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Fluctuation conductivity and normal resistivity in $YBa_2Cu_3O_y$

Makoto Hikita and Minoru Suzuki

Nippon Telegraph and Telephone Corporation, Opto-Electronics Laboratories, Tokai, Ibaraki 319-11, Japan

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Magnetic-field-dependent fluctuation conductivity in the *a-b* plane of single-crystal $YBa_2Cu_3O_y$ is evaluated using recent theories for layered superconductors considering both Aslamozov-Larkin and Maki-Thompson terms. The normal resistivity, without fluctuation conductivity effects, is also estimated in the temperature region above T_c using the parameters obtained from the analysis of the field-dependent fluctuation conductivity. The result is substantially different from previously estimated values that are linearly extrapolated from high-temperature data.

Because of the intrinsically short coherence lengths of high-T_c superconductors, the thermodynamic fluctuation of the order parameter observably manifests itself on the excess conductivity near T_c, as reflected by the rounding of resistivity above T_c. It is well known that fluctuation conductivity is the sum of two parts which were independently proposed by Aslamazov and Larkin[1] (AL) and by Maki[2] and Thompson[2] (MT). In an earlier stage, Freitas, Tsuei, and Plaskett[3] have evaluated excess conductivity of sintered $YBa_2Cu_3O_y$ samples above T_c, and found that its temperature dependence behaves like that of the three-dimensional (3D) AL model. Subsequent experimental studies on temperature-dependent excess conductivity, however, produced various conclusions involving the power-law functional forms for 3D (Ref. 4) and 2D (Refs. 5 and 6) AL terms, the logarithmic dependence[7] of 2D MT, or the 2D-to-3D crossover type[8,9] proposed by Lawrence and Doniach[10] (LD). All their results have been derived from the analysis of the excess conductivity in the absence of a magnetic field. In the evaluation, the excess conductivity σ_{ex} was determined from both the measured conductivity σ_{meas} ($=1/\rho_{meas}$) and the normal resistivity ρ_n ($=1/\sigma_n$) defined as the linear extrapolation from high-temperature data, i.e., $\sigma_{ex}=\sigma_{meas}-\sigma_n$. However, ρ_{meas} in the *a-b* plane of single-crystal $YBa_2Cu_3O_y$ is not exactly linear in the high-temperature region, and then it is difficult to estimate precise ρ_n above T_c. Furthermore, it is noted that the empirical definition of ρ_n by linear extrapolation has no reliable basis. Thus, a more reliable method has become necessary for estimating precise ρ_n near T_c in order to discuss flux motions, resistive transition curves, and other electrical properties, because ρ_n is one of the fundamental parameters in these studies.

Recently, another approach has been presented theoretically[11-13] and experimentally,[14,15] which evaluates magnetic-field-dependent fluctuation conductivity above T_c. The advantage of this method is that magnetoconductivity, $\Delta\sigma(H)=\sigma(H)-\sigma(0)$, can be precisely determined from the values for $\sigma(0)$ and $\sigma(H)$ obtained by applying a magnetic field H. The results of this approach reveal that both AL and MT terms significantly contribute to the fluctuation conductivity of $YBa_2Cu_3O_y$. This implies that excess conductivity is significant in a considerably wide temperature range and that its temperature dependence cannot be expressed by a simple relation such as proposed earlier. In a previous paper, we obtained anisotropic coherence lengths of single-crystal $YBa_2Cu_3O_y$ from magnetoconductivity above T_c arising from fluctuation conductivity using preliminary calculations considering the AL term.[15] This method has another advantage of avoiding flux motions in the estimation of coherence lengths.

This Rapid Communication describes the evaluation of field-dependent fluctuation conductivity of the *a-b* plane in detail based on the recent theories[11-13] and the previously reported data of single-crystal $YBa_2Cu_3O_y$ (Ref. 15). We also estimate normal resistivity ρ_n above T_c without fluctuation conductivity using the obtained parameters and demonstrate that the linear extrapolation from the high-temperature values does not give a good approximation for ρ_n.

