Fractional quantum Hall effect in very-low-density $GaAs/Al_xGa_{1-x}As$ heterostructures

T. Sajoto, Y. W. Suen, L. W. Engel, M. B. Santos, and M. Shayegan
Department of Electrical Engineering, Princeton University, Princeton, New Jersey 08544-5263
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We report an experimental study of the quantum Hall effect in very-high-quality $GaAs/Al_xGa_{1-x}As$ heterostructures in an extremely-low-electron-density regime. At a temperature of ~28 mK, the fractional quantum Hall states at Landau-level filling factors $\nu=\frac{2}{3}$ and $\frac{1}{3}$ are observed at a density as low as 7.0×10^9 cm^{-2}, and the integral quantum Hall states at $\nu=2$ and 1 are observed at a density as low as 4.0×10^9 cm^{-2}. The possibilities of the even-denominator quantization in the lowest ($\nu<2$) and second ($2<\nu<4$) Landau levels in the very-low-density regime have also been investigated. Near the fractional filling factors $\nu=\frac{1}{2}$, $\frac{3}{2}$, $\frac{3}{4}$, and $\frac{5}{8}$, we observe weak inflection points in the Hall resistivity which lie on the classical Hall line, but we do not observe an activated temperature dependence for the diagonal resistivity. The previously reported even-denominator quantization at $\nu=\frac{5}{2}$ is observed at a density as low as 1.3×10^{11} cm^{-2}. We also report evidence for the development of new fractional quantum Hall states at $\nu=\frac{8}{11}$ and $\frac{8}{13}$. The $\nu=\frac{8}{13}$ state belongs to a new (inner) branch of higher-order hierarchical states emanating from the $\nu=\frac{2}{3}$ parent state.

I. INTRODUCTION

The discovery of the fractional quantum Hall effect[1] (FQHE) has generated a great deal of interest in the many-body physics of interacting electrons in the fractionally filled Landau levels. The FQHE is presently understood as being the consequence of the condensation of the two-dimensional electron system (2DES) into an incompressible quantum fluid state at special values of the Landau-level filling factor $\nu=p/q$ ($\nu=nh/eB$, with n being the areal density, eB/h the Landau-level degeneracy, and B the magnetic field).[2-4] Despite ample experimental evidence for the observation of exclusively odd-denominator FQHE states (q odd), the recent discovery[5] of the even-denominator FQHE state at filling factor $\nu=5/2$ has confirmed the long-standing conjectures that the FQHE is not a phenomenon exclusive to odd-denominator quantum numbers.[6] The restriction to odd-denominator fractions arises from the antisymmetry requirement for the wave functions describing the fully spin-polarized Fermi quantum-liquid states.[7] In addition, the spin Zeeman energy was previously believed to be so large that all fractional states could be safely assumed to be fully polarized. However, as first noted by Halperin,[8] the Zeeman gap for reversed-spin states may not be large enough, due to the small g factor in GaAs, to safely assume full polarization of the FQHE states at low magnetic fields. In fact, recent experimental evidence[9-16] and theoretical studies[17-27] seem to suggest that the spin degree of freedom may indeed play an important role in forming the condensed ground state as well as its quasiparticle excitations in the low-field regime. The formation of the spin-reversed pairs of electrons at low fields has been suggested as a mechanism for even-denominator quantization, although its stability has been questioned in the presence of a physically realistic Hamiltonian. Other exotic states have also been proposed for these even-denominator filling factors; they include a Wigner-crystal-like ground state,[28] charge density wave,[29] and the formation of electron clusters.[30] Despite mounting theoretical predictions for the interesting possibilities at even-denominator filling factors and different spin configurations for the FQHE ground states as well as the quasiparticle excitations at low fields, no experimental studies have been done, prior to this work, to probe the extremely-low-density regime.

In this paper, we present an experimental study of the integral quantum Hall effect (IQHE) and FQHE at low temperatures ($T\geq28$ mK) in the extremely-low-density regime ($4.0\times10^9\leq n\leq1.3\times10^{11}$ cm^{-2}), which is at least a factor of 3 lower than the previously reported studies.[5,9-16,31-43] The possibilities of the even-denominator quantization in the lowest ($\nu<2$) and second Landau levels ($2<\nu<4$) at low magnetic fields have also been investigated. For a sample with a density of 1.3×10^{11} cm^{-2}, the FQHE at $\nu=\frac{5}{2}$ is observed with a developing plateau in the Hall resistivity (ρ_{xy}) and accompanied by a relatively deep minimum in the diagonal resistivity (ρ_{xx}). This density is less than half of the density for which a $\nu=\frac{5}{2}$ state with comparable strength was originally reported.[5] Unlike the $\nu=\frac{5}{2}$ case, no quantization of the Hall plateau to the correct value ($h/\nu e^2$) is observed at $\nu=\frac{1}{2}$, $\frac{3}{2}$, $\frac{3}{4}$, and $\frac{5}{8}$ at a temperature as low as ~28 mK. However, weak inflection points in ρ_{xy} (minima in $d\rho_{xy}/dB$) which lie on the classical Hall line are observed near $\nu=\frac{1}{2}$, $\frac{3}{2}$, $\frac{3}{4}$, and $\frac{5}{8}$. The ρ_{xx} minima at these filling factors, however, do not show an activated temperature dependence. Although these observations do not prove the existence of FQHE states at $\nu=\frac{1}{2}$, $\frac{3}{2}$, $\frac{3}{4}$, and $\frac{5}{8}$, they may suggest the development of such even-denominator FQHE states or perhaps different many-particle states in very-high-quality 2DES. Presumably, future experiments on samples of higher quality will show whether the observed features are indeed indicative of FQHE states (weak inflection points in ρ_{xy} would eventually develop into quantized Hall plateaus and ρ_{xx} minima would vanish as temperature decreases, similar to the case of $\nu=\frac{5}{2}$ in the second Landau level[14]), or whether they are of fundamentally different origins. We also report the observation of new higher-order FQHE states at $\nu=\frac{8}{11}$ and $\frac{8}{13}$ which are accompanied by the corroborating ρ_{xy} plateau development.

II. SAMPLE STRUCTURES AND EXPERIMENTAL DETAILS

The data reported here are all obtained from very-high-quality 2DES at the interface of GaAs/$Al_xGa_{1-x}As$ heterostructures grown by molecular-beam epitaxy (MBE). Details of the MBE system, wafer preparation, growth procedures, and growth parameters were reported elsewhere.[34,38,44] As shown in Fig. 1, the structures of the three samples studied are similar. The two major differences among these three structures are the $Al_xGa_{1-x}As$ spacer layer thickness separating the 2DES from the first dopant (Si) layer and the density of this Si layer.

Contacts to the 2DES were made by alloying In in a hydrogen atmosphere at 400 °C for about 10 min. The low-temperature 2D carrier concentration was varied by either illuminating the sample with a red-light-emitting diode (LED) or applying a back-gate voltage in the dark. The diagonal and Hall resistivities were measured in a van der Pauw geometry with magnetic field perpendicular to the sample plane in a top-loading Oxford TLM-200 dilution refrigerator with a base temperature of ~24 mK. Magnetotransport measurements were typically performed using 3.7 Hz excitation with 10 nA current for ρ_{xx} and 1 nA current for ρ_{xy} without any observable electron heating. However, lower excitation current values of 1 and 0.25 nA were also used to routinely check and avoid the possibility of electron heating, especially for extremely low densities and high resistivities at very low temperatures. In general, if the ρ_{xx} value is not negligible compared to the ρ_{xy} value then the ρ_{xx} admixture contribution to the ρ_{xy} must be considered. For this reason, and also since the FQHE states are known to be very sensitive to the presence of inhomogeneity in the sample, the experimental observations were occasionally checked and found reproducible for different contact pairs, current directions, magnetic field directions, as well as upon subsequent thermal recycling, reillumination, and the application of a gate voltage.

Figure 2 shows the dependence of the low-temperature electron mobility (μ) on the 2DES carrier concentration for samples M73 and M97, as well as the variation of the 2DES density with the application of a back-gate voltage for M73. The dependence of μ on n for M73 (for $n \geq 1\times10^{10}$ cm^{-2}) indicates $\mu \sim n^{\alpha}$ behavior with $\alpha\approx0.6$.

