Localization problem in optics: Nonlinear quasiperiodic media

S. Dutta Gupta
School of Physics, University of Hyderabad, Central University P.O., Hyderabad 500 134, Andhra Pradesh, India

Deb Shankar Ray
Department of Physical Chemistry, Indian Association for the Cultivation of Science, Jadavpur, Calcutta 700 032, West Bengal, India

(Received 23 June 1989)

We have presented a detailed numerical study of the localization problem in a nonlinear quasiperiodic structure for normal incidence of plane-polarized light. The main conclusions are as follows. Strong surface localization, which is observed for forbidden states (transmission coefficient $T \cong 0$) in the linear theory, is strongly affected even by weak nonlinearity, resulting in inhibition of localization, while the extended states corresponding to allowed regions ($T \cong 1$) in the linear theory retain their distribution pattern. Depending on nonlinearity, forbidden regions may exhibit critical-state behavior. The evidence of bulk localization is apparent from the nature of solitonlike field distributions for allowed regions. The bulk localization, which is a result of a delicate interplay between dispersion and nonlinearity, persists for a very large number of layers in contrast to linear theory. The bulk localized states have been shown to be self-similar.

I. INTRODUCTION

Quasiperiodic structures, i.e., structures that are intermediate between periodic and random systems, have been the subject of considerable theoretical investigation in recent years. The problem of the propagation of electrons in one-dimensional quasiperiodic structures[1–12] has revealed new kinds of generic exotic features, such as weak localization and scaling, and concurrently depending on the strength of coupling and total energy the role of localized, critical, and extended states has come into the picture in the context of conductivity and the associated phenomena. These theoretical studies have received early impetus from the experimental discovery of the quasicrystal phase in metallic alloys,[13] which was followed by another realization of quasiperiodic superlattice structure by Merlin *et al.*[14]

Recently an analogous problem in optics was addressed by Kohmoto, Sutherland, and Iguchi[15] in which the propagation of transverse electromagnetic waves through a stack of dielectric layers, which is constructed using two types of dielectrics arranged in a Fibonacci sequence, is considered. The transmission coefficient as a function of optical path length was demonstrated to be multifractal and displayed scaling behavior (i.e., scaling of dispersion). Subsequently, some variants[16,17] of this linear theory have also been reported. In a recent communication[18] we have presented a nonlinear generalization of this theory in terms of a nonlinear-characteristic-matrix formalism[19] taking full account of the nonlinearity of the boundary conditions.[20] The theory is extremely simple and, in principle, can handle any number of layers. The theory was applied to two coupled nonlinear Fabry-Perot resonators[19] and as a general case to a nonlinear Fibonacci multilayer[18] to calculate the power-dependent transmission coefficient in the context of optical bistability and multistability. For weak incident intensity one can get back the linear results of Kohmoto, Sutherland, and Iguchi.[15] However, when the incident light intensity comes into play, nonlinearity profoundly affects the system because of multistability. In fact, the transmission coefficient as a function of input intensity offers a wide variation ranging from almost null transmission to complete transmission for different values of linear optical path length.[15,18] Also if we note that since we are taking care of both dispersion (only dispersion was considered by Kohmoto *et al.*[15]) and nonlinearity, one can expect to observe a delicate interplay between the two in some restricted situations.[21] With this in mind we address the following questions. What is the nature of state (by state we mean spatial distribution) of the electric field intensity corresponding to null and complete transmissions? How do these states change as functions of light intensity?

Before any further elaboration, we bring forth the analogy with the electronic problem where we have encountered localized, extended, and critical states depending on the strength of coupling and total energy. Let us recall that by an exponential or critical state we mean that if $\psi(x)$, the wave function for an electron, or E, an electric field, describes an envelope function, then $|\psi(x)|^2$ or $|E(x)|^2$ varies asymptotically as $\exp(-\xi x)$ for an exponentially localized state or as $x^{-\xi}$ ($\xi > 0$) for a critical state. Another way of realizing these states is in terms of Landauer resistivity.[22] $R/(1-R)$, where R is the reflectivity for a classical electron. It has been shown that $R/(1-R)$ becomes exponential or power-law bounded function as a function of material length x for localized and critical states, respectively. For an extended state the transmission coefficient is given by a constant or a bounded function of x. It is quite natural that in optical problems, optical analog of Landauer resistivity $R/(1-R)$ (or R/T, since $T = 1-R$, where T is the transmission coefficient) is being employed.[17]

Now the question regarding the nature of electric field distribution (or state) has been asked in the context of linear theory by a number of workers[15,17] in the recent past and it has been shown that in the limit when the number of layers become large all the observable states studied are exponentially bounded surface states. However, for a reduced number of layers some states appear to be critical (i.e., power-law bounded) but cross over to exponentially localized as the number of layers increases. Hence, although the theory indicates the existence of allowed states in such material, the energy width of such states goes over to zero as the length of the material goes over to infinity. Allowed states form a Cantor set of Lebesque measure zero.[15]

With these linear results[15,17] in mind we now investigate the nature of field localization within a given optical quasiperiodic structure in the nonlinear regime. We show that nonlinearity has a variety of significant effects on the field intensity distribution corresponding both to almost null and complete transmissions. Moreover, the delicate interplay between dispersion and nonlinearity leads to the emergence of bulk localization of a given structure which, in sharp contrast to linear theory,[17] stays even in the limit of a large number of layers. Some preliminary results regarding these bulk localized states and their self-similarity were reported earlier in the context of systems with smaller number of layers (55 and 233). In this paper we supplement those results with calculations involving as many as 2584 layers.

