}$$

where $\Delta k_\parallel$ is the height of the cylinder (assumed to be 0.05 Å^{-1}). The field is given in teslas. For comparison we also show in Fig. 1 the amplitude of the second harmonic without frequency mixing due to long-range antiferromagnetic order, i.e., for $\cos\chi_{2A_1}=0$. Due to the heavy masses involved the frequency mixing effect is not dramatic, but it could be observable at large fields.

In Fig. 2 the amplitudes $f(2A_+)$ and $f(A_+\pm A_-)$ are displayed as a function of temperature for a fixed field of 8 T. For a comparison we also show the $f(2A_+)$ amplitude in the absence of the mixing effect due to antiferromagnetic order ($\cos\chi_{2A_1}=0$). Above T_N the two $f(2A_+)$ are identical, while $f(A_+\pm A_-)$ is zero. In curve 2 there is a small change of slope due to the mass renormalization by the order parameter Δ. At T_N there is a discontinuity but no divergence of the amplitudes, since the factor $(1-T/T_N)$ in α is cancelled by the temperature dependence of Δ^2 arising from the a_i and b_i.

For the sake of simplicity we have neglected critical fluctuations of the antiferromagnet and restricted ourselves to a mean-field approach. Critical fluctuations appear at temperatures close to T_N, both above and below T_N. As a consequence the effect of frequency mixing should be observable above T_N as well. Long-range order is actually not a necessary condition for the effect, but the frequency mixing is only effective if the correlation length divided by the Fermi velocity is much larger than the time interval required to complete the orbit.

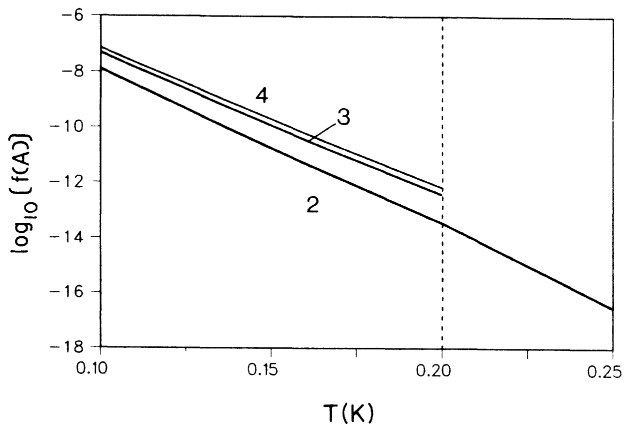

FIG. 2. The amplitudes of the oscillatory free energy for the example given in the text as a function of temperature. The field is 8 T and $T_N=0.2$ K. The ordinates are in arbitrary units. Curves 2 and 3 are the amplitudes of the second harmonic without and with antiferromagnetic frequency mixing ($\cos\chi_{2A_1}=0$) and $\cos\chi_{2A_1}=1$), respectively, and curve 4 corresponds to the mixed frequencies.

III. SHOENBERG EFFECT

Another mechanism giving rise to frequency mixing is the Shoenberg effect. In the classical theory of the dHvA effect the applied field **H** gives rise to oscillatory contributions to the magnetization. In the presence of electron interactions (i.e., beyond the independent electron approximation) the electrons experience the total magnetic induction, $\mathbf{B}=\mathbf{H}+4\pi\mathbf{M}$, rather than the applied field **H**. As has been shown by Shoenberg[19] and Pippard[26] this gives rise to a nonlinear self-consistent equation for the magnetization. Its solution modifies the content of the higher harmonics, i.e., their amplitudes are changed, and gives rise to frequency mixing if the Fermi surface has more than one extremal closed orbit. This magnetic interaction effect has been observed in various metals, e.g., in Ag (Ref. 27) and In (Ref. 28). We briefly discuss this effect in the context of heavy fermions.

