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Positron lifetime in a resonating-valence-bond superconductor

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The behavior of the annihilation rate of positrons in a strongly correlated two-dimensional electron system in which the spin and charge degrees of freedom are decoupled is examined, and it is found that the positron lifetime becomes longer in the proposed resonating-valence-bond paired-boson superconductor when a gap exists for the charged excitations.

Positron-annihilation studies of the copper oxide superconductors have produced results which, in view of the usual insensitivity of positrons to superconductivity, are surprising. The insensitivity to the superconductivity of conventional superconductors is a consequence of the smallness of the energy gap $\Delta \sim k_B T_c \leq 1$ meV compared to the characteristic electronic energies, the Fermi energy, or the electron-positron correlation energy, which are both ~ 10 eV. However, the initial lifetime studies of Jean *et al.*[1] indicated that positrons are sensitive to superconductivity in the copper oxide superconductors. Further work showed[2-4] that in reasonably defect-free samples of superconducting $La_{2-x}Sr_xCuO_4$, $YBa_2Cu_3O_{7-x}$, and $Tl_{2.2}$-$Ca_2Ba_2Cu_3O_{10+x}$, the (bulk) positron lifetime τ is temperature-independent in the normal state, but below the superconducting transition temperature T_c it becomes longer as the temperature is decreased. The reported increases in τ between T_c and $T=0$ are $\sim(5-10)\%$. No similar lengthening of the lifetime is observed either in conventional superconductors or in the nonsuperconducting parent compounds. In presumably less-perfect samples, different temperature-dependent behaviors of the lifetimes are seen and are tentatively associated with positron trapping at defects in the materials.[5,6] Temperature-dependent changes in the electron momentum spectra as measured by positrons have also been reported.[5,7] These include[5] a decrease in the intensity of low momentum annihilations in $YBa_2Cu_3O_{7-x}$ on cooling superconducting ($x \approx 0,0.4$) samples below T_c, with no similar decrease observed in nonsuperconducting ($x \approx 0.7$) material.

If the observed lifetime lengthening below T_c is due to a stiffening of the charge response to the positron in the superconducting state,[8] then positron annihilation probes the charged excitations of the new superconductors, and these results give yet more support for suggestions that a novel mechanism is responsible for the superconductivity. Because of this, it is of interest to explore if, and under what conditions, proposed novel mechanisms can give an altered charge response and a measurable change in the positron lifetime on cooling below T_c. The purpose of this Rapid Communication is to report the temperature dependence of the positron lifetime below T_c that results from one proposed radically new mechanism, the pairing[9,10] of the charged-bosonic excitations (holons) of the resonating-valence-bond (RVB) picture.[11] The reasons for selecting this particular model are that it is one of the more quantitative proposals, so that although the model itself continues to evolve, calculations can at least be done within the simplest version,[9,12] and also that significant temperature dependence is more likely to arise from bosonic than from fermionic excitations.

The positron-annihilation rate is proportional to the electron density at the position of the positron. Since, in a metallic environment, the net screening charge surrounding a positron is $1e$, a simple criterion for the qualitative behavior of this rate should be the behavior of the screening length $l_s \equiv q_s^{-1}$, with $\tau \sim l_s^d$. As a test of this proposal, Fig. 1 shows a comparison of q_{TF}^3 with the electron-gas annihilation rate calculated by Arponen and Pajanne.[13] Fitting at $r_s=3$ gives, over the metallic density range, a rate which is $\approx 10\%$ high at $r_s=2$ and $\approx 20\%$ low at $r_s=5$, although the rate itself changes by a factor of 3 and the electron density by more than an order of magnitude over this interval. The proportionality $\tau \sim l_s^d$ gives a remarkably simple and reasonably accurate description of the density dependence of the lifetime.