One important aspect regarding the problem of localization in nonlinear media may be in order. Although the problem bears its origin in solid-state physics in connection with the theory of electrical conductivity in disordered and in quasiperiodic media, subsequent realization that almost any wave equation with random (or quasiperiodic) potential may possess localized (or critical) solutions has made the field quite general. For the electronic problem the wave equation we deal with is the usual Schrödinger equation which is essentially linear in nature. For the analogous problem in optics we employ the classical Maxwell equation and as such we are not necessarily restricted to linear situations. Because of this nonlinearity we expect the Maxwell wave equation in nonlinear quasiperiodic media to admit a variety of new physical features in localization problems. The self-similar bulk localized solitionlike field states (in addition to localized, critical, and extended states) and the enhancement of delocalization due to the increase in nonlinearity are two such features as discussed in this paper.

The outline of the paper is as follows. In Sec. II we review the nonlinear transfer-matrix method[18,19,21] for the propagation problem and give the relevant quantities, such as expression for field intensities, transmission coefficient, and Landauer resistivity. We present detailed numerical results for the problem in Sec. III. The paper is concluded in Sec. IV.

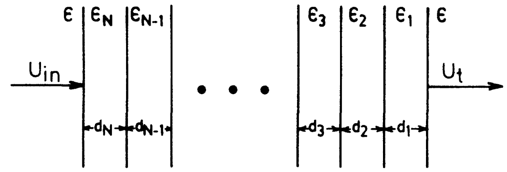

FIG. 1. A schematic view of the multilayered medium.

II. TRANSMISSION FROM A NONLINEAR MULTILAYERED MEDIUM

In this section we review the results obtained earlier for the reflection and transmission coefficients from a nonlinear multilayered medium.[18,19] The nonlinear multilayered medium consisting of N nonlinear plane-parallel slabs is embedded in a linear medium of dielectric constant ϵ (see Fig. 1). Let a *TE*-polarized plane wave be incident on the Nth slab from the left. Let the nonlinearity of the jth slab given by the following displacement vector:[23]

$$\mathbf{D}_j^{\mathrm{NL}}=\epsilon_j\chi_j[\tilde{A}_j\mathbf{E}_j(\mathbf{E}_j\cdot\mathbf{E}_j^*)+\tilde{B}_j\mathbf{E}_j^*(\mathbf{E}_j\cdot\mathbf{E}_j)] , \quad (2.1)$$

where ϵ_j and χ_j are the linear dielectric constant and the constant of nonlinear interaction, respectively; $\mathbf{E}_j$ is the electric field vector and the constants $\tilde{A}_j$ and $\tilde{B}_j$ define the strength and the type of nonlinear interaction. In the slowly varying envelope approximation,[24] the solutions of Maxwell equations for the electric field E_j in the jth slab can be expressed as follows:

$$E_j=A_{j+}\exp(ik_{j+}x)+A_{j-}\exp(-ik_{j-}x) . \quad (2.2)$$

In Eq. (2.2), A_{j+} (A_{j-}) is the constant amplitude of the forward (backward) wave. k_{j+} (k_{j-}) is the field-dependent forward (backward) wave vector given by[19,25]

$$k_{j\pm}=k_0(\epsilon_j)^{1/2}(1+U_{j\pm}+2U_{j\mp})^{1/2} \quad (2.3)$$

with $k_0=\omega/c$ and

$$U_{j\pm}=\chi_j(\tilde{A}_j+\tilde{B}_j)|A_{j\pm}|^2=\alpha_j|A_{j\pm}|^2 . \quad (2.4)$$

Henceforth we assume that the slabs have the same nonlinearity constants, i.e., $\alpha_j=\alpha$ for all j. Following the method discussed in Ref. 19, one can calculate the dimensionless intensities $U_{j\pm}$ by solving the coupled sets of nonlinear equations given by

$$\begin{bmatrix} U_{j+} \\ U_{j-} \end{bmatrix} = \left| \begin{bmatrix} 1 & 1 \\ k_{j+}/k_0 & -k_{j-}/k_0 \end{bmatrix}^{-1} M_{j-1}M_{j-2}\cdots M_2M_1 \begin{bmatrix} 1 \\ (\epsilon)^{1/2} \end{bmatrix} \right|^2 U_t , \quad (2.5)$$

where U_t is the normalized transmitted intensity (treated as a given parameter in our theory); M_k is the nonlinear characteristic matrix defining the propagation properties of the kth slab. The explicit expression for the characteristic matrix has been given in Ref. 19. Equation (2.5) enables one to obtain the characteristic matrices recursively for all the slabs. The total characteristic matrix is given by

$$M = M_N M_{N-1} \cdots M_2 M_1 \ . \tag{2.6}$$

The relevant experimental quantities which are required for subsequent discussion are the reflection (R) and transmission (T) coefficients which can be expressed in terms of the elements of the total characteristic matrix M.[19]

The optical analog of the Landauer resistivity $\bar{R}$ and the incident intensity U_{in} are given by

$$\bar{R} = R/T \tag{2.7}$$

and

$$U_{\text{in}} = U_t/T \ . \tag{2.8}$$

In order to obtain the linear results we allow U_t to assume vanishingly small values.