The magnetization is obtained via $M=-\partial F/\partial H$ from Eq. (3)

$$M_{\rm osc}(H)=-\frac{2k_BT}{H}\sum_{i,n}(-)^n\left[\frac{eH}{2\pi nc\hbar}\right]^{3/2}\frac{1}{\sinh(2\pi^2nk_BTm_i/\hbar\omega_c)}|A_i''|^{-1/2}[\alpha_{in}^2+\beta_{in}^2]^{1/2}\cos\left[\frac{\hbar cn}{eH}A_i+\theta_{in}'\right] , \tag{19}$$

where θ_{in}' is a phase and

$$\alpha_{in}=\hbar cnA_i/eH, \quad \beta_{in}=2\pi^2nk_BTm_i/\hbar\omega_c+\tfrac{3}{2} . \tag{20}$$

We have used in Eq. (19) that $\beta_{in}\gg 1$. In order to take into account the interaction among the electrons, we replace **H** in Eqs. (19) and (20) by $\mathbf{H}+4\pi\mathbf{M}$. The magnetization of heavy-fermion systems consists of two contributions: (i) a nonoscillating enhanced Pauli susceptibility and (ii) the oscillating part, which is essentially given by Eq. (20). The renormalization of the field due to the spin paramagnetism is in general small (of the order of 1%) and will be neglected in the following. Keeping only the oscillating terms we approximate

$$M_{\rm osc}(H+4\pi M_{\rm osc})\simeq M_{\rm osc}(H)+4\pi M_{\rm osc}(H)(dM_{\rm osc}/dH) , \tag{21}$$

where the second term generates the desired frequency mixing. The amplitudes for the first few most important oscillating contributions to the magnetization are now straightforwardly obtained:

$$f(A_1)=2\frac{k_BT}{H}\left[\frac{eH}{2\pi c\hbar}\right]^{3/2}(\alpha_1^2+\beta_1^2)^{1/2}\frac{|A_1''|^{-1/2}}{\sinh(2\pi^2k_BTm_1/\hbar\omega_c)}\ , \tag{22}$$

$$f(2A_1)=\frac{k_BT}{2H}\left[\frac{eH}{\pi c\hbar}\right]^{3/2}(\alpha_1^2+\beta_1^2)^{1/2}\frac{|A_1''|^{-1/2}}{\sinh(4\pi^2k_BTm_1/\hbar\omega_c)}\left|1+4\pi\frac{k_BT}{H^2}\left[\frac{eH}{\pi c\hbar}\right]^{3/2}(\alpha_1^2+\beta_1^2)|A_1''|^{-1/2}\cos\chi'_{2A_1}\right|\ , \tag{23}$$

$$f(A_1\pm A_2)=2\pi(k_BT)^2\left[\frac{e}{\pi c\hbar}\right]^3\frac{|A_1''A_2''|^{-1/2}}{\sinh[(2\pi^2k_BT/\hbar\omega_c)(m_1+m_2)]}(\alpha_1^2+\beta_1^2)^{1/2}(\alpha_2^2+\beta_2^2)^{1/2}[(\alpha_1\pm\alpha_2)^2+(\beta_1+\beta_2)^2]^{1/2}\ , \tag{24}$$

where

$$\alpha_i=\hbar cA_i/eH,\quad \beta_i=2\pi^2k_BTm_i/\hbar\omega_c+\tfrac{3}{2}\ . \tag{25}$$

In the calculation of the amplitudes we used the following simplification, $\beta_i\sim\beta_i-\frac{3}{2}$, which is not a dramatic approximation, since $\alpha_i\gg\beta_i\gg\frac{3}{2}$ for the parameters of interest. Note that for the Shoenberg effect the amplitudes for the mixed frequencies $|A_1-A_2|$ and A_1+A_2 are different.

In Fig. 3 we show the relevant amplitudes for the simple band model discussed in Sec. II but in the absence of antiferromagnetic order, i.e., for $\Delta=0$. The extremal orbits are the two circles of radii, $k_\perp^\pm=k_0\pm m_\perp^*E_0/\hbar^2k_F$. In order to compare the results to those in Fig. 1 we chose the same set of parameters. The temperature is 100 mK, while $k_0=0.2$ Å^{-1} and $k_0=0.8$ Å^{-1} in Figs. 3(a) and 3(b), respectively. The amplitudes displayed correspond to the frequencies associated with the areas A_+, $2A_+$ (with and without Shoenberg effect), (A_++A_-) and (A_+-A_-). As a consequence of the large mass enhancements, the Shoenberg effect only leads to small modifications of the oscillation spectrum. By increasing the areas the frequency mixing amplitudes grow relatively to the ones of the fundamental frequencies. Reducing the temperature does not dramatically alter the situation, i.e., all amplitudes grow but their relative strength is roughly invariant.