The magnetoconductivity of the AL- and the MT-orbital terms has been calculated for layered superconductors by Hikami and Larkin[11] and independently by Maki and Thompson,[12] where 2D superconductors are weakly coupled along the c axis. This model implicitly expresses both cases for anisotropic-3D and quasi-2D superconductors in the temperature range near T_c (Ref. 16), and accordingly this is adopted for $YBa_2Cu_3O_y$, which has the anisotropic coherence lengths.

In the following, we use the equations that can be applied in a wide range of magnetic fields, i.e., Eq. (1) in Ref. 15 for the AL-orbital term and the following equation for the MT-orbital term:[11]

$$\sigma_{\rm MTO}(H)=\frac{e^2}{8\hbar}\frac{1}{d\varepsilon(\alpha/\delta-1)}\int_0^{2\pi}[\psi(\tfrac{1}{2}+A)-\psi(\tfrac{1}{2}+B)]\frac{dx}{2\pi}, \quad (1)$$

$$A=\frac{\varepsilon}{2h}[1+\alpha(1-\cos x)],\quad B=\frac{\alpha\varepsilon}{2h}\left(\frac{1}{\delta}+1-\cos x\right),$$

where

$$h = \ln\frac{T_c(0)}{T_c(H)} = \frac{2|e|}{\hbar c}\xi_{ab}^2(0)H ,$$

$$\alpha = \frac{2\xi_c^2(T)}{d^2} = \frac{2\xi_c^2(0)}{d^2\varepsilon} , \quad \delta = \frac{16\xi_c^2(0)k_B T\tau_\phi}{\pi d^2 \hbar} .$$

ξ_{ab} and ξ_c are anisotropic coherence lengths, $\varepsilon = \ln(T/T_c) \approx (T-T_c)/T_c$, d is the distance between conducting layers, τ_ϕ is the phase relaxation time, and ψ is the di-γ function. Recently, Aronov, Hikami, and Larkin[13] have proposed novel factors for field-dependent fluctuation conductivity due to Zeeman effects on the AL and the MT terms. Adding conventional orbital effects on the AL and the MT terms, the total magnetoconductivity coming from fluctuation conductivity consists of four terms,[17] as follows:

$$\Delta\sigma(H) = \Delta\sigma_{\mathrm{ALO}}(H) + \Delta\sigma_{\mathrm{MTO}}(H) + \Delta\sigma_{\mathrm{ALZ}}(H) + \Delta\sigma_{\mathrm{MTZ}}(H) , \quad (2)$$

where $\Delta\sigma(H)$ is the total magnetoconductivity, $\Delta\sigma_{\mathrm{ALO}}(H)$ is the AL-orbital term, $\Delta\sigma_{\mathrm{MTO}}(H)$ is the MT-orbital term, $\Delta\sigma_{\mathrm{ALZ}}(H)$ is the AL-Zeeman term, and $\Delta\sigma_{\mathrm{MTZ}}(H)$ is the MT-Zeeman term. For the equations valid in a weak magnetic field, we used the AL- and MT-Zeeman terms expressed by Eqs. (12) and (5) in Ref. 13, respectively.[18] The applied field limit of 12 T in our experiment is low enough for the Zeeman terms in a weak-field approximation, however, this is not low for the orbital terms.

