PHYSICAL REVIEW B VOLUME 41, NUMBER 11 15 APRIL 1990-I

Influence of localization on the Hall effect in narrow-gap, bulk semiconductors

Ramesh G. Mani

Joint Program for Advanced Electronic Materials, Department of Physics, University of Maryland and Laboratory for Physical Sciences, College Park, Maryland 20742

(Received 13 December 1989)

Our transport study of the narrow-gap, bulk (three-dimensional) semiconductors $Hg_{1-x}Cd_xTe$ and InSb reveals incipient, *nonquantized* "Hall plateaus" which coincide with minima of the Shubnikov–de Haas oscillations, analogous to the quantum Hall effect in two-dimensional systems. We attribute this effect to the existence of quasi-mobility-gaps at the bottom of each Landau level, originating from localization due to shallow hydrogenic donors and disorder.

Although low-temperature Shubnikov–de Haas (SdH) oscillations of the magnetoresistance in three-dimensional (3D) systems have been explained,[1] concurrent oscillatory deviations in the Hall effect, from the classical behavior ($R_{xy} \sim H$), are still not well understood. Previous investigators[2] have attributed Hall-effect oscillations to the following mechanisms: First, an oscillating carrier density resulting from a pinned Fermi level.[3] Second, a contribution to the Hall effect from σ_{xx} through the relation $R_{xy} \sim \sigma_{xy}/(\sigma_{xx}^2+\sigma_{xy}^2)$.[4] Here, R_{xy} is the Hall resistance, σ_{xx} (σ_{xy}) is the diagonal (off-diagonal) conductivity tensor component, and H is the applied magnetic field. Finally, Hall oscillations have also been attributed to higher-order, scattering corrections to σ_{xy}.[5,6] However, these explanations suffer two principal flaws: First, it is difficult to invoke a pinned Fermi level for every system in which Hall oscillations have been observed. Second, scattering contributions to the Hall effect are higher-order corrections while experiment frequently indicates large Hall oscillations, in low-doped samples, as in our studies.

The effect of localization upon the Hall effect in 3D semiconductor systems has received limited attention even though it is known that localization is the key factor which influences the width of the integral quantum Hall plateaus in 2D systems.[7] Finite width of the quantum Hall plateaus and dissipationless current flow in 2D systems has been attributed to the existence of localized states in the tails of the 2D Landau bands, which creates a mobility gap at the Fermi energy for near-integral filling factors. In contrast, the simple, ideal 3D Landau-level (LL) spectrum does not allow for mobility gaps between Landau levels since each LL extends to infinite energy. Thus, dissipationless transport is not expected in 3D semiconductor systems. Also, quantization of R_{xy} to values which depend only upon fundamental constants would not occur in 3D systems since parallel conduction in the third dimension introduces a geometrical factor, the sample thickness, which varies from sample to sample.[8] Our study of quantum oscillations in the narrow-gap semiconductors InSb and $Hg_{1-x}Cd_xTe$ reveals incipient, *nonquantized* "Hall plateaus" which coincide with the SdH minima. We suggest that these results are evidence for the existence of quasi-mobility-gaps even in 3D, bulk semiconductor systems.

In our transport studies, the four-terminal resistance R_{xx} and the Hall effect were measured with a constant dc current I applied to Hall-bar-type samples in the transverse configuration $\mathbf{I} \perp \mathbf{H}$. The samples were immersed in pumped liquid He^4 for $1.5\ K < T < 4.2\ K$ and they were in contact with a He^3 bath for $T < 1.5\ K$. The data were collected with a computer. Here, we report results for an InSb sample and a pair of $Hg_{1-x}Cd_xTe$ samples, labeled A ($x=0.206\pm0.004$) and B ($x=0.20\pm0.01$).[9,10] In Fig. 1, we plot the low-temperature transport data for the InSb sample, $n(4.2\ K)=5\times10^{14}\ cm^{-3}$ and μ(4.2