With the aid of this proportionality, it is easy to see why no lifetime lengthening below T_c is observable in conven-

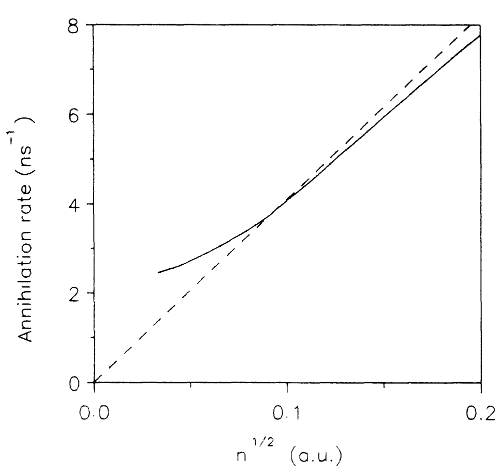

FIG. 1. Comparison of the density dependence of the positron-annihilation rate in an interacting electron gas as calculated by Arponen and Pajanne (Ref. 13) (solid curve) with the density dependence of the cube of the Thomas-Fermi screening vector. The dashed curve (5.24 ns^{-1}) $(q_{TF}a_0)^3$ is fitted to the calculated rate at $r_s=3$.

tional superconductors. BCS theory gives[14]

$$\frac{\epsilon_0 q_s^2}{e^2} = \left(\frac{\partial \bar{n}}{\partial \mu}\right)_\Delta = \int d\varepsilon g(\varepsilon) \frac{\Delta^2}{2E^3} \quad (1)$$

at $T=0$, where $E=[(\varepsilon-\mu)^2+\Delta^2]^{1/2}$, and g is the density of states.[15] The rate of change of the mean electron density $\bar{n}$ with chemical potential μ is evaluated at constant gap Δ because the correlation length exceeds the screening length. The deviation of q_s^3 from its normal-state value is shown in Fig. 2 as a function of Δ/μ, and there is little change for small values of Δ/μ. For the lifetime change to be $\sim 5\%$, as observed in the copper oxide superconductors, where probably[7] fewer than half the annihilations are with the CuO_2 plane "conduction" electrons, a change at least $\sim 10\%$ in the rate of annihilation with those electrons would be needed, suggesting $\Delta/\mu \sim 1$. This value is absurdly large in the conventional BCS context.[16]

The RVB model encompasses the class of models of a Mott-Hubbard system in which the elementary excitations of the spin-liquid ground state exhibit decoupling of the charge and spin degrees of freedom, although their excitation spectra and statistics remain controversial. One suggested mechanism for superconductivity is the pairing[9,10] of the charged-bosonic excitations (holons). The two models of holon pairing, that of Rice and Wang[9] (RW) and that of Wheatley, Hsu, and Anderson[10] (WHA), differ in their treatment of the interparticle repulsion. The RW model is the simpler of the two, and is used here. The screening length for $d=2$ is given by

$$\frac{2\epsilon_0 q_s}{e^2} = \frac{2\epsilon_0}{e^2 l_s} = \frac{1}{g_0}\left(\frac{\partial \bar{n}}{\partial \mu}\right)_\Delta , \quad (2)$$

where g_0 is the density of states. The mean holon areal density is[9,10]

$$\bar{n} = g_0 \int d\varepsilon_{\mathbf{k}} \langle n_{\mathbf{k}} \rangle \quad (3)$$

with occupation numbers

$$\langle n_{\mathbf{k}} \rangle = \frac{1}{2}\left(\frac{\varepsilon_{\mathbf{k}}-\mu}{E_{\mathbf{k}}}\coth(\beta E_{\mathbf{k}}/2) - 1\right) \quad (4)$$

where now $E_{\mathbf{k}}=[(\varepsilon_{\mathbf{k}}-\mu)^2-\Delta_{\mathbf{k}}^2]^{1/2}$. The energy $\varepsilon_{\mathbf{k}}$ contains a Hartree contribution from the holon-holon repulsion.[9] Differentiation yields the dimensionless inverse screening length

$$\frac{1}{g_0}\left(\frac{\partial \bar{n}}{\partial \mu}\right)_\Delta = \left(\frac{\varepsilon_0}{k_B T_0} + \frac{1}{\langle n_0 \rangle}\right)^{-1} , \quad (5)$$

where T_0 is the degeneracy temperature given by $g_0 k_B T_0 = \bar{n}$. The $\mathbf{k}=0$ value ε_0 of the unpaired holon energy is simply the Hartree energy, and the temperature dependence of (5) is determined by the $\mathbf{k}=0$ value $\langle n_0 \rangle$ of the holon occupation number (4). The corresponding result for the normal state is given by the same expressions with $\Delta=0$.