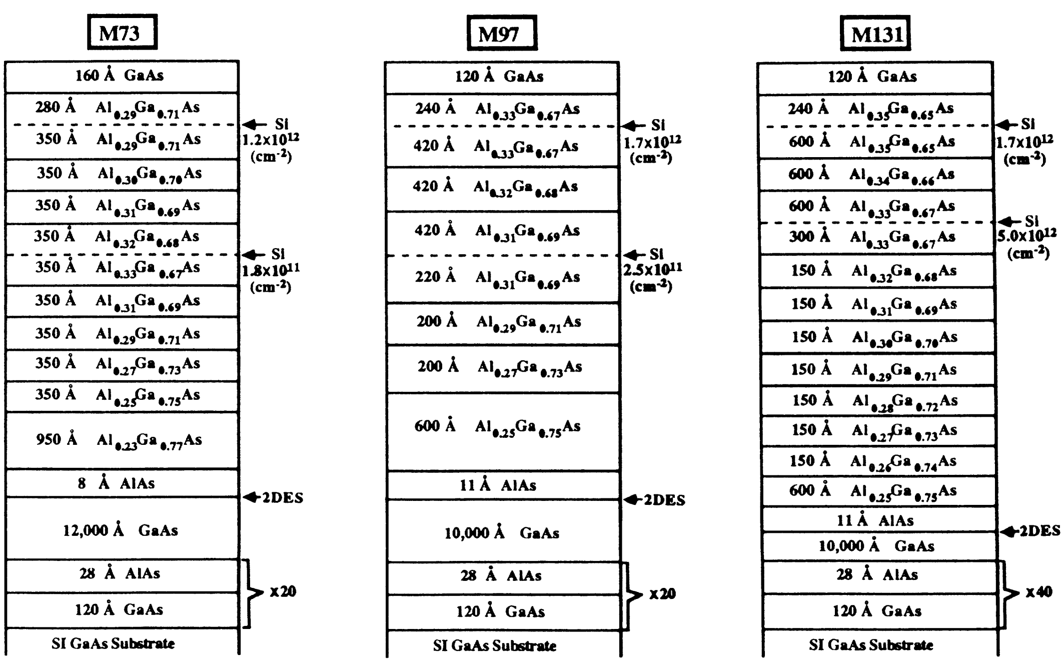

FIG. 1. Details of the structures of the samples used in this study are shown.

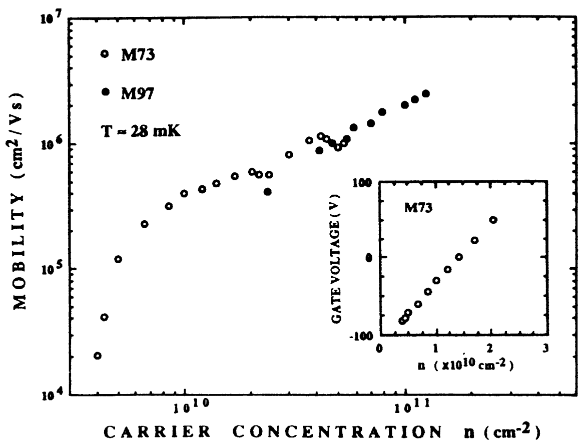

FIG. 2. The dependence of the low-temperature electron mobility on the 2DES carrier concentration is shown. The open circles correspond to the densities obtained by varying the back-gate voltage in the dark and by illuminating the sample with a red LED for sample M73. The solid circles correspond to the densities obtained in the dark and by illuminating the sample with a red LED for sample M97. The dependence of the 2DES carrier concentration on the back-gate voltage for sample M73 is shown in the inset.

A similar power-law dependence was reported previously[34,45] and was attributed to the dominant scattering by the residual impurities in the GaAs channel and the $Al_xGa_{1-x}As$ spacer rather than the screened remote-ionized impurity scattering. For M97, $\alpha \approx 1$ is observed; the difference may be attributed to the higher density range for sample M97, the use of a thinner spacer layer for M97 ($\sim$1200 Å) compared to M73 ($\sim$2700 Å), as well as the use of higher intentional Si doping level near the GaAs channel for M97. These observations are consistent with the results of a systematic investigation by Jiang, Tsui, and Weimann.[36] In Fig. 2, μ decreases drastically as n decreases below $\sim 1\times10^{10}$ cm^{-2}. In this extremely-low-density regime, the rapid drop in μ can no longer be explained by Stern's model[46] which takes into account various scattering mechanisms (Ref. 36). This rapid decrease in μ at low n was recently attributed to the strong influence of the density fluctuations at low n.[36,47]

III. IQHE AND FQHE IN THE VERY-LOW-ELECTRON-DENSITY REGIME

Figure 3 shows the magnetotransport coefficients ρ_{xx} and ρ_{xy} as a function of the magnetic field for sample M73. Despite the drastic decrease of the mobility at low n, it is quite remarkable that the IQHE states at $\nu=2$ and 1 are observed down to $n=4.0\times10^9$ cm^{-2} [the data for $n=4.3\times10^9$ cm^{-2} are shown in Fig. 3(a)]. Such a drastic decrease of the mobility in this extremely-low-density regime suggests a more severe localization due to the disorder at lower densities.[48] The importance of disorder is also evident from the fact that the observed ρ_{xx} background increases as density decreases or magnetic field increases, and more importantly, the ρ_{xx} minima for the IQHE states at $\nu=2$ and 1 do not go to zero for $n<5.0\times10^9$ cm^{-2}. The ρ_{xy} trace shown in Fig. 3(a) is the average of the two ρ_{xy} traces obtained from contact pairs (1-3) and (2-4) with the corresponding current of 1 nA passed through contact pairs (2-4) and (1-3), respectively. By averaging the two ρ_{xy} traces, the ρ_{xx} admixture is almost completely removed, and the ρ_{xy} plateaus for $\nu=2$ and 1 appear to be quantized at the correct values to better than 1%. We did not do such averaging for the rest of the traces in Fig. 3 (since the ρ_{xy} traces for different contact pairs or current directions were nearly identical) except for the ρ_{xy} data shown in Fig. 3(g) where the ρ_{xy} traces before the averaging start to deviate from the classical Hall line for $\nu \leq 1/5$.

In Figs. 3(b)–3(d), magnetotransport data at successively higher n are shown. The FQHE states at $\nu=\frac{2}{3}$ and $\frac{1}{3}$ are observed at a density as low as 7.0×10^9 cm^{-2}. At $\nu=\frac{2}{3}$ and $\frac{1}{3}$, the ρ_{xx} minima go to zero and the Hall plateaus are quantized at the correct values of $h/\nu e^2$ to better than 0.5% for a density as low as 1.4×10^{10} cm^{-2}. The ρ_{xx} minimum and the inflection point in ρ_{xy} for $\nu=\frac{4}{3}$ start to appear at a density of $n\sim1.0\times10^{10}$ cm^{-2}, and the ρ_{xy} plateau becomes fully quantized at a density of $n\sim2.5\times10^{10}$ cm^{-2}. In Fig. 3(c), well-resolved FQHE states at $\nu=\frac{2}{5}$ and $\frac{3}{5}$ can be seen at a density as low as 1.7×10^{10} cm^{-2}, and they disappear at densities below $\sim1.3\times10^{10}$ cm^{-2}. In Fig. 3(d), ρ_{xx} minima for FQHE states at $\nu=\frac{2}{7}, \frac{3}{7}, \frac{4}{9}, \frac{4}{7}, \frac{5}{3}$, and $\frac{8}{5}$ (Ref. 49) start to appear for a density as low as 2.2×10^{10} cm^{-2}.