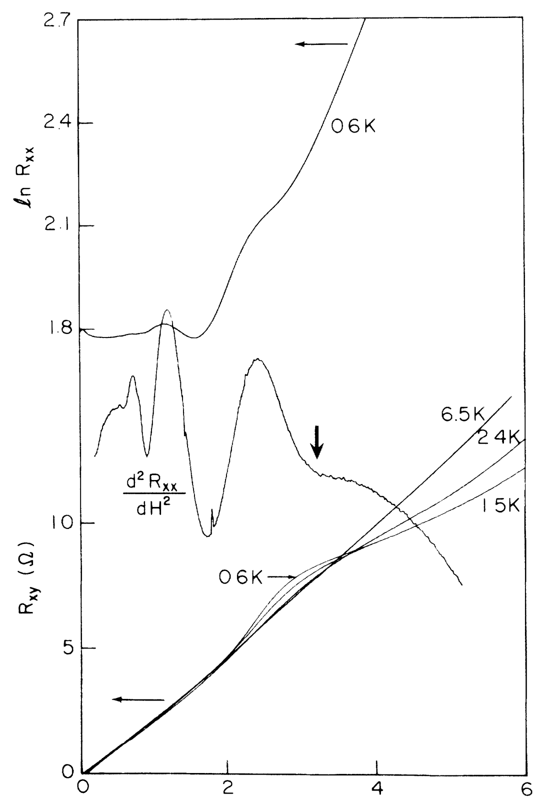

FIG. 1. As R_{xx} exhibits weak SdH oscillations in InSb (top), the oscillatory part, d^2R_{xx}/dH^2, of R_{xx} is shown in the center. The bold arrow indicates the last SdH minimum. R_{xy} deviates from classical behavior with decreasing T (bottom) and shows a plateau at the lowest temperatures.

K) = 125 000 cm^2/V s. Figure 1 (top) shows that R_{xx} increases rapidly versus H with weak SdH oscillations superimposed upon the background. The SdH oscillations were enhanced using standard field-modulation techniques and the results are also shown in Fig. 1 (center). The Hall effect, shown in Fig. 1 (bottom), exhibits classical behavior for $T > 6.5$ K and develops inflections with decreasing temperatures which result in a Hall plateau for $H \sim 3.5$ kOe. Note that the Hall plateau coincides with the last SdH minimum but its value *does not* equal $h/e^2 = 25812$ Ω as would be the case for a $\nu = 1$ quantum Hall plateau in a 2D system.[7] Finally, magneto-optical studies on the same sample indicate impurity cyclotron resonance in addition to free-electron cyclotron resonance *at fields as low as* $H = 2.5$ kOe for $\hbar\omega = 3$ meV while SdH and Hall oscillations persist to ~4 kOe.[11]

The transport results for the $Hg_{1-x}Cd_xTe$ sample B, $n(4.2\ K) \sim 3\times10^{15}\ cm^{-3}$ and $\mu(4.2\ K) = 300000$ cm^2/V s, are shown in Fig. 2. The plot shows that the SdH minima (R_{xx}) coincide with inflections in R_{xy}. The oscillatory components ΔR_{xx} and ΔR_{xy} have similar amplitudes (see lower part of Fig. 2) and there is a ~90° phase difference between these two oscillatory components. We point out that the quantum Hall effect (QHE) in $GaAs/Al_xGa_{1-x}As$ heterostructures is characterized by a similar 90° phase difference, especially when the oscillatory part of the Hall resistance $\Delta R_{xy} = R_{xy} - H/ne$ is compared with SdH (R_{xx}) oscillations. In the conventional percolation picture of QHE, there are no current-carrying states at E_F away from the sample edges, for near-integral filling factors, due to localization of states at the Landau subband edges. Thus, scattering is suppressed and $R_{xx} \to 0$ as $T \to 0$. The Laughlin-Halperin arguments show that the Hall resistance becomes quantized to $R_{xy} = h/\nu e^2$ for near-integral filling factors when E_F lies among localized states.[7] Thus, the 90° phase difference in $GaAs/Al_x$- $Ga_{1-x}As$ reflects the existence of a mobility gap in 2D systems. Similarly, the 90° phase difference observed in low-doped bulk semiconductors (Fig. 2) can be interpreted as evidence for the existence of quasilocalized states in these systems.