For a given quasiperiodic structure one can have evident field localization in two ways: by calculating $\log_{10}(\bar{R})$ as a function of the number of layers N with all other parameters fixed, or by following the intensity distribution throughout the structure. We define the sum intensity U_j as

$$U_j = U_{j+} + U_{j-} \tag{2.9}$$

and consider it as a measure of intensity in the jth slab. Note that in the slowly varying envelope approximation both U_{j+} and U_{j-} for fixed j are constants, and can be calculated using Eq. (2.5). In the next section we resort to both of these methods for our numerical investigation.

III. NUMERICAL RESULTS AND DISCUSSION

Let us now apply the theory reviewed in the last section to a Fibonacci multilayer constructed recursively by two nonlinear slabs A and B (with linear dielectric constants ϵ_A and ϵ_B and widths d_A and d_B, respectively) as

$$S_{j+1} = (S_{j-1} S_j) \tag{3.1}$$

with $S_0 = (B)$ and $S_1 = (A)$. Thus $S_2 = (BA)$. $S_3 = (ABA)$, $S_4 = (BAABA)$, etc. We also assume that the medium on the left as well as on the right is linear with dielectric constant ϵ and also that the linear optical paths are the same for both the slabs, i.e.,

$$\delta_A = k_0(\epsilon_A)^{1/2} d_A = \delta_B = k_0(\epsilon_B)^{1/2} d_B = \delta \ . \tag{3.2}$$

We consider various generations S_n constructed using (3.1) and define ϵ_i as $\epsilon_1 = \epsilon_A$, $\epsilon_2 = \epsilon_B$, $\epsilon_3 = \epsilon_A$, $\epsilon_4 = \epsilon_A$, and so on. For all of our numerical calculations we have assumed the following parameter values: $\epsilon_A = 4$, $\epsilon_B = 9$, and $\epsilon = 1$ (same as in Ref. 15 except ϵ).

Examination of transmission spectra as a function of linear optical path length (δ) serves as a guideline for where to expect exponential, critical, and extended state behavior (as an illustration, Fig. 2 of Ref. 18 may be noted). Drawing again an analogy with electronic wave function one might observe that a band of energies corresponding to almost null transmission ($T \cong 0$) may be con-

