Quantum theory of the Faraday magneto-optical effect in paramagnetic media

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In some paramagnetic media, the effective field H_i, which includes an effective exchange field and an applied field, can cause a splitting of the ground level. In the temperature region $T > T_c$ (T_c is the Curie temperature), there are certain probability distributions of electrons on two energy levels due to ground-state splitting. This induces the ground level for a double transition. It is shown that there are several temperature behaviors for the magneto-optical Faraday effect in paramagnetic media. If the applied field is not too large, the ratio of the Verdet constant to the magnetic susceptibility is $V/x = A(1+BT)$ in some media, or $V/x = A(1+B/T)$ in other media. It is also indicated that both the real part θ' and imaginary part θ'' of the Faraday rotation are proportional to the effective field H_i, and show similar temperature behaviors beyond the region of optical absorption.

I. INTRODUCTION

With the wide use of magneto-optical materials and devices, more and more attention has been paid to the study of magneto-optical materials. Up to now, many works have been reported on the theoretical research of the magneto-optical effects in paramagnetic magneto-optical media.[1,2] It is generally believed that the magneto-optical effects in paramagnetic media are derived from the first Zeeman effect, and the ground level is degenerate, or if not degenerate, all electrons are concentrated on the lowest ground level since the temperature is much lower. By this theory only the origin and the frequency dependence of the magneto-optical effects can be roughly explained, but the complicated temperature behavior cannot be explained at the same time. For example,[3] it has been discovered that the temperature dependence of the Verdet constant V is obviously different from that of the magnetic susceptibility x in paramagnetic media, and V/x is proportional to the temperature T in some paramagnetic media such as NdF_3, PrF_3, or proportional to $1/T$ in others, such as CeF_3.

In our opinion, there are two aspects that have not been taken into account in the previous works: (1) There are various interactions in some paramagnetic media in which the exchange interaction and the crystal field are especially important, and these interactions can be equivalent to the effective field which causes the splitting of the ground level together with the applied field H_e. (2) The effective exchange field H_v can be expressed as νM, and its value can be compared with H_e in general. But in the temperature regions above the Curie point T_C, $kT > \nu M$, there are certain distributions of electrons on the two lowest-energy levels of the ground state. So it is not proper to consider that only the electrons on the lowest-energy level of the ground state may contribute to the electron transition.

After taking into account the above two aspects, the Faraday effect in paramagnetic media has been theoretically analyzed in this paper, and it is shown by calculation that there are several temperature dependences for the Faraday effect and some analogous behaviors for the real part and the imaginary part of the Faraday rotation in the media. This theoretical analysis agrees with the experimental results and is more comprehensive than others obtained by classical theory.[4]

II. QUANTUM THEORY

With the neglect of the local field acting on the electric dipole, the dielectric constant tensor elements can be written as

$$\epsilon_{ij} = \epsilon_0(\delta_{ij} + N\alpha_{ij}) , \tag{1}$$

where α_{ij} is the ij component of polarizability α, δ_{ij} is the Dirac function, and N is the atomic or ionic number per unit volume.

By supposing that one electron is attached to an atom or ion in the media, the ground level and the excited level are labeled by a and b, respectively, ω is the frequency of incident light, $\hbar\omega_{ab}$ is the separation between energy levels a and b, and Γ_{ab} is the linewidth. The components of polarizability are given by Condon and Shortley as[5]

$$\alpha_{ij} = \sum_a \frac{\beta_a}{\hbar} \sum_b \left[\frac{p^i_{ab} p^j_{ba}}{\omega_{ab} + \omega - i\Gamma_{ab}} + \frac{p^j_{ab} p^i_{ba}}{\omega_{ab} - \omega + i\Gamma_{ab}} \right] , \tag{2}$$

where $i,j = x,y,z$, β_a is the probability of an electron lying in the level a, and $\beta_{max} = 1$, $P^x_{ab} = \langle \psi_a | ex | \psi_b \rangle$ is the electric dipole matrix element. Substituting Eq. (2) into (1) and defining $p^{\pm}_{ab} = p^x_{ab} \pm i p^y_{ab}$, $p^{\pm}_{ab} = p^{\mp}_{ba}$, then