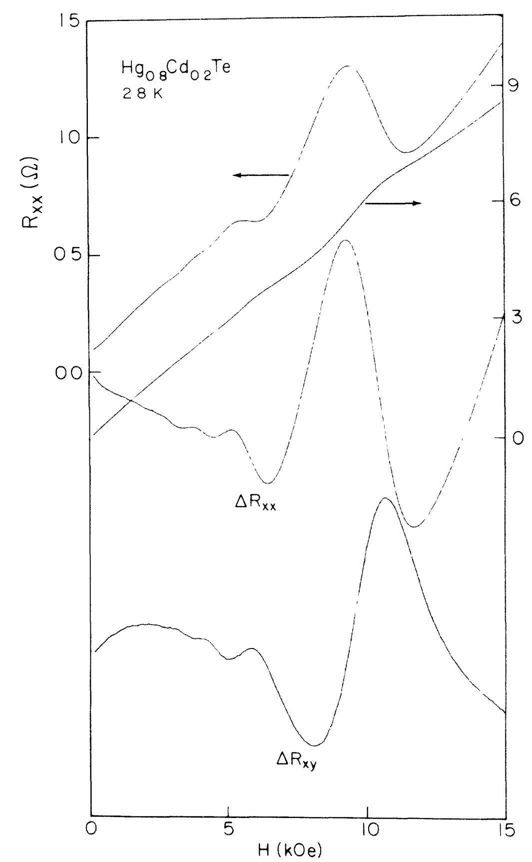

FIG. 2. R_{xx} and R_{xy} are shown for the $Hg_{0.80}Cd_{0.20}Te$ sample B. Minima of SdH oscillations coincide with inflections in R_{xy}. Note the ~90° phase difference between ΔR_{xx} and ΔR_{xy}.

Figure 3 shows the transport data for sample A, $n(4.2\ K) = 1.1\times10^{15}\ cm^{-3}$ and $\mu(4.2\ K) = 300000$ cm^2/V s. The data suggest large R_{xy} oscillations as in the other samples. Also, the SdH oscillations show anomalous behavior versus T: The last minimum gets *deeper* with decreasing T while the height of the last maximum is roughly independent of T. The standard picture of 3D, $I \perp H$, SdH oscillations associates peaks in R_{xx} with an enhancement in the scattering, $R_{xx} \sim \sigma_{xx}/\sigma_{xy}^2$, as the Fermi level E_F sweeps through the singularity in the density of states (DOS) at the bottom of each Landau level.[1] In this picture, σ_{xx} becomes more sensitive to the singularity in the DOS at lower T when the Fermi-Dirac distribution "sharpens-up." Thus, the peak SdH amplitude is usually expected to increase with decreasing T. However, our observation of a deeper SdH minimum with decreasing T

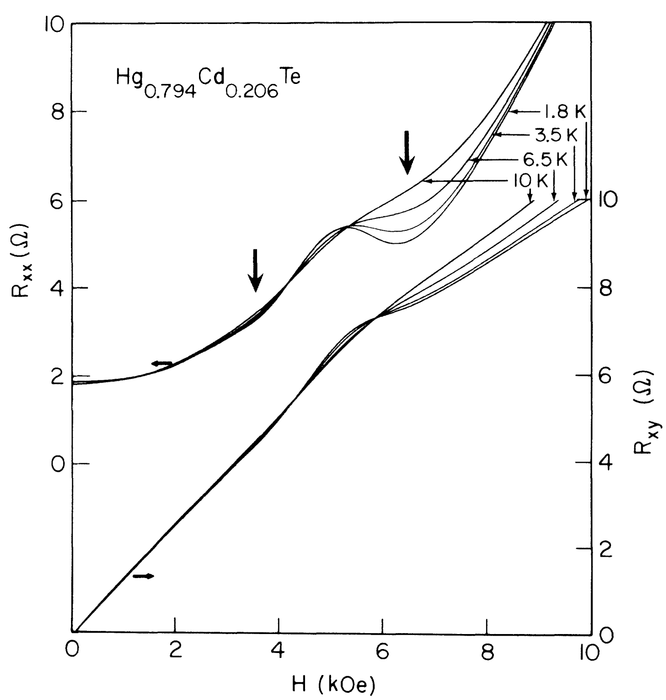

FIG. 3. The temperature dependence of R_{xx} and R_{xy} are shown for the $Hg_{0.794}Cd_{0.206}Te$ sample A. Note the anomalous temperature dependence of the SdH minimum, indicated by the bold arrows.