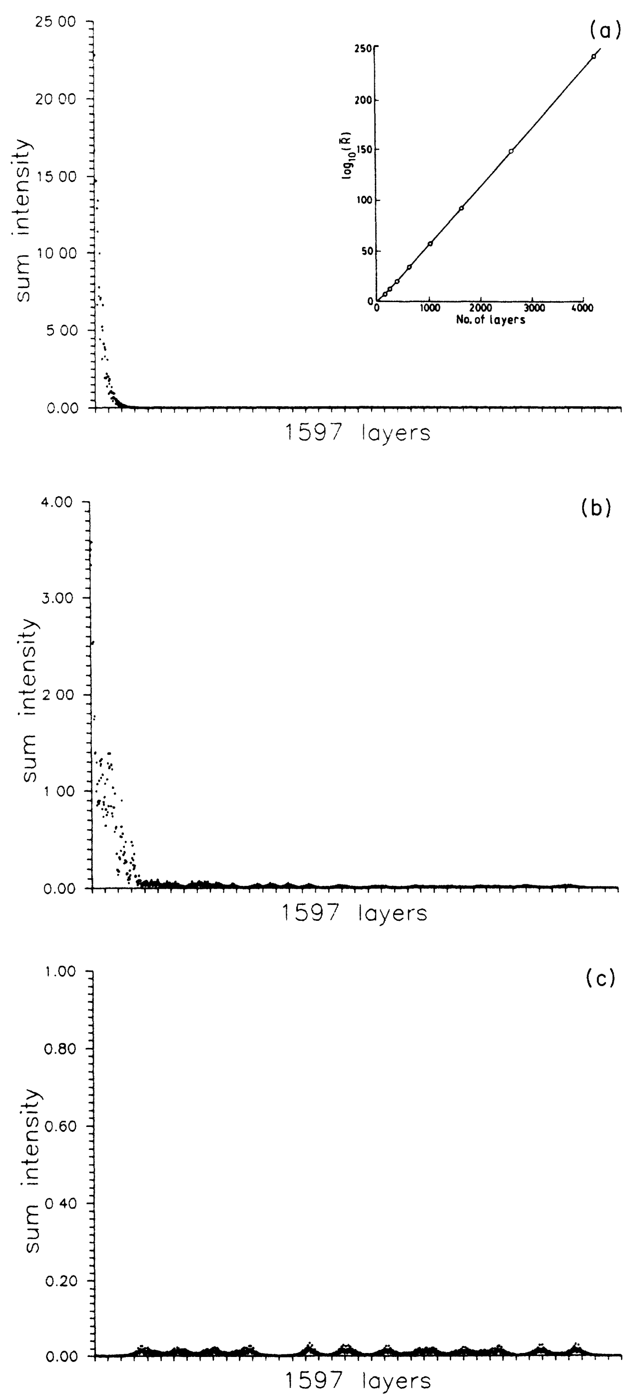

FIG. 2. Sum intensity distribution in a forbidden region for $\delta = 1.23\pi$ and for various U_t values. (a) Linear case, $U_t = 1.0\times10^{-39}$; (b) nonlinear case, $U_t = 1.0\times10^{-5}$; and (c) $U_t = 2.0\times10^{-5}$ (scale in arbitrary units). The inset is $\log_{10}(\bar{R})$ as a function of Fibonacci layer numbers.

sidered to be a forbidden region or band gap where the electric field intensity is expected to decay exponentially with increasing distance into the structure producing a surface-localized state. On the contrary, a band of energies corresponding to almost complete transmission ($T \cong 1$) is considered to be an allowed region where the extended states are likely to be observed. The transition regions correspond to critical states.

A. Linear results

In order to make a fair comparison with nonlinear results, let us first consider the linear properties of the system. This implies that we assume U_t, the normalized transmitted intensity, to be vanishingly small. Figure 2(a) shows the spatial distribution of the sum intensity U_j over 1597 layers for a typical forbidden region (i.e., $T \cong 0$) corresponding to $\delta = 1.23\pi$ and $U_t = 1.0 \times 10^{-39}$. The envelope in Fig. 2(a) exhibits an exponential decay as expected. The inset in the figure displays the plot of the logarithm of optical Landauer resistivity for the same δ as a function of the numbers of Fibonacci layers. The plot is a straight line indicating the strong surface localization of the field. In Fig. 3(a) the spatial distribution of the sum intensity U_j over 1597 layers is plotted for an allowed region ($T \cong 1$) corresponding to $\delta = 1.913\,997\,3\pi$ and for $U_t = 1.0 \times 10^{-10}$ (i.e., for weak nonlinearity). It is immediately apparent that the field distribution is close to periodic over the length of the material. The calculation of the transmission coefficient shows that it is finite and fluctuates around a constant mean value as a function of Fibonacci layer number indicating the extended nature of the state.

The linear results displayed here are in conformity with those of the earlier workers.[15,17] It has been shown[17] that in the limit when the number of layers becomes large all the observable states studied are exponentially bounded surface states. For a reduced number of layers some states appear to be critical, i.e., power-law bounded, but cross over to exponentially bounded states as the number of layers is increased. Secondly, although the theory indicates the existence of allowed states in such material, the energy width of such states goes to zero as the length of the material goes to infinity and all observable states in the material are surface states. Also the allowed states form a Cantor set of Lebesque measure zero.[15]

B. Nonlinear results

In order to obtain the nonlinear results we turn on the transmitted intensity parameter U_t to assume an appreciably larger value. Figure 2(b) shows the spatial distribution of the sum intensity U_j over 1597 layers for the same forbidden region (i.e., the region forbidden in linear case corresponding to $\delta = 1.23\pi$) for $U_t = 1 \times 10^{-5}$. It is apparent that the envelope function is no longer an exponentially decaying function [compare with Fig. 2(a)]. A further twofold increase in the value of U_t [Fig. 2(c)] clearly shows the inhibition of strong localization observed in the linear case.

Although nonlinearity affects significantly the localized states, it has little or almost no effect on the distribution of the extended states corresponding to allowed regions ($T \cong 1$) as can be seen from Fig. 3(b), where U_t has been increased to assume a value of 1.0×10^{-4} for the same δ as in Fig. 3(a).

Before we go into further calculations, special attention should be given to the multivalued character of the input-output dependence. Note that in most of our calculations we treat U_t as a parametrically given quantity. In moderately nonlinear systems (i.e., for moderate or higher values of U_t) the output characteristics can be multivalued as a function of the input intensity. It follows that certain ranges of the values of U_t define the unstable branches of the input-output characteristics. These branches are not realizable in an experiment. Therefore, for any set of calculations, we make sure that the chosen value or range of values of U_t does not correspond to the unstable branches. In what follows, we pay attention to the input-output characteristics of the nonlinear system (for a typical curve see, for example, Fig. 3 of Ref. 18). It is interesting to see how the intensity distributions corresponding to different forbidden regions