The weakening of the FQHE states with decreasing density appears to be caused by the potential fluctuations and localization due to disorder rather than the low magnetic fields at which these FQHE states are observed. In the *absence of disorder,* the energy gap separating the $\nu=\frac{1}{3}$ and $\frac{2}{3}$ ground states from their quasiparticle excitations approximately scales with the magnetic field as $2\Delta_0=Ce^2/(4\pi\epsilon_0\epsilon l_0)$ where C is a constant of order 0.1 and $l_0=(h/2\pi eB)^{1/2}$ is the magnetic length.[7,50] Since the energy gap $2\Delta_0$ (in the absence of disorder) is large compared to the temperature of our measurements (28 mK) even at magnetic fields as low as 0.1 T, the FQHE should be observable down to very low magnetic fields. It has been experimentally observed, however, that disorder reduces the energy gap of the FQHE states,[1,33,35,48,50] although the dependence of the energy gap on the disorder is not known. Therefore, it is likely that disorder is responsible for the observed weakening of the $\nu=\frac{1}{3}$ and $\frac{2}{3}$ FQHE states with decreasing density, and the eventual collapse of these states for $n<7.0\times10^9$ cm^{-2}.

Figure 3(e) shows the magnetotransport data for a density of 5.0×10^{10} cm^{-2} which exhibit well-resolved and well-developed FQHE states over a wide range of filling factors.[51] The ρ_{xx} minima for the FQHE states at $\nu=\frac{2}{5}$, $\frac{3}{7}, \frac{4}{9}, \frac{4}{7}, \frac{3}{5}, \frac{2}{3}, \frac{5}{7}, \frac{9}{7}, \frac{4}{3}, \frac{7}{5}, \frac{8}{5}, \frac{5}{3}, \frac{7}{3}$, and $\frac{8}{3}$ are remarkably well developed at such low fields. At this density, the Hall plateaus for FQHE states at $\nu=\frac{2}{5}, \frac{3}{7}, \frac{4}{7}, \frac{3}{5}, \frac{2}{3}, \frac{4}{3}, \frac{7}{5}, \frac{8}{5}$, and $\frac{5}{3}$ are quantized at the correct values to better than 0.5%.

In Fig. 3(f), the ρ_{xx} data at 96 mK are shown up to 12 T for $n=5.0\times10^{10}$ cm^{-2}. We observe ρ_{xx} minima at $\nu=\frac{2}{7}$, $\frac{3}{11}$, $\frac{2}{9}$, $\frac{1}{5}$, and $\frac{2}{11}$. Note that the depth of the ρ_{xx} minimum at $\nu=\frac{1}{5}$ is only $\sim8\%$ of the background at $T\approx96$ mK.[52] Our preliminary measurements of the temperature dependence of the ρ_{xx} minimum at $\nu=\frac{1}{5}$ give an activation energy $\Delta\approx750$ mK which is roughly a factor of 2 higher than the previously reported value at about twice higher magnetic field.[35] Another strong indication for the low disorder in sample M73 is the observation of the delicate $\nu=\frac{1}{7}$ FQHE state shown in Fig. 3(g), which has only been observed in samples with extremely high quality. This observation of $\nu=\frac{1}{7}$ confirms the previously reported result by Goldman, Shayegan, and Tsui[37] on other samples from the same wafer, as well as a later result by Willett *et al.*[53] on a sample from an entirely different wafer.

Figure 4 shows the magnetotransport data for M97 with a density of 1.3×10^{11} cm^{-2}. Well-resolved ρ_{xx} minima at higher-order fractions $\nu=\frac{5}{11}$, $\frac{6}{11}$, $\frac{7}{13}$, and $\frac{13}{9}$ are observed in Fig. 4(a). In addition, a ρ_{xx} feature accompanied by the corroborating Hall plateau development

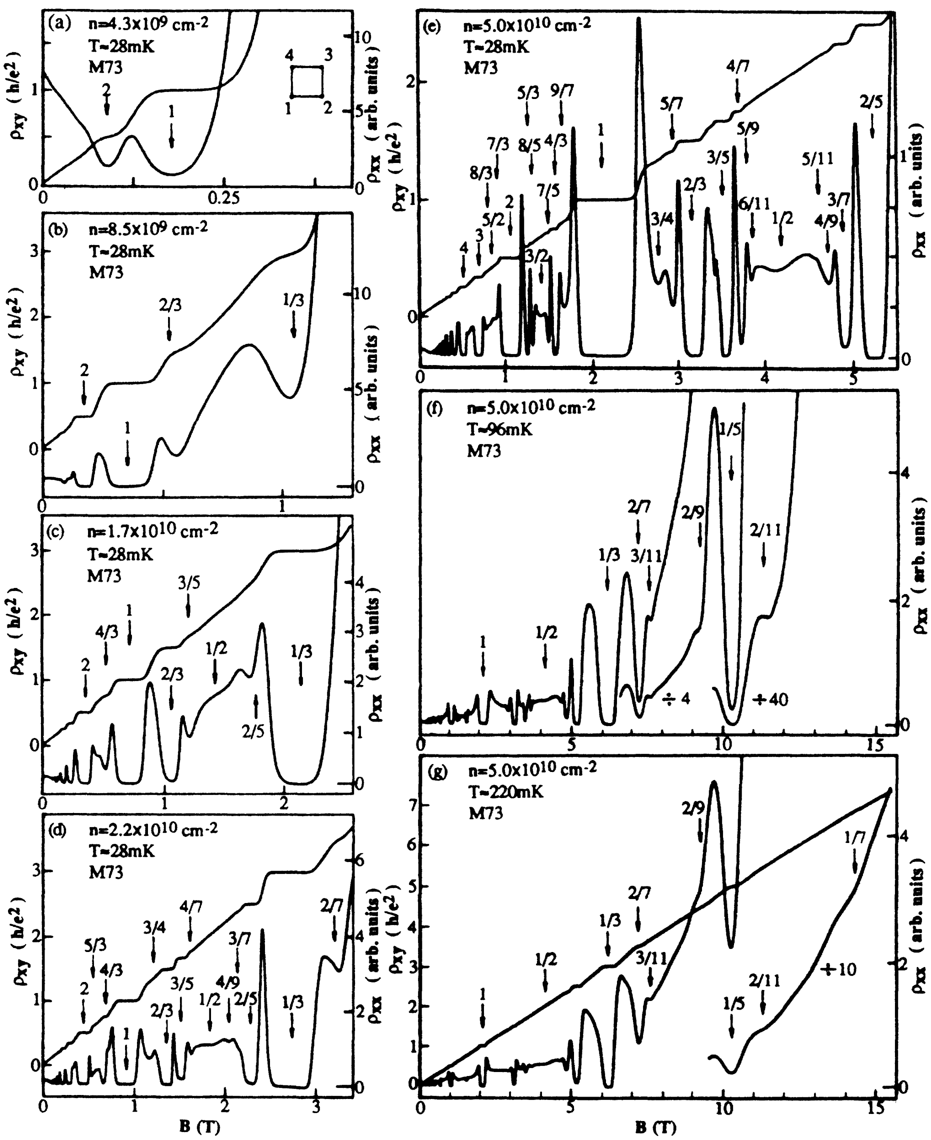

FIG. 3. The diagonal resistivity (ρ_{xx}) and Hall resistivity (ρ_{xy}) for sample M73 are shown for several different densities. The vertical arrows indicate the Landau-level filling factors (ν) at which the integral or fractional quantum Hall effect is observed. Also shown in Fig. 2(a) is the van der Pauw geometry used in these measurements.