suggests that scattering is *suppressed* for particular filling factors, i.e., bands of localized states which do not carry current sweep across the Fermi level versus H and the lack of current-carrying states at E_F suppresses dissipation.

An illustration of our model for these transport effects is shown in Fig. 4. The ideal, disorder-free, 3D, DOS for $H=0$ ($dN/dE_{H=0} \sim E^{1/2}$) and $H\neq 0$ ($dN/dE_{H\neq 0}$) are shown in Fig. 4(a) while the effects of impurities and disorder are illustrated in the rest of the figure. Previous magneto-optical studies of InSb and $Hg_{1-x}Cd_xTe$ have revealed transitions between donor-bound states [inset Fig. 4(b)] associated with the $N=0$ and the $N=1$ Landau levels, i.e., impurity cyclotron resonance, in addition to the free-electron cyclotron resonance (CCR).[12] The transition energy between donor states of different Landau levels is only slightly greater than the Landau-level separation $\hbar\omega_c$ since the hydrogenic-binding energy R^* is small compared to $\hbar\omega_c$ in these narrow-gap systems.[12,13] For our transport studies, these results imply the existence of a reservoir of quasilocalized states below each Landau level [see Fig. 4(b)]. Also, we expect that spatial fluctuations of the band edge due to disorder would also effectively localize a fraction of the zero-transverse-momentum, i.e., $\hbar k_z \sim 0$, states at the bottom of each Landau level [see Fig. 4(b)]. We assume that these quasilocalized states do not contribute to R_{xy}.

In a finite-sized system, the formation of a bound state reduces the volume available for extended states. Thus, the number of current-carrying states decreases as the number of quasilocalized states increases. Also, the large value of $\gamma = \hbar\omega_c/2R^*$ in these narrow-gap systems imply that the disorder-broadened impurity band can overlap the Landau levels. These points suggest that our transport results can be modeled using the DOS shown in Fig. 4(c), which is similar to the ideal 3D case shown in Fig. 4(a), except that each Landau level includes a mobility edge E_m to separate the quasilocalized states from the current-carrying states.

We have simulated the Hall effect using the model DOS shown in Fig. 4(c), neglecting nonparabolicity and finite-T effects. As in the percolation picture of the QHE, we have also assumed that current-carrying states "speed up" and exactly compensate for the lost current-carrying capability due to localization.[7] Thus, in our picture, $R_{xy} = H/e\sum n^i_{ex} S^i$. Here, n^i_{ex} is the number of extended states below E_F associated with the ith Landau level, S^i is the speed-up factor which assures that the plateau resistance is independent of $E_m/\hbar\omega$, and E_F satisfies charge neutrality by counting the extended states and quasilocalized states at each value of H. The simulated DOS at E_F, which is $\sim R_{xx}$, exhibits SdH oscillations versus H [Fig. 5(a)]. Figure 5(b) shows R_{xy} for various values of $E_m/\hbar\omega_c$. The figure indicates that R_{xy} develops "pla-