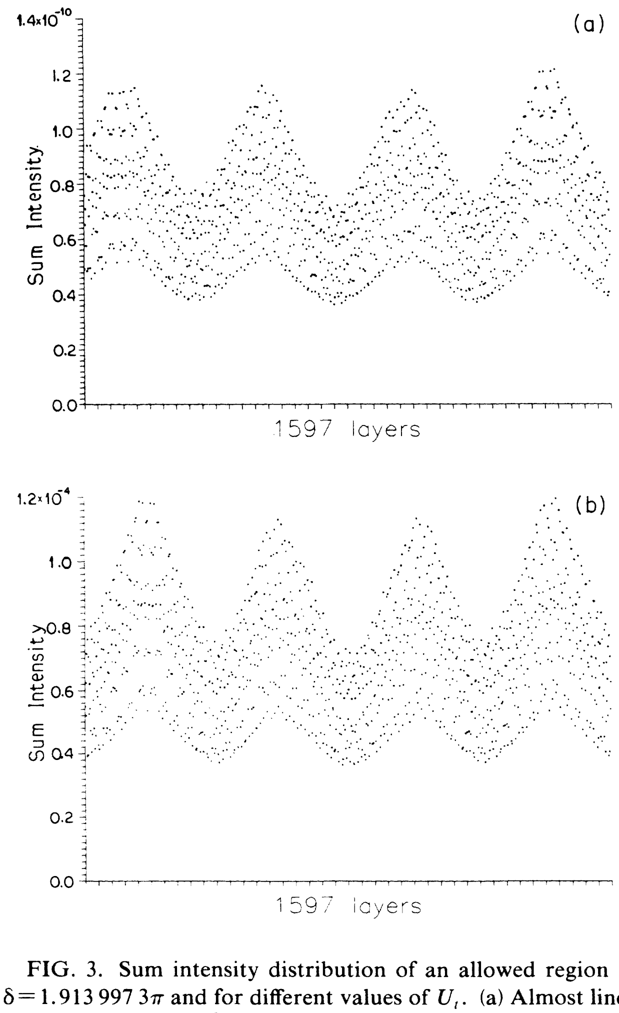

FIG. 3. Sum intensity distribution of an allowed region for $\delta = 1.913\,997\,3\pi$ and for different values of U_t. (a) Almost linear case, $U_t = 1.0 \times 10^{-10}$; (b) nonlinear case, $U_t = 1.0 \times 10^{-4}$ (scale in arbitrary units).

($T \cong 0$) of the nonlinear system change as U_t is changed keeping the optical path fixed. To put this more precisely, let us note, for example, that for 2584 layers and $\delta = 1.16\pi$ the forbidden regions ($T \cong 0$) appear for $U_t = 0.372\,33 \times 10^{-2}$, $0.441\,83 \times 10^{-2}$, $0.498\,67 \times 10^{-2}$, etc. The question is what is the nature of distribution in these cases. Figures 4(a)–4(c) illustrate these distributions. It is interesting to note that these are like critical states. This conclusion can also be inferred from the insets in the figures, which show that optical analog of Landauer resistivity $\bar{R}$ is a power-law bounded function. Also compare Figs. 4(a)–4(c) with Figs. 3(b) and 3(c) of Ref. 17 and note the difference in the scale of the vertical axis of the insets.