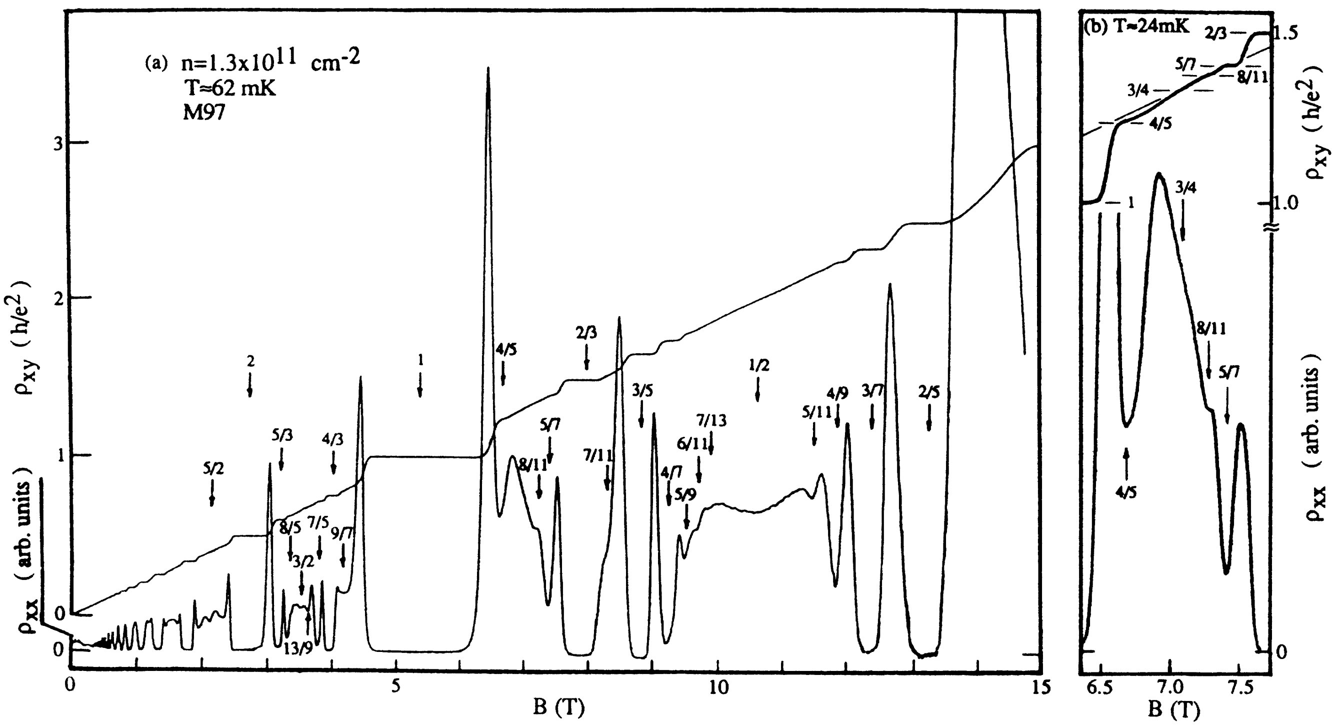

FIG. 4. The ρ_{xx} and ρ_{xy} data are shown for sample M97. (b) shows an enlargement of the data in the vicinity of $\nu=\frac{8}{11}$ at $T\approx 24$ mK.

for the higher-order FQHE state at $\nu=\frac{8}{11}$ is observed for the first time as shown in Fig. 4(b). This new FQHE state at $\nu=\frac{8}{11}$ is observed in the temperature range of $24\leq T\leq 125$ mK. The ρ_{xx} feature at $\nu=\frac{7}{11}$, which was previously reported by Goldman and co-workers[38,39,42] is also observed at 60 mK $\leq T\leq 0.4$ K. Below 60 mK, the ρ_{xx} feature at $\nu=\frac{7}{11}$ becomes weaker. The weakening of the ρ_{xx} feature at $\nu=\frac{7}{11}$ as temperature decreases may be attributed to the competition between the formation of the ρ_{xx} minimum at this filling factor and the rising of the adjacent flanks as well as the broadening of the adjacent ρ_{xx} minima.[54]

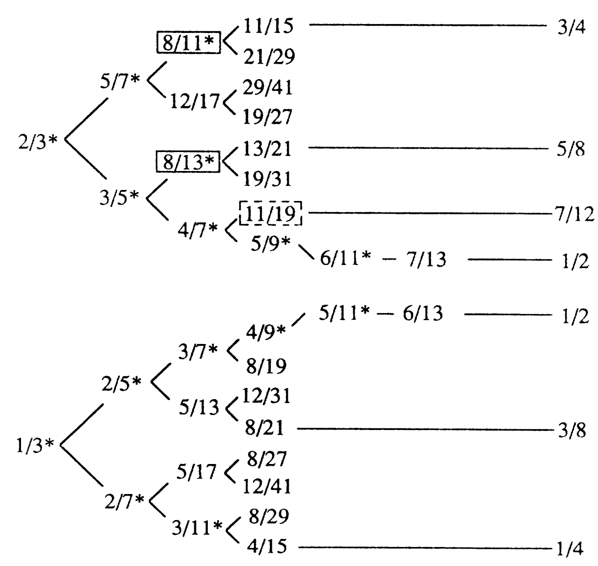

FIG. 5. The Haldane-Halperin-Laughlin spin-polarized hierarchical sequences of rational filling factors emanating from the primitive $\nu=\frac{1}{3}$ and $\frac{2}{3}$ parent states are shown. Two new higher-order FQHE states at $\nu=\frac{8}{11}$ and $\frac{8}{13}$ are observed for the first time. The FQHE states at filling factors with asterisks have been previously observed.

Based on the standard Haldane-Halperin-Laughlin spin-polarized hierarchical model,[55–57] higher-order FQHE states may occur at a sequence of rational filling factors given by

$$\nu=\cfrac{1}{q+\cfrac{\alpha_1}{s_1+\cfrac{\alpha_2}{s_2+\cdots}}}\,, \tag{1}$$

where $q=1,3,5,\ldots$, $s_i=2,4,6,\ldots$, and $\alpha_i=0$ or ± 1. In Fig. 5, the hierarchies originating from the $\frac{1}{3}$ and $\frac{2}{3}$ states [using $s_1=2$ and $\alpha_1\pm 1$ in Eq. (1)] are shown.[58] Within this hierarchical picture, the $\nu=\frac{8}{11}$ state is a daughter state of the $\nu=\frac{5}{7}$ parent state whose higher-order descendants along this branch converge towards the even-denominator filling factor $\nu=\frac{3}{4}$. The commonly observed broad ρ_{xx} minimum around $\nu=\frac{3}{4}$, however, is not observed at this density.

IV. MAGNETOTRANSPORT DATA NEAR $\nu=\frac{1}{2}$, $\frac{3}{2}$, AND $\frac{5}{2}$

We now focus on the behavior of ρ_{xx} and ρ_{xy} near the even-denominator fractional filling factors $\nu=\frac{1}{2}$, $\frac{3}{2}$, and $\frac{5}{2}$. In Fig. 6, the region in the vicinity of $\nu=\frac{1}{2}$ is expanded to show the magnetotransport coefficients more clearly.[51] The derivative of the Hall resistance with respect to the magnetic field ($d\rho_{xy}/dB$), which has a similar characteristic to ρ_{xx} as noted previously by Chang and Tsui,[59] is also shown in Fig. 6. Despite the noise caused by the numerical differentiation, it is clear from the derivative data that for $n=1.7\times10^{10}$ cm^{-2}, there is an inflection point in ρ_{xy} near $\nu=\frac{1}{2}$. For sample M73, the weak inflection point in ρ_{xy} disappears for densities lower than 1.4×10^{10} cm^{-2} where the broad basin in ρ_{xx} near $\nu=\frac{1}{2}$ is barely discernible. For higher densities, the inflection point appears to become weaker compared with the neighboring higher-order odd-denominator FQHE states such as $\frac{2}{5}$, $\frac{3}{5}$, etc. [Fig. 6(b)]. The positions (in B) of the minima in ρ_{xx} and $d\rho_{xy}/dB$ are both shifted by a few percent from the expected position of $\nu=\frac{1}{2}$. It is important to mention that the weak inflection point near $\nu=\frac{1}{2}$ always appears regardless of how the density is obtained and measured in sample M73 (dark values, illuminating the sample with a red LED, applying different back-gate voltages, different contact pairs, and different current directions), as well as upon subsequent thermal recycling and reillumination of the sample. Therefore, it is unlikely that this inflection point is due to the inhomogeneity in the sample. Since the ρ_{xx} value is not negligible compared to the ρ_{xy} value, however, the ρ_{xx} admixture contribution to the ρ_{xy} must be considered. This contribution, which increases as density or temperature decreases due to the increase of the ρ_{xx} background, can be estimated from the magnitude of the ρ_{xx} near the $\nu=\frac{1}{2}$. For the densities reported here, $\rho_{xx}\approx1800$ and 7900 $\Omega/\square$ for $n=5.0\times10^{10}$ cm^{-2} and 1.7×10^{10} cm^{-2}, respectively, at $T\approx28$ mK. Based on our estimate of the admixture contribution, the inflection point in ρ_{xy} near $\nu=\frac{1}{2}$ does not appear to be the consequence of the ρ_{xx} admixture; however, we cannot entirely rule out this possibility. Finally, we do not observe an activated temperature dependence for the minimum in either ρ_{xx} or $d\rho_{xy}/dB$ at $\nu=\frac{1}{2}$ for $T\geq28$ mK, but the minimum in $d\rho_{xy}/dB$ near $\nu=\frac{1}{2}$ appears to become slightly stronger with lower temperatures ($T\leq180$ mK).