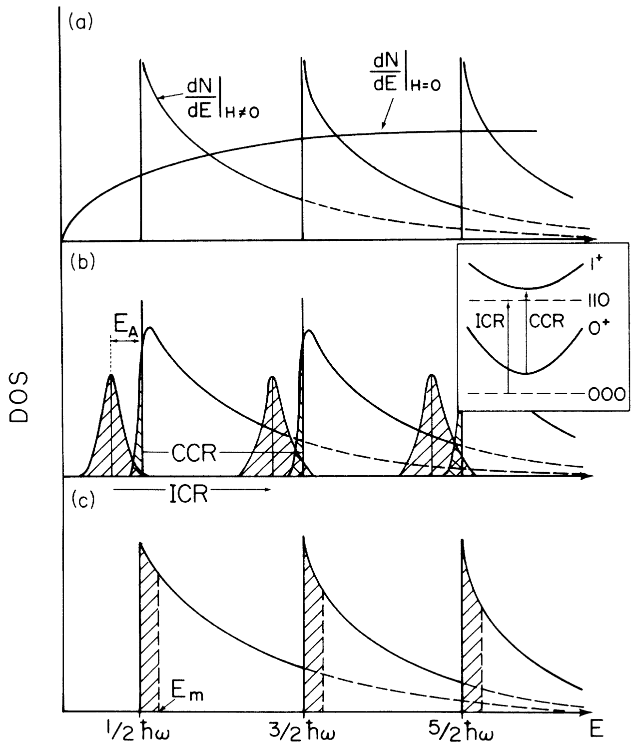

FIG. 4. (a) The ideal, 3D DOS's are shown for $H=0$ and $H\neq 0$. (b) The DOS, including excited hydrogenic bound states and disorder. The inset in (b) illustrates transitions between hydrogenic bound states (ICR) and cyclotron resonance (CCR). (c) The model DOS used to simulate the Hall effect. E_m denotes the mobility edge which separates the current-carrying states from the quasilocalized states [shaded regions in (c)].

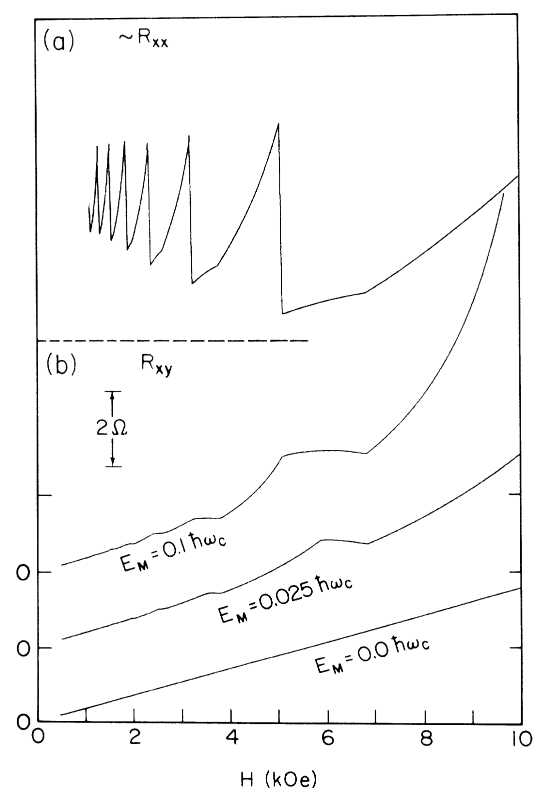

FIG. 5. The DOS at E_F which is $\sim R_{xx}$ is shown in (a). In (b), R_{xy} is shown for three values of E_M, where E_M is the mobility edge shown in Fig. 4(c). Note that $\sim R_{xx}$ minima coincide with the nonquantized Hall plateaus.

teaus" versus H which become wider as $E_m/\hbar\omega_c$ is increased. This behavior is similar to the observed correlation between the quantum Hall plateau width and increased disorder (localization) in 2D systems.[7] However, note that the R_{xy} plateaus *are not* quantized to h/ve^2 as in the quantum Hall effect. Also, minimal scattering or minimum R_{xx} occurs when R_{xy} develops plateaus. However, R_{xx} does not vanish because there are current-carrying states associated with lower Landau levels at E_F, even when E_F is pinned in the mobility gap of a particular Landau level.[14] Finally, we point out two principal effects of finite T which can influence the Hall plateaus. First, for $T > 0$, carriers can be thermally activated out of the quasilocalized states that are in the vicinity of E_F. Second, current-carrying states, which are within k_BT of E_F, can contribute to the Hall resistance. These effects would tend to smear out the Hall plateaus with increasing temperatures for fixed disorder $(E_m/\hbar\omega)$ as in the data of Fig. 1. As E_m is related to the donor-binding energy in our picture and the donor-binding energy varies strongly with the field in narrow-gap semiconductors, based on the strong field studies of field-induced localization in low-doped InSb and $Hg_{1-x}Cd_xTe$ (Refs. 15 and 16), we expect the Hall plateaus to be most pronounced at the lowest T and filling factors.