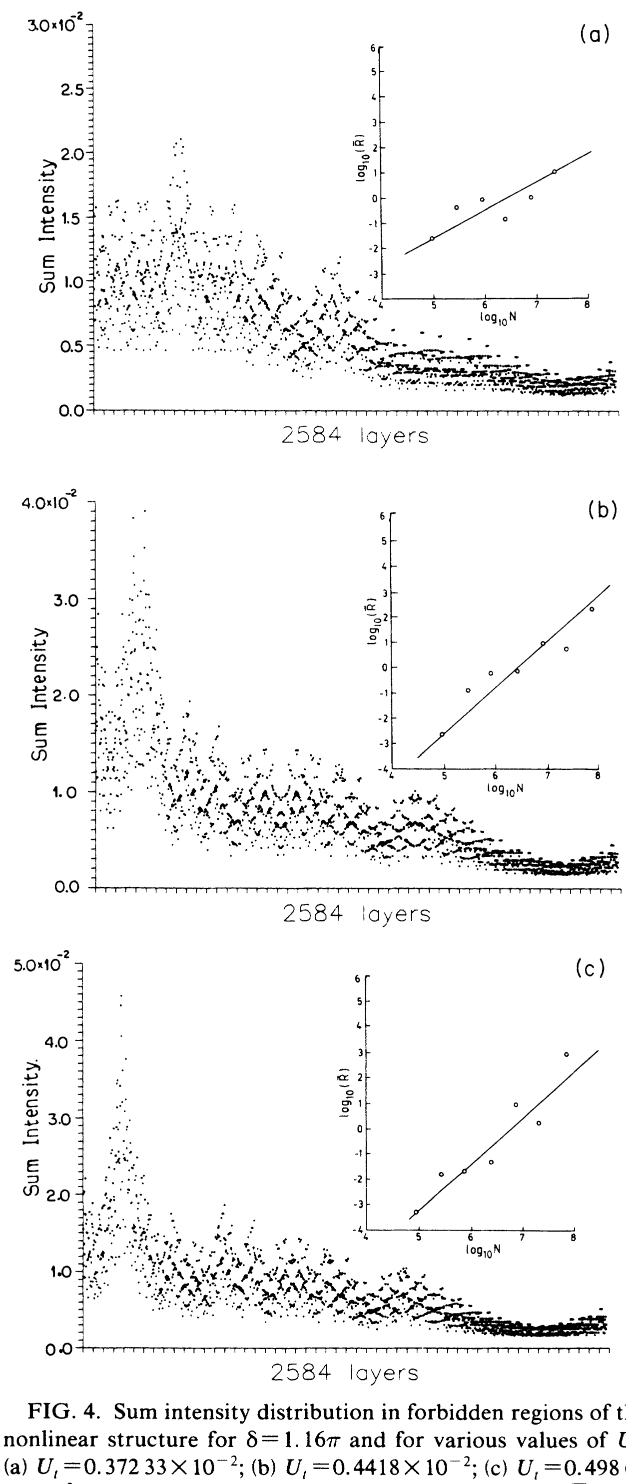

FIG. 4. Sum intensity distribution in forbidden regions of the nonlinear structure for $\delta = 1.16\pi$ and for various values of U_t. (a) $U_t = 0.372\,33 \times 10^{-2}$; (b) $U_t = 0.4418 \times 10^{-2}$; (c) $U_t = 0.498\,67 \times 10^{-2}$ (scale in arbitrary units). The insets are $\log_{10}(\bar{R})$ as a function of Fibonacci layer number.

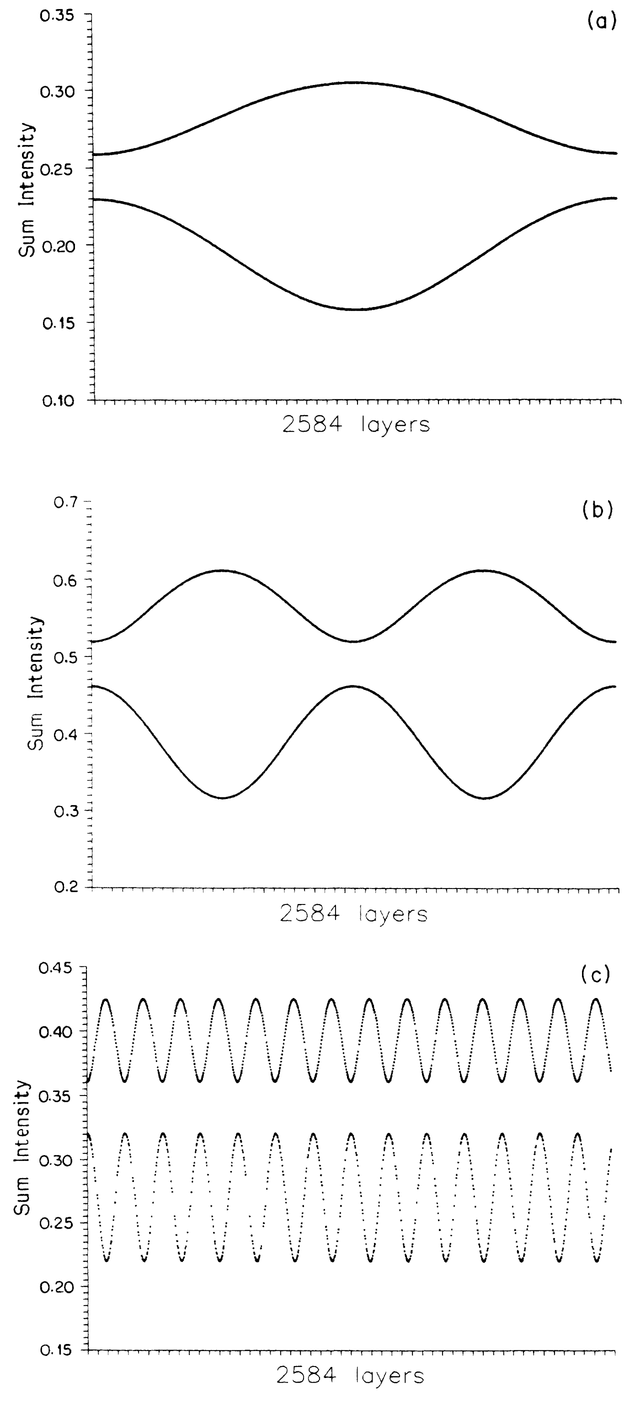

FIG. 5. Sum intensity distribution in allowed regions of the nonlinear structure for $\delta = 2\pi$ and for various values of U_t. (a) $U_t = 0.413\,36 \times 10^{-3}$; (b) $U_t = 0.83 \times 10^{-3}$; (c) $U_t = 0.577\,096 \times 10^{-2}$ (scale in arbitrary units).