Figure 1 shows the best fits to the experimentally obtained magnetoconductivity in the *a*-*b* plane of single-crystal $YBa_2Cu_3O_y$ under a field perpendicular to the *a*-*b* plane. The T_c of the crystal is 90.83 K and the resistivity at 100 K is less than 50 $\mu\Omega$ cm. The precise characteristics and the method of the crystal growth were described in the previous paper.[15] The four terms of Eq. (2) are used for the fitting, where adjustable parameters are h and α, namely $\xi_{ab}(0)$ and $\xi_c(0)$. In the evaluation, we adopted the value for τ_ϕ of 10^{-13} s at 100 K and τ_ϕ is proportional to $1/T$, according to the results obtained by Matsuda and co-workers.[14,19] We discuss this assumption later. The values obtained for $\xi_{ab}(0)$ and $\xi_c(0)$ are 15–16 and 2.8–3.0 Å (Ref. 20), respectively, where the distance between layers is assumed as the lattice constant of the *c* axis of 11.7 Å. These values for $\xi_{ab}(0)$ and $\xi_c(0)$ are 20% and 50% larger than those previously obtained,[15] respectively. However, this causes no change in our basic arguments that $YBa_2Cu_3O_y$ behaves like quasi-2D superconductors and has very short $\xi_{ab}(0)$. We discuss each contribution of the four terms in the following section.

In the parameter fitting, $\xi_{ab}(0)$ strongly depends on the shape of the curve, while $\xi_c(0)$ depends on the absolute value of magnetoconductivity. When considering only the AL-orbital term, the tendency of the curve saturation is slightly faster than that of the total magnetoconductivity, as shown in Fig. 1. This is because the contribution of the MT-orbital and the AL-Zeeman terms gradually increases in a higher magnetic field, although the MT-Zeeman term is negligibly small in this temperature region below $t = 1.04$, where $t = T/T_c$. The small difference between the present and the previous [$\xi_{ab}(0) = 13$ Å, $\xi_c(0) = 2$ Å] (Ref. 15) results is ascribed to the shape of the curve, particularly at higher fields. Magnetoconductivity of the MT-orbital and the AL-Zeeman terms, on the other hand, is negligibly small in a low field in the same temperature range.

Figure 2 describes the temperature dependence of fluctuation conductivity in the absence of a field. The AL and