In Fig. 7, the region in the vicinity of $\nu=\frac{3}{2}$ is expanded to show the magnetotransport coefficients more clearly for $n=5.0\times10^{10}$ cm^{-2}.[51] The derivative of the Hall resistance with respect to the magnetic field ($d\rho_{xy}/dB$) shows a weak inflection point in ρ_{xy} near $\nu=\frac{3}{2}$. The development of the ρ_{xx} minimum near $\nu=\frac{3}{2}$ is also similar to $\nu=\frac{1}{2}$ and does not show an activated temperature dependence for $T\geq28$ mK.

In general, the data reported in literature for most high quality 2DES's show broad ρ_{xx} minima around $\nu=\frac{1}{2}$ and $\frac{3}{2}$ similar to the data shown in Figs. 3, 4, 6, and 7. In a few samples, however, the minima in ρ_{xx} around these filling factors are fairly deep and persist to higher temperatures (e.g., see Fig. 3 in Ref. 38). Recently, Jiang *et al.*[43] reported the observation of relatively sharp cusps at $\nu=\frac{1}{2}$ and $\frac{3}{2}$ in the deep minima around these filling factors. A similar observation was also previously made by Gold-

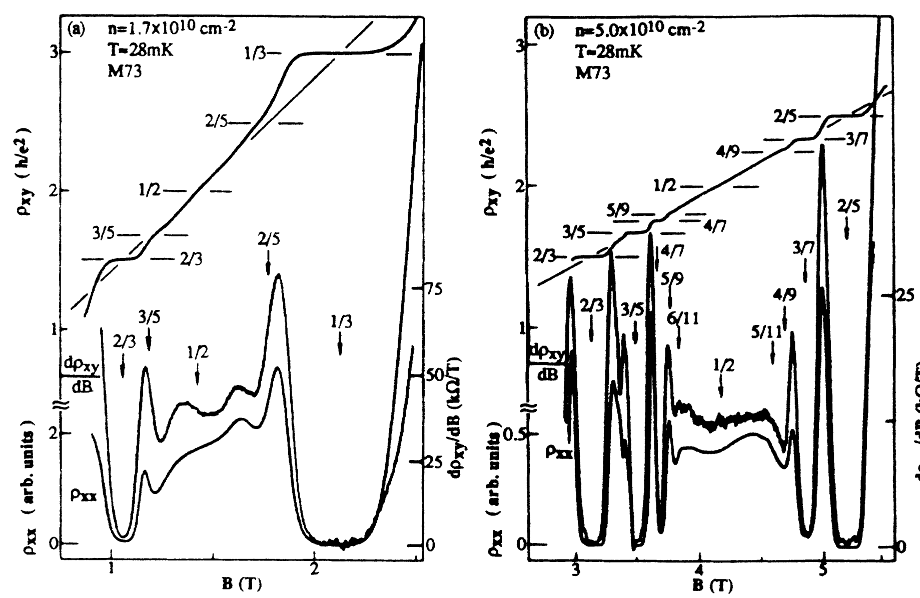

FIG. 6. The ρ_{xy}, its derivative with respect to the magnetic field ($d\rho_{xy}/dB$), and ρ_{xx} data are shown in detail in the vicinity of $\nu=\frac{1}{2}$ for sample M73. The vertical arrows correspond to the filling factors and the horizontal lines correspond to the quantum number p/q.

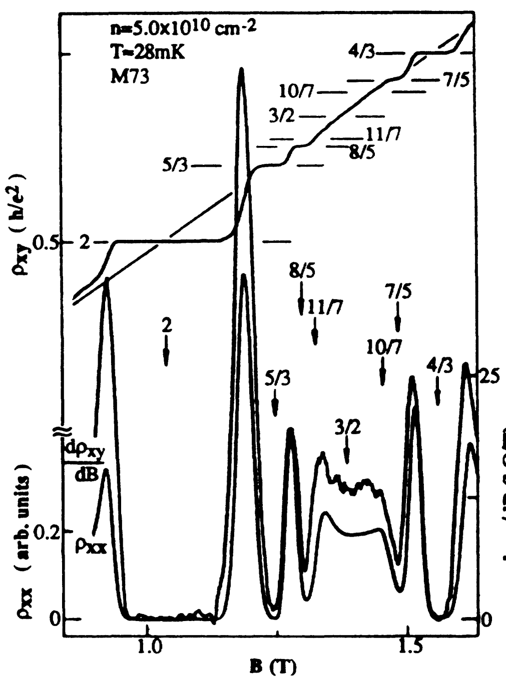

FIG. 7. The ρ_{xy}, $d\rho_{xy}/dB$, and ρ_{xx} data are shown in detail in the vicinity of $\nu=\frac{3}{2}$ for sample M73.

man *et al.*[60] Figure 8 shows a similar behavior for sample M131. As also noted by Jiang *et al.*,[43] this ρ_{xx} feature is very peculiar in that the deep minima around $\nu=\frac{1}{2}$ and $\frac{3}{2}$ persist up to temperatures of several degrees Kelvin (Fig. 8), whereas the relatively sharp cusps at these filling factors do not become stronger for $T \leq 0.6$ K (Figs. 8 and 9). It is important to mention that although the deep ρ_{xx} minima and the cusps at $\nu=\frac{1}{2}$ and $\frac{3}{2}$ are reproducible in a given sample, not all the samples from the same wafer show this behavior (some samples show the commonly observed broad ρ_{xx} minima around $\nu=\frac{1}{2}$ and $\frac{3}{2}$). The different behavior for samples from the same wafer has been seen so far in three wafers with different structures and quality.[60] We do not know the origin of this different behavior.

We have also investigated the behavior of ρ_{xy} near $\nu=\frac{1}{2}$ and $\frac{3}{2}$ in sample No. 3. Unlike the results of Jiang *et al.*[43] where no ρ_{xy} plateau development was observed down to $T=90$ mK for a density of $n=1.7\times10^{11}$ cm^{-2}, we observe weak inflection points in ρ_{xy} at $\nu=\frac{1}{2}$ and $\frac{3}{2}$ for the studied density range ($6.9\times10^{10}\leq n \leq 8.6\times10^{10}$ cm^{-2}). These inflection points are reproducible and become slightly stronger as temperature decreases as shown in Fig. 9. The trace shown in Fig. 9(a) is the average of two ρ_{xy} scans measured with opposite magnetic field directions (both scans show nearly identical inflection points). It is important to note that the ρ_{xx} value near $\nu=\frac{1}{2}$ is nearly temperature independent for $T\leq 0.6$ K, whereas the ρ_{xy} inflection point becomes stronger with decreasing temperature.