A decrease in $\langle n_0 \rangle$ is needed for stiffening of the charge response. This only occurs for strong coupling, when there is an energy gap ($E_0>0$ as $T \rightarrow 0$) for charged excitations. To illustrate the various possible temperature variations of the lifetime, Fig. 3 shows $l_s(T)^2/l_s(T_c)^2$ vs T/T_c for three sets of parameter values. For curve A, the values of the strength V of the pairing interaction, the cutoff frequency ω_0, and the Hartree-Fock energy ε_0 were chosen to give a 10% change in l_s^2, compatible with the observed change in τ if half the annihilations are attributed to CuO_2 plane "conduction" electrons. Curve B illustrates that the onset of lifetime lengthening need not occur at T_c. Curve C is for one set of the weak coupling parameter values used by Wang and Rice in their discussion[12] of the magnetic penetration depth, and illustrates the absence of lifetime lengthening in the weak coupling limit.

Although the values of curve A reproduce the observed overall change in τ below T_c, inconsistencies remain. The

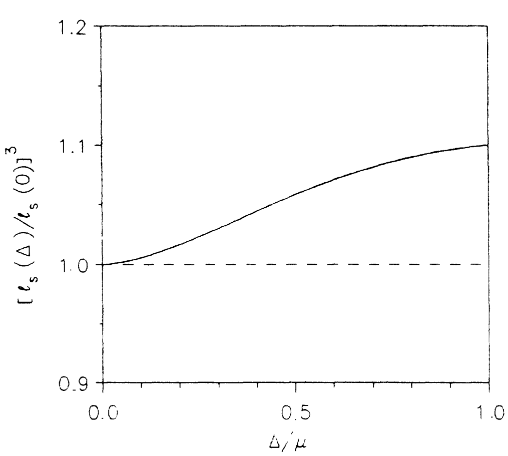

FIG. 2. The cube of the screening length of a BCS superconductor vs the ratio of the gap parameter to the Fermi energy. The positron lifetime is expected to be approximately proportional to l_s^3.

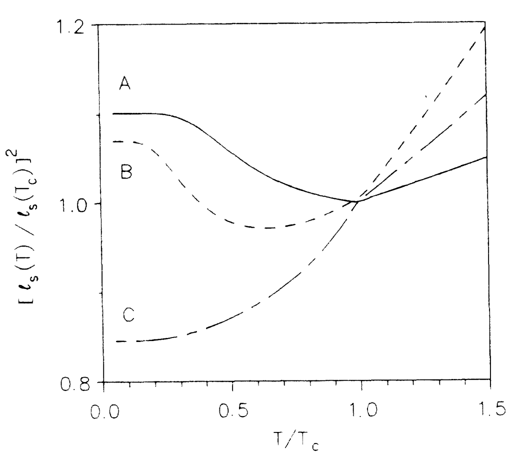

FIG. 3. The square of the screening length vs reduced temperature T/T_c for the Rice-Wang model of boson pairing. The parameters are, in multiples of $k_B T_0$, for curve A $g_0 V=5$, $\omega_0=10$, $\varepsilon_0=200$; for curve B $g_0 V=3$, $\omega_0=5$, $\varepsilon_0=20$; and for curve C $g_0 V=0.6$, $\omega_0=5$, $\varepsilon_0=10$. The positron lifetime is expected to be approximately proportional to l_{TF}^2.

$T=0$ "charge gap"[9,10] needed for l_s to increase as $T \to 0$ only appears for V greater than a threshold value in the RW model, but the T_c exceeds the degeneracy temperature T_0. Also, the T dependence above T_c is inconsistent with the observed T-independent lifetime in the normal state. However, in the WHA model a charge gap exists for all values of the coupling strength, and l_s appears to be only weakly temperature dependent in the normal state. Consequently, these inconsistencies may disappear with a more sophisticated treatment of the holon-holon repulsion.

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[15]For small values of Δ/μ, the leading order terms in the expansion of the fractional change in q_s^2 from its normal-state value are

$$\frac{(q_s^2)_S-(q_s^2)_N}{(q_s^2)_N}=\frac{1}{8}\left(\frac{\Delta}{\mu}\right)^2\left[\ln\left(\frac{\Delta}{\mu}\right)+C\right],$$

where $C=3(\sqrt{2}-1)-\ln[2(1+\sqrt{2})]\simeq-0.332$.

[16]A more complete discussion of positron-annihilation characteristics in BCS superconductors is given by H. B. Schüttler and R. Benedek, Phys. Rev. B (to be published).