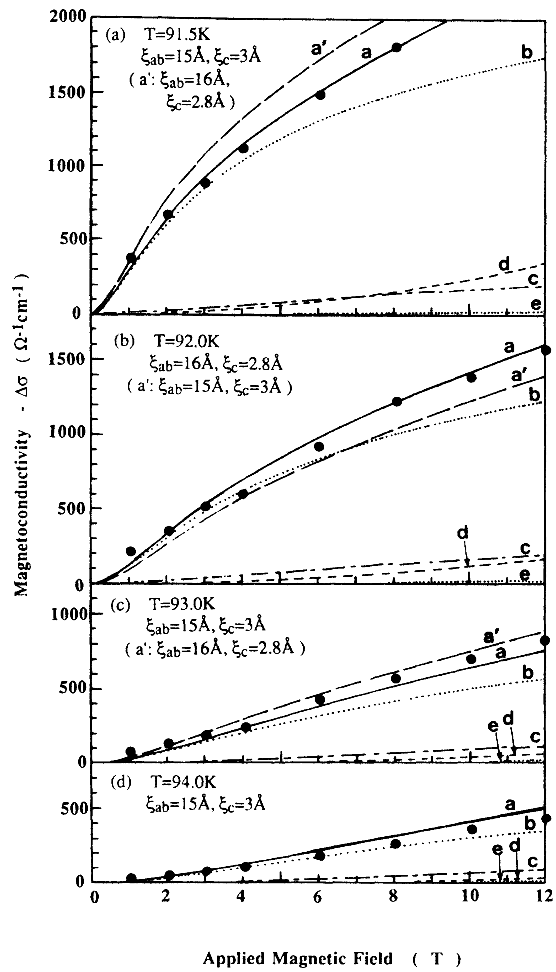

FIG. 1. Magnetoconductivity in the *a*-*b* plane of single-crystal $YBa_2Cu_3O_y$ ($T_c = 90.83$ K) under a field perpendicular to the *a*-*b* plane and the best fitting results of the fluctuation conductivity calculation. The black circles are the experimental data. The lines are calculation results where curve *a* is the total magnetoconductivity [$\Delta\sigma(H)$], curve *b* is the AL-orbital term [$\Delta\sigma_{\mathrm{ALO}}(H)$], curve *c* is the MT-orbital term [$\Delta\sigma_{\mathrm{MTO}}(H)$], curve *d* is the AL-Zeeman term [$\Delta\sigma_{\mathrm{ALZ}}(H)$], and curve *e* is the MT-Zeeman term [$\Delta\sigma_{\mathrm{MTZ}}(H)$]. (a) $T = 91.5$ K, (b) $T = 92.0$ K, (c) $T = 93.0$ K, (d) $T = 94.0$ K. The values for the parameters ξ_{ab} and ξ_c are also shown in the figure. The notation *a'* indicates the total magnetoconductivity [$\Delta\sigma(H)$] obtained from the other set of ξ_{ab} and ξ_c shown in each parentheses.

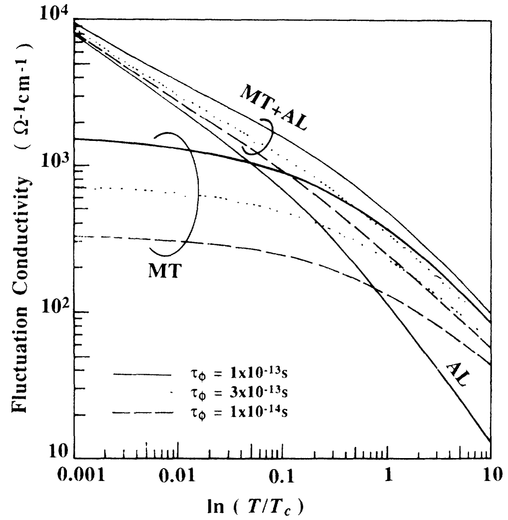

FIG. 2. Temperature dependence of fluctuation conductivity in the absence of a field calculated from Eqs. (3) and (4) for the AL and MT terms. The notations AL+MT, AL, and MT indicate $\sigma_{fl}(0)$, $\sigma_{AL}(0)$, and $\sigma_{MT}(0)$, respectively. The $\sigma_{MT}(0)$ values are calculated for three different values for τ_ϕ at 100 K.

MT terms are given as

$$\sigma_{AL}(0)=\frac{e^2}{16\hbar d}\frac{1}{\varepsilon(1+2\alpha)^{1/2}}, \quad (3)$$

$$\sigma_{MT}(0)=\frac{e^2}{8\hbar d\varepsilon(1-\alpha/\delta)}\ln\left[\frac{\delta}{\alpha}\frac{1+\alpha+(1+2\alpha)^{1/2}}{1+\delta+(1+2\delta)^{1/2}}\right], \quad (4)$$

where the total fluctuation conductivity $\sigma_{fl}(0)$ is that $\sigma_{fl}(0)=\sigma_{AL}(0)+\sigma_{MT}(0)$. Three cases of τ_ϕ with $\xi_c(0)=3$ Å are used for the calculation, as shown in Fig. 2. When τ_ϕ is larger than 3×10^{-14} s at 100 K, the MT term is large enough to contribute the total conductivity of $\sigma_{fl}(0)$ even at a temperature of $\varepsilon=0.01$. Therefore, the temperature dependence of $\sigma_{fl}(0)$ is not simple compared with that of the AL term alone. This contrasts with the case where magnetoconductivity of the MT term is negligibly small in a low field at a small ε.