The ρ_{xx} minima at $\nu=\frac{1}{2}$ and $\frac{3}{2}$ do not show an activated temperature dependence, regardless of the shape of the ρ_{xx} features around these filling factors. Also, an inflection point in ρ_{xy} is observed for both samples (M73 and M131), although the general shape of the ρ_{xx} features are different. It is not clear at this point whether this difference is related to the difference in sample quality and/or homogeneity. Nor is it obvious whether the observed inflection points in ρ_{xy} at $\nu=\frac{1}{2}$ and $\frac{3}{2}$ will develop into FQHE states (i.e., fully quantized Hall plateaus accompanied by vanishing ρ_{xx} as $T\rightarrow0$) with the improvement of sample quality. On the theoretical side, many interesting possibilities have been suggested for $\nu=\frac{1}{2}$ such as the formation of a spin-singlet FQHE state,[18,21] Wigner-crystal-like ground state,[28] charge density wave,[29] and the formation of electron clusters.[30] The results of the finite-size numerical calculations for the half-filling even-denominator states, however, were found to depend on the model and the number of particles used in the calculation. In the absence of a realistic model which takes

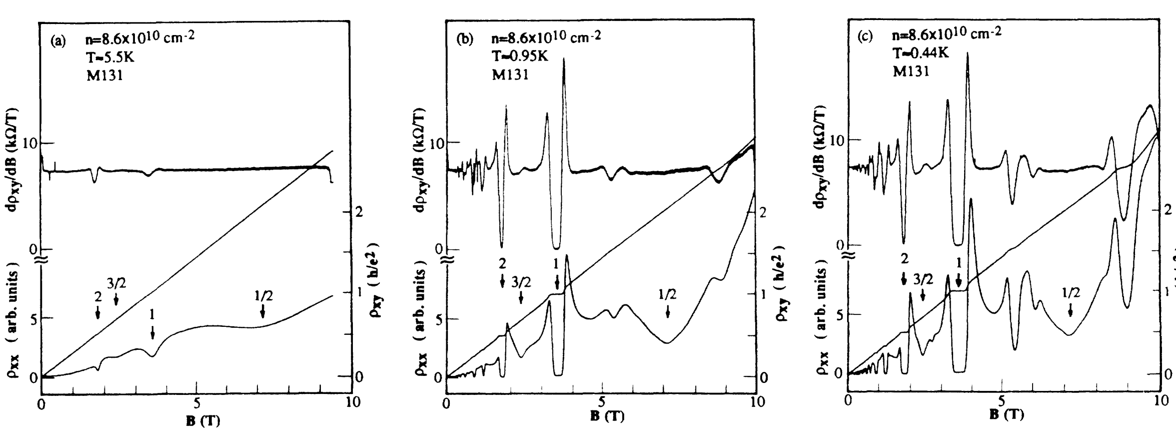

FIG. 8. High-temperature behavior of the ρ_{xx} and ρ_{xy} data near $\nu=\frac{1}{2}$ and $\frac{3}{2}$ is shown for sample M131. At this density, the low-temperature mobility μ is $\sim 1\times10^6$ cm^2/V s.

into account the presence of the disorder in a real sample, these theoretical results cannot be expected to give a correct and complete picture and more experimental studies are certainly needed to shed some light on this interesting but puzzling behavior.

Figure 10 shows the even-denominator FQHE state at $\nu=\frac{5}{2}$ at a magnetic field as low as $2.1T$. The strength of the $\nu=\frac{5}{2}$ ρ_{xx} minimum and the quantization of the ρ_{xy} plateau is similar to the previously reported result by Willett *et al.*[5] obtained from a sample with nearly twice higher density. Also shown in Fig. 10 is the temperature dependence of the ρ_{xx} minimum at $\nu=\frac{5}{2}$ for $T\leq 125$ mK. The ρ_{xx} minimum at $\nu=\frac{5}{2}$ does not show an activated temperature dependence for $T\geq 24$ mK. This behavior, which was also observed by Willett *et al.*, was attributed to the competition between the formation of the ρ_{xx}

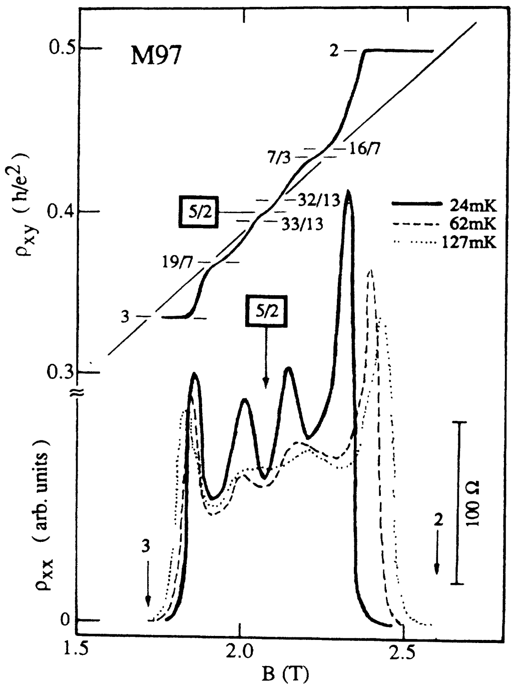

FIG. 10. The ρ_{xx} and ρ_{xy} data are shown in detail in the vicinity of $\nu=\frac{5}{2}$ for sample M97. The temperature dependence of ρ_{xx} is also shown.

minimum and the rising of the adjacent flanks as temperature decreases.[5] Recent magnetotransport data of Eisenstein *et al.*[14] on a higher quality sample with $n=2.3\times10^{11}$ cm^{-2} show a typical activated temperature dependence for the ρ_{xx} minimum at $\nu=\frac{5}{2}$, and $\rho_{xx}\rightarrow 0$ as $T\rightarrow 0$ rather than remaining roughly fixed as found earlier. This suggests that the development of the even-denominator FQHE state at $\nu=\frac{5}{2}$ improves with better sample quality. It remains to be seen whether the behavior at $\nu=\frac{1}{2}$ and $\frac{3}{2}$ in the lowest Landau level will follow a similar trend with further improvement of the sample quality.

V. MAGNETOTRANSPORT DATA NEAR $\nu=\frac{3}{4}$, $\frac{5}{8}$ AND $\frac{7}{12}$

For sample M73, the behavior of the magnetotransport coefficients near $\nu=\frac{3}{4}$ is qualitatively similar to what we observe for $\nu=\frac{1}{2}$ and $\frac{3}{2}$.[61] For $2.2\times10^{10}\leq n\leq 5.0\times10^{10}$ cm^{-2}, we observe an inflection point in ρ_{xy} (minimum in $d\rho_{xy}/dB$) which becomes stronger with decreasing temperatures (Fig. 11).[51] The increase of the strength does not appear to be caused by the ρ_{xx} admixture since the ρ_{xx} minimum at $\nu=\frac{3}{4}$ stays roughly fixed whereas the inflection point in ρ_{xy} clearly becomes stronger as temperature decreases. Similar to the case of $\nu=\frac{1}{2}$ and $\frac{3}{2}$, no temperature activated behavior is observed for the broad ρ_{xx} minimum near $\nu=\frac{3}{4}$. In addition, the minima in both ρ_{xx} and $d\rho_{xy}/dB$ are shifted to lower fields towards $\nu=\frac{4}{5}$ (see Fig. 11). We do not have

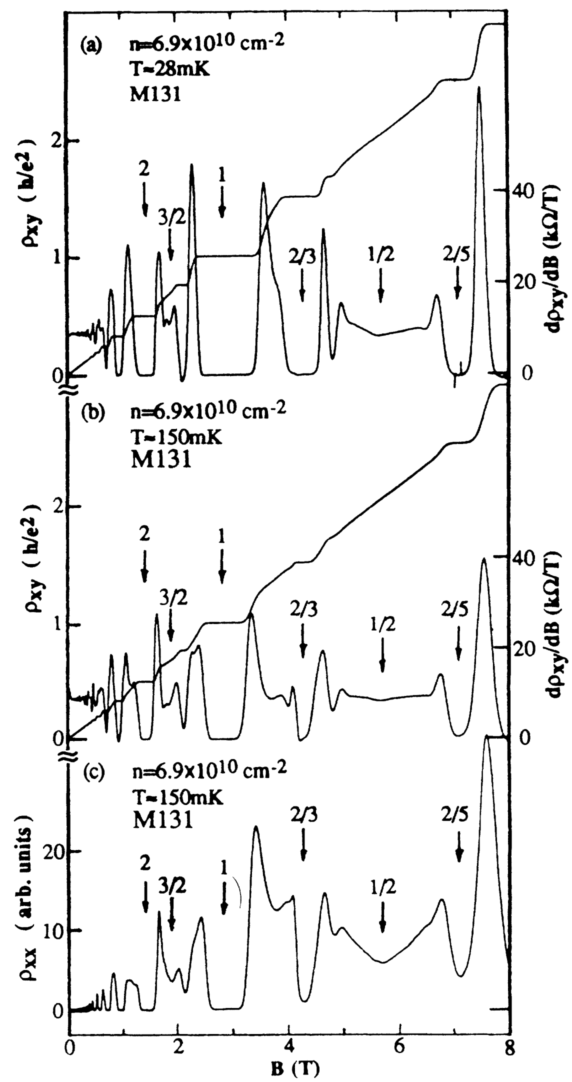

FIG. 9. Low-temperature behavior of the ρ_{xx}, ρ_{xy} and $d\rho_{xy}/dB$ is shown for sample M131. At this density, the low-temperature mobility μ is $\sim 0.9\times10^6$ cm^2/V s.

an explanation for this observation except that the shift may be due to a competition between the FQHE state at $\nu=\frac{4}{5}$ and a possible state at $\nu=\frac{3}{4}$.