In summary, our comparative study of SdH oscillations and the Hall effect in the narrow gap, bulk semiconductors InSb and $Hg_{1-x}Cd_xTe$ indicates incipient Hall plateaus and anomalous T dependence of the SdH oscillations which we have attributed to localization effects.

We acknowledge useful discussions with L. Ghenim, H. A. Fertig, D. Belitz, F. C. Zhang, and S. Das Sarma. Special thanks to Professor J. R. Anderson for much guidance and advice. This work was supported by the U.S. Army Research Office and the U.S. Defense Advanced Research Projects Agency under Grant No. DAA G29-85-K-0052.

[1]L. M. Roth and P. N. Argyres, *Semiconductors and Semimetals,* edited by R . K. Willardson and A. C. Beer (Academic, New York, 1966), Vol. 1, p. 175.

[2]A. I. Ponomarev *et al.*, Fiz. Tekh. Poluprovodn. **11**, 45 (1977) [Sov. Phys. Semicond. **11**, 24 (1977)]; V. I. Ivanov-Omskii *et al.*, Fiz. Tekh. Poluprovodn. **7**, 715 (1973) [Sov. Phys. Semicond. **7**, 496 (1973)]; L. M. Bliek *et al.*, Phys. Status Solidi **31**, 115 (1969); G. A. Antcliffe *et al.*, Phys. Lett. **20**, 119 (1966); T. O. Yep *et al.*, Phys. Rev. **156**, 939 (1967).

[3]E. N. Adams *et al.*, J. Phys. Chem. Solids **10**, 254 (1959).

[4]S. T. Pavlov *et al.*, Zh. Eksp. Teor. Fiz. **48**, 701 (1965) [Sov. Phys. JETP **21**, 1049 (1965)].

[5]G. I. Guseva *et al.*, Phys. Status Solidi **25**, 775 (1968).

[6]I. G. Kuleev *et al.*, Fiz. Nizk. Temp. **2**, 123 (1976); **3**, 308 (1977) [Sov. J. Low Temp. Phys. **2**, 64 (1976); **3**, 147 (1977)].

[7]For a review, see *The Quantum Hall Effect,* edited by R. E. Prange and S. M. Girvin (Springer-Verlag, New York, 1987).

[8]B. I. Halperin, Jpn. J. Appl. Phys. **26**, Suppl. 26-3, 1913 (1987).

[9]Thanks to J. B. Choi and H. D. Drew (D. A. Nelson and Honeywell, Inc.,) for the InSb ($Hg_{1-x}Cd_xTe$) sample.

[10]R. G. Mani *et al.*, Phys. Rev. B **36**, 9146 (1987).

[11]J. B. Choi, Ph.D thesis, University of Maryland, 1989 (unpublished).

[12]W. S. Boyle *et al.*, Phys. Rev. **107**, 903 (1957); E. Gornik *et al.*, Phys. Rev. Lett. **40**, 1151 (1978); V. J. Goldman *et al.*, *ibid.* **56**, 968 (1986).

[13]For InSb ($Hg_{0.8}Cd_{0.2}Te$), $m^*/m = 0.014$ (0.005), and $R^* = 0.7$ meV (0.3 meV).

[14]The coexistence of quasilocalized states and extended states at the same energy originates from the accidental degeneracy of states associated with different Landau levels.

[15]R. G. Mani, Phys. Rev. B **40**, 809 (1989).

[16]M. Shayegan *et al.*, Phys. Rev. B **38**, 5585 (1988).