Figure 3 shows normal resistivity ρ_n $(=1/\sigma_n)$ without fluctuation conductivity estimated from the measured value ρ_{meas} $(=1/\sigma_{meas})$, where $\sigma_{meas}=\sigma_n+\sigma_{fl}(0)$. The obtained curve is significantly different from the linear extrapolation from the high-temperature region. It may appear peculiar that the excess resistivity $\Delta\rho$ $(=\rho_n-\rho_{meas})$ does not decrease with an increase in temperature but is almost constant. However, it is reasonable when considering the temperature dependence of ρ_{meas}. The $\Delta\rho$ can be expressed as $\Delta\rho=\sigma_{fl}(0)\rho_{meas}\rho_n\approx\sigma_{fl}(0)\rho_{meas}^2$. From Fig. 2, $\sigma_{fl}(0)$ is approximately proportional to $T^{-1/2}$ in the temperature range of $0.1<t<1$ (100–200 K) while ρ_{meas} is roughly proportional to T in the same range. Then it follows that $\Delta\rho$ is almost constant, as shown in Fig. 3. This contrasts with the case of conventional low-T_c superconductors where the T dependence of $\Delta\rho$ directly reflects that of $\sigma_{fl}(0)$ since ρ_n is almost constant, as commonly observed. Thus the main reason of constant $\Delta\rho$ is attributable to the temperature dependence of ρ_n, or more specifically electron scattering time τ_e, i.e., $\tau_e\propto1/T$, where it is assumed that τ_e is the same as τ_ϕ. This assumption is discussed in the next section. The estimated ρ_n in Fig. 3 has a slight rounding near T_c and we believe that it is not intrinsic. One of the reasons for this may be ascribed to a little ambiguity in the T_c determination because the crystal has a small amount of inhomogeneity as reflected by the finite transition width, even though the crystal is probably of the highest quality available. Another may be found in the oversimplified model for a real layered structure of $YBa_2Cu_3O_y$. We believe that the temperature dependence of the correct ρ_n is much more linear in the temperature range discussed.

In this paper, the following three basic assumptions are implicitly included: (i) τ_ϕ is approximately equal to τ_e, (ii) $YBa_2Cu_3O_y$ is an s-wave superconductor, i.e., fluctuation conductivity is calculated for s-wave superconductors, and (iii) all observed magnetoresistivity comes from fluctuation conductivity. In this section, we discuss the reliability of these assumptions. Larkin[21] has proposed that τ_ϕ in the MT term corresponds to electron inelastic scattering time. Then the assumption of $\tau_\phi\approx\tau_e$ can be regarded as a good approximation in the high-temperature region above 90 K. In fact, the value and the temperature dependence of τ_ϕ obtained by Matsuda and co-workers[14,19] are in good agreement with τ_e obtained from the optical reflectivity by Thomas *et al.*[22] On the other hand, it has been reported[23] that the behavior of the s-wave superconductor has been observed in the temperature dependence

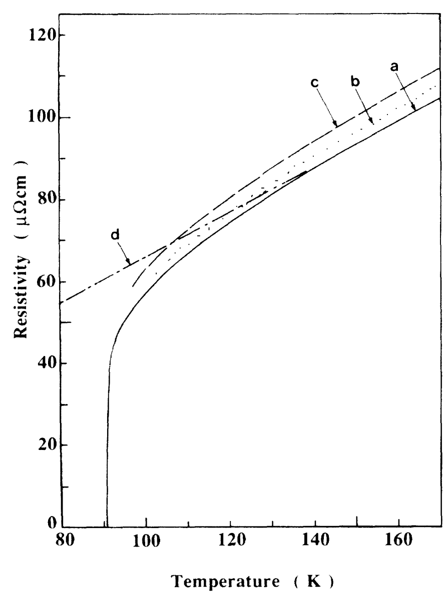

FIG. 3. Temperature dependence of normal resistivity. The notations: curve a, measured value ρ_{meas}; curve b, estimated normal resistivity ρ_n $[\sigma_n=\sigma_{meas}-\sigma_{AL}(0)]$ considering only the AL term $(\sigma_{MT}(0)=0)$; curve c, estimated normal resistivity ρ_n $\{\sigma_n=\sigma_{meas}-\sigma_{fl}(0)=\sigma_{meas}-[\sigma_{AL}(0)+\sigma_{MT}(0)]\}$ considering both the AL and the MT terms, where $\xi_c=3$ Å and $\tau_\phi=10^{-13}$ s at 100 K; curve d, linear extrapolation value from the high-temperature data.