We have extended the above numerical analysis to study the allowed regions ($T \cong 1$) also. We observe that for 2584 layers and $\delta = 2\pi$, the allowed regions appear for $U_t = 0.413\,36 \times 10^{-3}$, 0.83×10^{-3}, $0.577\,096 \times 10^{-2}$, etc. In Figs. 5(a)–5(c) sum intensity distributions have been plotted for several values of U_t for all of which T is almost equal to unity. For comparison, we have reproduced Fig. 2(b) of Ref. 21 in Fig. 6, which shows the same distribution. It is interesting to observe that for lower values of U_t (i.e., nonlinearity) bulk localization appears in the structure [see Figs. 5(a) and 6]. As one goes over to higher values of U_t the states acquire more and more extension in character. It was pointed out in Ref. 21 that the distributions corresponding to the lowest nonzero value of U_t and $T \cong 1$ can be fitted by a sech^2 dependence. The fit is almost exact. Thus the total transparency of the nonlinear system is explained in terms of the excitation of these solitonlike distributions. Note that excitation of such objects are known in nonlinear periodic systems. In fact, theoretical proof of the existence of solitons in a nonlinear periodic system for normal incidence was given by Sipe and Winful.[26] Solitonlike objects have also been reported in nonlinear periodic systems for oblique incidence.[27] However, the transparency of the system for oblique incidence is explained in terms of the nonlinear supermodes of the structure.

Figures 5(a) and 6 answer one pertinent question that may arise in the context of whether bulk localization can persist even if we increase the number of layers for same δ but for different values of U_t scaled appropriately according to system size. For example, we note that at $\delta = 2\pi$, $T \cong 1$ regions appear for 55, 233, and 2584 layers for U_t values 0.0195, 0.0046, and $0.413\,36 \times 10^{-3}$, respectively. The following relation for U_t holds:

$$\frac{1/U_t^{55}}{55} \cong \frac{1/U_t^{233}}{233} \cong \frac{1/U_t^{2584}}{2584}, \quad \text{etc.} \tag{3.3}$$

It is immediately apparent from Figs. 5(a) and 6 that bulk localization of a given structure can persist for allowed regions for appropriately scaled nonlinearity, i.e., U_t, even if we increase the number of layers to be very large. Note that we have gone up to as many as 2584 layers. This is in sharp contrast to linear theory,[17] where it has been shown that bulk localization goes over to a state localized at the surface with the increase in the number of layers. The persistence of bulk localization characterized by a solitonlike distribution is a typical signature of an interplay between nonlinearity and dispersion. Another interesting feature which should be noted from Figs. 5(a) and 6 is their self-similarity. We have shown recently[21] that the width of the bulk localized states scales as the system size. Such properties are missing in nonlinear periodic systems and presumably this is a consequence of quasiperiodicity.

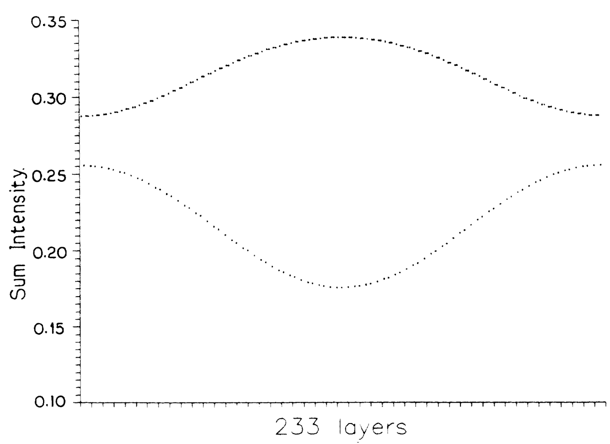

FIG. 6. Same as in Fig. 5, but for 233 layers and for $U_t = 0.0046$.

IV. CONCLUSION AND SUMMARY

In this paper we have presented a detailed numerical study of a localization problem in a nonlinear quasiperiodic structure for normal incidence of plane-polarized light. The main conclusions can be summarized as follows.

(a) Stong surface localization, which is observed for the forbidden states ($T \cong 0$) in the linear theory, is significantly affected even by weak nonlinearity resulting in inhibition of localization. However, the extended states corresponding to allowed regions ($T \cong 1$) in the linear theory remain almost unaffected by it (except for a scaling of the amplitude).

(b) The field distribution corresponding to different values of U_t (i.e., nonlinearity) for forbidden regions (i.e., for those U_t for which $T \cong 0$) appears to exhibit critical state behavior.

(c) The evidence of bulk localization is apparent from the nature of field distribution corresponding to lower values of U_t (i.e., nonlinearity) for allowed regions (i.e., for those values of U_t for which $T \cong 1$). For appropiately scaled U_t values the bulk localization persists even if one increases the number of layers. This is in sharp contrast to linear theory and the evidence of bulk localization is an indication of the delicate interplay between nonlinearity and dispersion and emergence of a new kind of structure.

(d) The bulk localized states can be self-similar.

The above discussion allows us to form a physical mechanism behind one of the main conclusions, roughly speaking, that increasing nonlinearity leads to decreasing localization. The feedback at each layer makes the field strongly dependent on the total phase shift. In the intense field regime, however, the dispersive nonlinearity renders the phase shift field dependent. Therefore, for each turn around every layer gets one erratic wandering of the field resulting in more and more loss of localization.