The experimentally observed low-temperature ($T \le 100$ mK) behavior of ρ_{xx} and ρ_{xy} near filling factors $\nu=\frac{3}{4}$, $\frac{4}{5}$, and $\frac{5}{7}$ (between $\nu=1$ and $\frac{2}{3}$) as a function of density in the *state of the art* samples[62] reported by various groups is noteworthy. At high densities ($n \sim 3.0\times10^{11}$ cm^{-2}), sharp ρ_{xx} minima and fairly-well-developed Hall plateaus have been reported for $\nu=\frac{4}{5}$ and $\frac{5}{7}$.[33] The $\nu=\frac{4}{5}$ state is observed at lower densities down to 9.0×10^{10} cm^{-2},[41] but it appears to become weaker with decreasing density,[63] and eventually seems to disappear at very low densities (Fig. 11). The $\nu=\frac{5}{7}$ state, on the other hand, appears to be absent in some intermediate density range of $1.7\times10^{11} \le n \le 2.1\times10^{11}$ cm^{-2},[64] but reemerges in the range of $5.0\times10^{10} \le n \le 1.3\times10^{11}$ cm^{-2},[65] before it disappears at very low densities [Fig. 11(a)]. The disappearance and emergence of the ρ_{xx} minimum as a function of density are also observed near filling factor $\nu=\frac{3}{4}$. In a high density range of $1.7\times10^{11} \le n \le 3.2\times10^{11}$ cm^{-2},[66] a relatively broad ρ_{xx} minimum is observed near $\nu=\frac{3}{4}$. In the intermediate density range of $9.0\times10^{10} \le n \le 1.5\times10^{11}$ cm^{-2},[67] the ρ_{xx} minimum disappears, while at yet lower densities[68] it reappears (as stated earlier, however, the position of this minimum is shifted towards $\nu=\frac{4}{5}$). At very low densities (below $\sim 2.0\times10^{10}$ cm^{-2}), no features are observed for $\frac{2}{3} < \nu < 1$ [see Fig. 3(c)].

The disappearance of the ρ_{xx} and ρ_{xy} features for $\frac{2}{3} < \nu < 1$ in the very-low-density regime may be attributed to the increasing importance of the potential fluctuations due to disorder with decreasing density. At higher densities, ρ_{xx} minima and ρ_{xy} plateaus start to emerge. The FQHE state at $\nu=\frac{5}{7}$ appears to be stable at low fields, unstable at intermediate fields (corresponding to $1.7\times10^{11} \le n \le 2.1\times10^{11}$ cm^{-2}), and stable again at very high fields. This observation may suggest the possibility of a phase transition from an unpolarized to a polarized ground state (similar to the recent experimental observations by Eisenstein *et al.*[11,15] for $\nu=\frac{8}{5}$ and Clark and coworkers[10,16] for $\nu=\frac{4}{3}$). Moreover, it may hint at the existence of a magnetic field region where the energy gap vanishes (similar to the recent theoretical results of Chakraborty[17] for $\nu=\frac{2}{3}$). We emphasize, however, that the experimental observations summarized in the preceding paragraph were made on different samples with different densities and qualities which have different correlation energies. Therefore, no definitive conclusions regarding the importance of the spin contribution can be made without doing an *in situ* tilted-field experiment. The same comment applies to the observations summarized for the $\nu=\frac{3}{4}$ filling factor, i.e., it is not clear whether the disappearance and emergence of the ρ_{xx} and ρ_{xy} features near $\nu=\frac{3}{4}$ at different magnetic field regions are caused by the difference in sample quality (disorder and homogeneity) or whether they are related to the intrinsic nature of the 2DES at this filling factor. More experimental data on high-quality samples and more theoretical work are certainly needed to identify the exact origin of the interesting behavior in the filling factor range of $\frac{2}{3} < \nu < 1$.

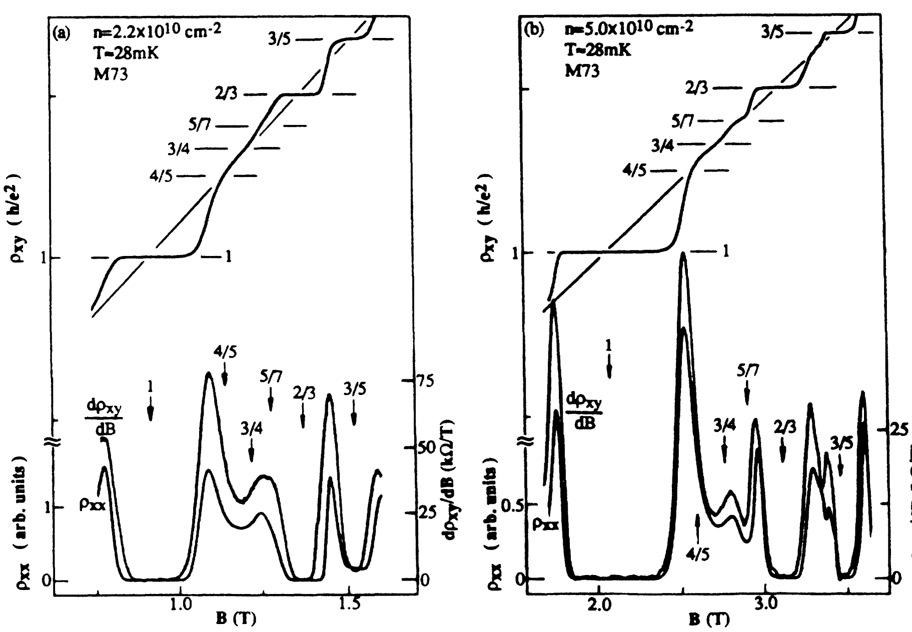

FIG. 11. The ρ_{xy}, $d\rho_{xy}/dB$, and ρ_{xx} data are shown in detail in the vicinity of $\nu=\frac{3}{4}$ for sample M73.

In Fig. 12, the region between $\nu=\frac{2}{3}$ and $\frac{3}{5}$ is expanded.[51] A ρ_{xx} minimum accompanied by a well-developed Hall plateau for a higher-order FQHE state at $\nu=\frac{8}{13}$ is observed for the first time. According to the hierarchical model of Eq. (1), the $\frac{8}{13}$ state originates from the $\frac{2}{3}$ state, but belongs to a new branch of the hierarchy (see Fig. 5). This is the first time that a FQHE state which does not belong to the outer two branches of the hierarchies originating from the $\frac{1}{3}$ and $\frac{2}{3}$ states (Fig. 5) is observed.[69] The inner branches evidently include states which are very delicate because of the following reasons. First, note that the branch starting with $\frac{8}{13}$ moves back towards $\frac{2}{3}$, (i.e., $\frac{3}{5}<\frac{8}{13}<\frac{2}{3}$). The $\frac{8}{13}$ state therefore falls in between the very strong $\frac{2}{3}$ and $\frac{3}{5}$ states. Second, the activation energy for the $\frac{8}{13}$ state is quite small because of its large denominator[70] (note that the denominator of the $\frac{8}{13}$ state is nearly twice as large as the denominator of the $\frac{4}{7}$ state which is the other daughter state of $\frac{3}{5}$). The observation of this new branch of more delicate higher-order states, therefore, further attests to the exceptional quality of sample M73.