IV. CONCLUDING REMARKS

We considered the possibility of frequency mixing in the dHvA oscillations of a heavy-electron system. By means of a simple model calculation we estimated the amplitudes arising from two mechanisms, namely, via the modulation of the antiferromagnetic order parameter and the Shoenberg effect. In both cases the mixing is caused by the interactions among the electrons. Since dHvA amplitudes decrease exponentially with the electron mass and temperature, the effects are expected to be small. This is indeed the case for the Shoenberg effect, but not necessarily for an itinerant antiferromagnet, where frequency mixing could be observable. Antiferromagnetic order is probably not necessary to observe the effect; it is sufficient to have an antiferromagnetic correlation length larger than the orbit of the electrons in real space.

The Néel temperature of an antiferromagnet is typically reduced by a magnetic field. In order to observe this effect the system should have a T_N of the order of 0.1 K in a field of 10 T. A possible candidate is the itinerant heavy-fermion antiferromagnet $CePb_3$ (Ref. 29).

As mentioned in the Introduction, there are other mechanisms that can lead to nonlinear dHvA oscilla-

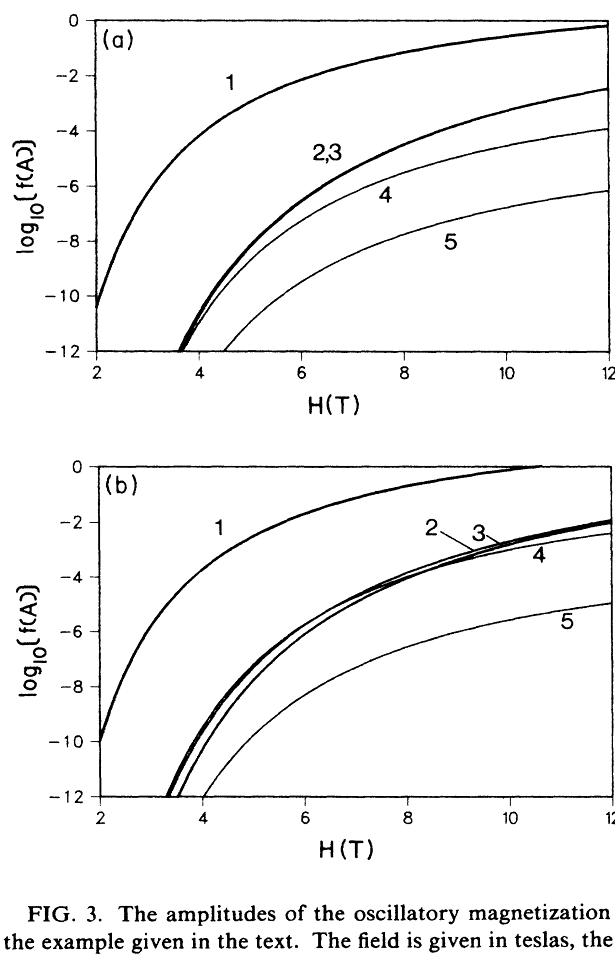

FIG. 3. The amplitudes of the oscillatory magnetization for the example given in the text. The field is given in teslas, the ordinates are in arbitrary units, the temperature is 100 mK and $k_0=0.2$ Å^{-1} in (a) and 0.8 Å^{-1} in (b). Curve 1 corresponds to the fundamental frequency, 2 and 3 to the second harmonic with and without Shoenberg effect, respectively, 4 is the mixing amplitude for the frequency A_++A_-, while 5 the amplitude for A_+-A_-.

tions, which can be classified as magnetic breakdown mechanisms, i.e., magnetic field induced tunneling between orbits. While magnetic breakdown mechanisms yield oscillations corresponding to sums of cross-sectional areas of the Fermi surface, the two mechanisms discussed here give rise to frequencies associated with differences of areas as well.

Note added in proof. We have considered nonlinear effects due to the modulation of the antiferromagnetic order parameter. Similar effects could be obtained with other forms of long-range-order, for instance, in a type II superconductor if the field is only slightly smaller than H_{c2}, so that field inhomogeneities due to the emerging vortices still do not affect the Landau levels. As in Fig. 2 the smallness of the order parameter values in H_{c2} should not prevent the observability of the effect.

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