One pertinent point regarding the present treatment needs consideration. Since a great majority of the results on the theory of localization are asymptotically valid, we have tried to deal with a large number of layers. For computational tractability we have relied on slowly varying envelope approximation which allows us to develop the scheme in terms of a transfer-matrix formalism (and in this respect the treatment is akin to those employed for quasiperiodic systems in the context of electronic problem[2-7]) and is a good approximation in the intensity range we are working. Nevertheless, it has to be noted that too much increase in intensity may lead to the breakdown of this approximation.

Notwithstanding the computational limitations for working with finite-size material, which puts restriction on prediction of the results in the asymptotic limit, one can hope to obtain a fair idea regarding the nature of electromagnetic field states in the nonlinear regime in the problem of localization in optics. Significant departures from the linear theory, which are apparently generic in nature, arise and one cannot conclude that all the observable states are strong surface localized states in the nonlinear theory. The possible emergence of new structure due to bulk localization which does not fit into any of the category of states such as exponentially localized, extended, or critical states is amenable to new theoretical interpretation.

ACKNOWLEDGMENTS

The authors would like to thank G. S. Agarwal for many helpful discussions. One of the authors (S.D.G) would also like to thank the Department of Science and Technology, Government of India for partial financial support. Another of the authors (D.S.R.) acknowledges financial support from the Council of Scientific and Industrial Research.

---

[1]B. Simon, Adv. Appl. Math. **3**, 463 (1982) and references given therein.

[2]S. Ostlund, R. Pandit, D. Rand, H. J. Schellnhuber, and E. D. Siggia, Phys. Rev. Lett. **50**, 1873 (1983).

[3]M. Kohmoto, L. P. Kadonoff, and C. Tang, Phys. Rev. Lett. **50**, 1870 (1983).

[4]S. Ostlund and R. Pandit, Phys. Rev. B **29**, 1394 (1984).

[5]M. Kohmoto and J. R. Banavar, Phys. Rev. B **34**, 563 (1986).

[6]J. P. Lu, T. Odagaki, and J. L. Birman, Phys. Rev. B **33**, 4809 (1986).

[7]M. Kohmoto, B. Sutherland, and C. Tang, Phys. Rev. B **35**, 1020 (1987).

[8]M. Casdagli, Commun. Math. Phys. **107**, 285 (1986).

[9]F. Delyon and D. Petritis, Commun. Math. Phys. **103**, 441 (1986).

[10]A. Suto, Commun. Math. Phys. **111**, 409 (1987).

[11]G. Gumbs and M. K. Ali, Phys. Rev. Lett. **60**, 1081 (1988).

[12]G. Gumbs and M. K. Ali, J. Phys. A **21**, 1517 (1988).

[13]D. Shechtman, I. Black, D. Gratias, and J. W. Cahn, Phys. Rev. Lett. **53**, 1951 (1984).

[14]R. Merlin, K. Bajima, R. Clarke, F. T. Juang, and P. K. Bhattacharyya, Phys. Rev. Lett. **55**, 1768 (1986).

[15]M. Kohmoto, B. Sutherland, and K. Iguchi, Phys. Rev. Lett. **58**, 2436 (1987).

[16]L. Rahman and H. G. Winful, Phys. Rev. A **38**, 4935 (1988).

[17]E. Sendler and D. G. Steel, J. Opt. Soc. Am. B **5**, 1636 (1988).

[18]S. Dutta Gupta and D. S. Ray, Phys. Rev. B **38**, 3628 (1988).

[19]S. Dutta Gupta and G. S. Agarwal, J. Opt. Soc. Am. B **4**, 691 (1987).

[20]G. S. Agarwal and S. Dutta Gupta, Opt. Lett. **12**, 829 (1987).

[21]S. Dutta Gupta and D. S. Ray, Phys. Rev. B **40**, 10 604 (1989).

[22]R. Landauer, Philos. Mag. **21**, 863 (1970).

[23]P. D. Maker, R. W. Terhune, and C. M. Savage, Phys. Rev. Lett. **12**, 507 (1964).

[24]For a detailed discussion of the method of solution using slowly varying envelope approximation, see, for example, Refs. 19 and 25. For exact treatments, see, for example, W. Chen and D. L. Mills, Phys. Rev. Lett. **58**, 160 (1987); Phys. Rev. B **36**, 6269 (1987); Th. Peschel, P. Dannberg, U. Langbein, and F. Lerder, J. Opt. Soc. Am. B **5**, 2530 (1988); U. Trutchel. F. Lederer, and M. Golz, IEEE J. Quantum Electron. **QE-25**, 194 (1989). Compared to the exact methods, slowly varying envelope approximation reduces the computer time drastically, which is crucial for large systems. Besides, in the intensity ranges considered in the paper the method is accurate enough and retains all of the essential physics.

[25]J. H. Marburger, in *Optical Phase Conjugation,* edited by R. A. Fisher (Academic, New York, 1983), p. 102.

[26]J. E. Sipe and H. G. Winful, Opt. Lett. **13**, 132 (1988).

[27]S. Dutta Gupta, J. Opt. Soc. Am. (to be published).