Unlike the descendants of the $\frac{4}{7}$ state (along the outer branch of the hierarchy in Fig. 5) which converge towards $\frac{1}{2}$, the descendants of the $\frac{8}{13}$ state (i.e., $\frac{13}{21}$, etc.) converge towards $\frac{5}{8}$. At $T=180$ mK, we observe a broad basin in ρ_{xx} and a very weak inflection point in ρ_{xy} near $\nu=\frac{5}{8}$ [Fig. 12(a)]. This inflection point becomes stronger at lower temperatures, but the ρ_{xx} minimum does not. This behavior is again similar to what we observe for $\nu=\frac{1}{2}$, $\frac{3}{2}$, and $\frac{3}{4}$. It is important to note that the inflection point near $\nu=\frac{5}{8}$ seems to be well clear of the nearest odd-denominator filling factors $\nu=\frac{13}{21}$, $\frac{12}{19}$, and $\frac{7}{11}$.

Another interesting feature is observed between $\nu=\frac{3}{5}$ and $\nu=\frac{4}{7}$ as shown in Fig. 12. On the right shoulder of the ρ_{xx} near $\nu=\frac{3}{5}$, a delicate ρ_{xx} feature is observed near filling factors $\nu=\frac{7}{12}$ and $\frac{11}{19}$. Based on the hierarchical model of Eq. (1), the $\nu=\frac{11}{19}$ state is a higher-order daughter state of the $\nu=\frac{4}{7}$ state. The higher-order descendants of the $\nu=\frac{4}{7}$ state along this inner branch of hierarchy converge towards the even-denominator filling factor $\nu=\frac{7}{12}$ (see Fig. 5). Although an inflection point in ρ_{xy} is observed in the derivative data $(d\rho_{xy}/dB)$ near these two filling factors for $T\leq 60$ mK, this inflection point does not lie on the classical Hall line and its position in ρ_{xy} appears to be well below the correct value of $h/\nu e^2$ for both filling factors. From Figs. 12(b) and 12(c), the position of the inflection point in the ρ_{xy} data appears to move towards the $\nu=\frac{3}{5}$ Hall plateau with decreasing temperature. The absence of the correct quantized Hall plateau precludes any definitive conclusion regarding the nature of the ground state near these two filling factors. However, considering the fact that this new branch of more delicate higher-order descendants falls in between two strong FQHE states at $\nu=\frac{3}{5}$ and $\frac{4}{7}$, it is possible that the broadening of the ρ_{xy} plateau at $\nu=\frac{3}{5}$ forces the inflection point to move below the classical Hall line.[54]

On the left shoulder of ρ_{xx} near $\nu=\frac{2}{3}$, a weak ρ_{xx} feature is observed at filling factor $\nu=\frac{9}{13}$ [Figs. 12(b) and 12(c)]. A similar observation was previously reported by Goldman and co-workers.[39,42] An inflection point in ρ_{xy} at $\nu=\frac{9}{13}$ is clearly observed in the derivative data $(d\rho_{xy}/dB)$ at $T=60$ mK [Fig. 12(b)]. This inflection point, however, does not lie on the classical Hall line and its position in ρ_{xy} appears to be well above the correct

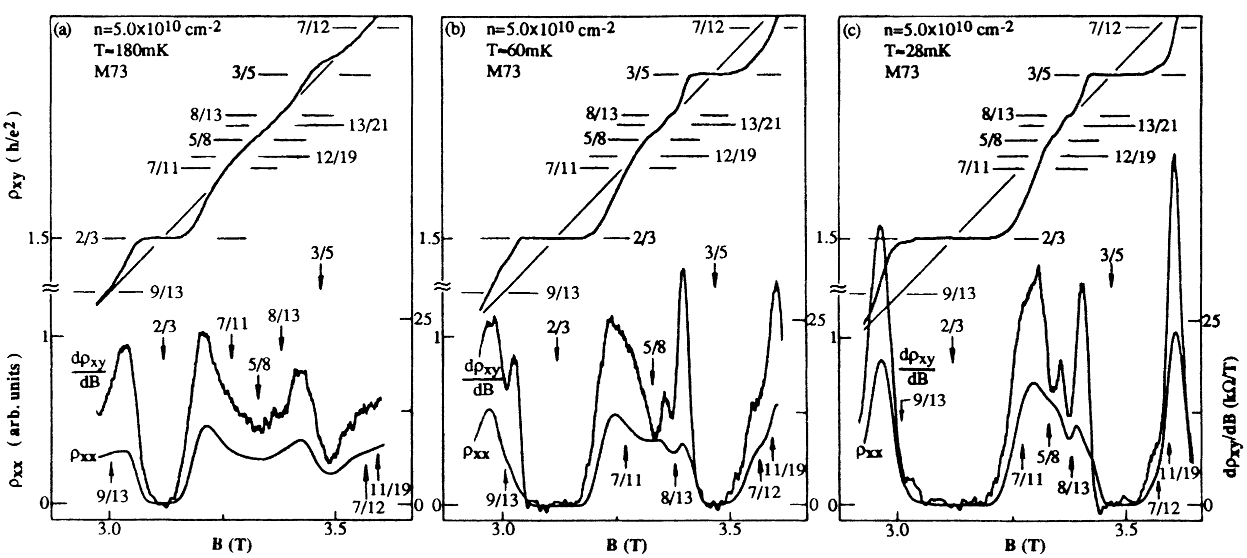

FIG. 12. The ρ_{xy}, $d\rho_{xy}/dB$, and ρ_{xx} data are shown in detail in the vicinity of $\nu=\frac{5}{8}$ for sample M73.

value of $13h/9e^2$. From Figs. 12(b) and 12(c), the position of the inflection point in the ρ_{xy} data appears to move towards the $\nu=\frac{2}{3}$ plateau with decreasing temperature. This behavior could perhaps be attributed to the broadening of the ρ_{xy} plateau at $\nu=\frac{2}{3}$ which forces the inflection point at $\nu=\frac{9}{13}$ to move above the classical Hall line.[54] The absence of the correct quantized Hall plateau again precludes any definitive statement concerning the nature of the ground state at $\nu=\frac{9}{13}$.

VI. SUMMARY

In summary, we have studied the IQHE and FQHE in the extremely-low-density regime from 4.0×10^9 cm^{-2} to 1.3×10^{11} cm^{-2}. The IQHE states at $\nu=2$ and 1 are observed at a density as low as 4.0×10^9 cm^{-2}, whereas the FQHE states at $\nu=\frac{2}{3}$ and $\frac{1}{3}$ are observed at a density as low as 7.0×10^9 cm^{-2}. Two new higher-order odd-denominator FQHE states at $\nu=\frac{8}{11}$ and $\frac{8}{13}$ are observed for the first time. For $\nu=\frac{1}{2}$, $\frac{3}{2}$, $\frac{3}{4}$, and $\frac{5}{8}$, weak inflection points in ρ_{xy} are observed. The strength of these inflection points increases slightly as temperature decreases, but we do not observe temperature-activated ρ_{xx} minima at these filling factors. The even-denominator quantization at $\nu=\frac{5}{2}$ is observed at a density as low as 1.3×10^{11} cm^{-2}. It is not clear at this point whether the ρ_{xx} minima at $\nu=\frac{1}{2}$, $\frac{3}{2}$, $\frac{3}{4}$, and $\frac{5}{8}$ will approach zero and the ρ_{xy} inflection points will develop into fully quantized plateaus as $T\rightarrow0$ with the improvement of sample quality, as is the case for $\nu=\frac{5}{2}$. More experimental and theoretical studies are needed to elucidate the exact nature of the ground states at these even-denominator fractional filling factors, and to determine whether they share a common origin which seems to be the case for the odd-denominator FQHE states.

Note added in proof. Recently, the fabrication[45,71] of 2DES with mobilities of the order of 10^7 $cm^2/V\,s$ and magnetotransport data[15,16,41,43,72] in such samples have been reported. We note here that the quality of our samples, deduced from the very narrow width of the IQHE and FQHE plateaus and the strength of the observed higher-order FQHE states, seems to be at least as high as that of samples (with similar density) which have much higher reported low-field mobility.[73] This observation implies that, in the case of very-high-quality 2DES, there are other as yet unknown factors, besides the low-field mobility, which determine the quality and suitability of the samples for FQHE experiments.

ACKNOWLEDGMENTS

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[65]Figures 4 and 11(b)–11(d) and Refs. 39 and 41–43.
[66]Refs. 31–33, 40, and 43.
[67]Figure 4, and Refs. 32, 40, and 41.
[68]Figure 11, and Refs. 39 and 42.
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