of the magnetic penetration depth of $YBa_2Cu_3O_y$. With respect to the last point, no origin can be interpreted in such a large magnetoresistivity near T_c except for fluctuation effects. This problem has been discussed in a previous paper.[15] Therefore, we believe that our estimation reflects realistic cases and is reliable at least in experimental and theoretical accuracy when allowing for the above-mentioned assumptions.

In seeking an explanation for the different experimental values of the temperature-dependent resistivity in $YBa_2Cu_3O_y$, the material-dependent parameter, factor C, has been introduced by Oh *et al.*[8] They proposed that the observed ρ_n is equal to the intrinsic value multiplied by the factor C due to the lack of sample uniformity, where the intrinsic $d\rho_n/dT$ is approximately 0.5–0.6 $\mu\Omega$ cm/K.[8] Although the idea of factor C is unnecessary for the value of 0.58 $\mu\Omega$ cm/K in this study,[15] we comment on the case for introducing factor C in the present fluctuation analysis, in order to apply this analysis more widely. The $\xi_{ab}(0)$ can be easily determined in the parameter fitting because $\xi_{ab}(0)$ depends on the shape of the magnetoconductivity curve as described in the former section. Conversely, the $\xi_c(0)$ is difficult to obtain precisely, and only the combined value of $\xi_c(0)$ and factor C is determined. However, when considering the wide temperature dependence of the magnetoconductivity in the parameter fitting, the determination of $\xi_c(0)$ is possible. Indeed, Matsuda *et al.*[19] successfully obtained a set of $\xi_{ab}(0)$ and $\xi_c(0)$ for the samples characterized by different C factors.

In conclusion, magnetoconductivity in the *a-b* plane of single-crystal $YBa_2Cu_3O_y$ is analyzed from the viewpoint of field-dependent fluctuation conductivity, considering both orbital and Zeeman effects on the AL and the MT terms. The estimated coherence lengths, $\xi_{ab}(0)=15$–16 Å and $\xi_c(0)=2.8$–3 Å, are slightly larger than previously obtained values by the authors.[15] This is explained by the contribution of the MT-orbital and the AL-Zeeman terms in a higher magnetic field. We also estimated ρ_n above T_c without fluctuation conductivity. It is strongly emphasized that our estimated ρ_n is substantially different from the previously estimated values that are linearly extrapolated from the data in the high-temperature range.

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[16] This model expresses a crossover from 2D to 3D, and the anisotropic-3D behavior appears in the temperature region below the crossover point, irrespective of the intrinsic dimensionality. The α in Eq. (1) is a key parameter for the determination of the dimensionality. The details are described in Refs. 11 and 15.

[17] Equation (2) is a good approximation only for a low field where the correlation between the Zeeman and the orbital effects can be neglected. When $H\rightarrow\infty$, magnetoconductivity becomes zero, i.e., $\sigma(H)$, $\sigma_{\rm ALO}(H)$, $\sigma_{\rm MTO}(H)$, $\sigma_{\rm ALZ}(H)$, $\sigma_{\rm MTZ}(H)\rightarrow 0$. From Eq. (2), then, when $H\rightarrow\infty$,

$$\begin{aligned}\Delta\sigma(H)&=-\sigma(0)\\&=-[\sigma_{\rm ALO}(0)+\sigma_{\rm MTO}(0)+\sigma_{\rm ALZ}(0)+\sigma_{\rm MTZ}(0)]\\&=-2[\sigma_{\rm AL}(0)+\sigma_{\rm MT}(0)].\end{aligned}$$

This relation for an infinite limit case is not correct.

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