$$\epsilon_{xy} = \frac{\omega_p^2}{2i} \sum_{a,b} \frac{\beta_a}{\omega_{ab}} \frac{(\omega - i\Gamma_{ab})(f^+_{ab} - f^-_{ab})}{\omega^2_{ab} - \omega^2 + \Gamma^2_{ab} + 2i\omega\Gamma_{ab}} , \tag{3}$$

where $\omega_p^2 = \epsilon_0 Ne^2/m$,

$$f^{\pm}_{ab} = \frac{m\omega_{ab}}{\hbar e^2} |p^{\pm}_{ab}|^2$$

are the transition probabilities of electrons excited from a to b by right and left circular polarized light, respectively, i.e., oscillator strengths.

In the crystals where the symmetry is higher than that of the orthorhombic system, or in the optical isotropic media, the Faraday rotation θ is

$$\theta=\theta'+i\theta''=\frac{\omega L}{2c}\frac{i\epsilon_{xy}}{n}\ , \tag{4}$$

where n is the index of refraction, L is the distance of the light passing through the medium, and c is the light velocity in vacuum. From Eqs. (3) and (4), the real part θ' and the imaginary part θ'' of θ are obtained

$$\begin{aligned}\theta'&=\frac{\omega_p^2 L\omega^2}{4nc}\sum_{a,b}\frac{\beta_a}{\omega_{ab}}\frac{(f_{ab}^{+}-f_{ab}^{-})(\omega_{ab}^2-\omega^2-\Gamma_{ab}^2)}{(\omega_{ab}^2-\omega^2+\Gamma_{ab}^2)^2+4\omega^2\Gamma_{ab}^2}\ ,\\ \theta''&=-\frac{\omega_p^2 L\omega}{4nc}\sum_{a,b}\frac{\beta_a}{\omega_{ab}}\frac{\Gamma_{ab}(f_{ab}^{+}-f_{ab}^{-})(\omega_{ab}^2+\omega^2+\Gamma_{ab}^2)}{(\omega_{ab}^2-\omega^2+\Gamma_{ab}^2)^2+4\omega^2\Gamma_{ab}^2}\ .\end{aligned} \tag{5}$$

III. FARADAY EFFECT

At temperatures above T_C, ferromagnetic, ferrimagnetic, and antiferromagnetic media will change into paramagnetic media. In the media there are exchange interactions between electron spins, and the ground level will be split by both H_v and H_e, as shown in Fig. 1. Near T_c, the energy of the exchange interaction is nearly equal to the energy of the thermal motion, $kT_c \approx H_v$. Therefore, when $T>T_c$ and H_e is not too large, kT will be larger than the splitting energy $2\hbar\Delta$ of the ground level. This means that there may be electron distributions on both of the split ground levels. This distribution can be expressed by a Boltzmann distribution

$$\begin{aligned}\beta_{a1}&=e^{-\beta E_1}\Big/\sum_l e^{-\beta E_l}\ ,\\ \beta_{a2}&=e^{-\beta E_2}\Big/\sum_l e^{-\beta E_l}=\beta_{a1}e^{-2\beta\hbar\Delta}\ ,\end{aligned} \tag{6}$$

where $E_2=E_1+2\hbar\Delta$, $\beta=1/kT$ and k is the Boltzmann constant.

Let the frequency shift caused by the ground level splitting be defined as

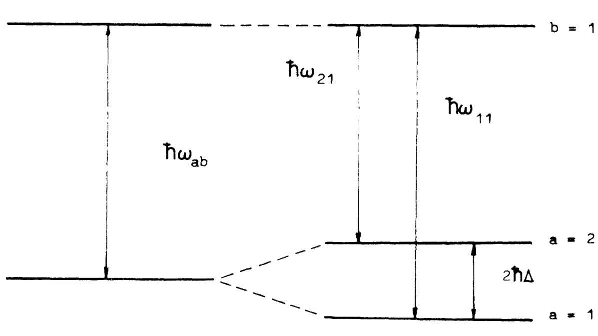

FIG. 1. The double transition with ground-state splitting.

$$\omega_{11}=\omega_0+\Delta,\quad \omega_{21}=\omega_0-\Delta\ .$$

Also,

$$\Gamma_{11}=\Gamma_{21}=\Gamma,\quad f_{11}^{+}=0,\quad f_{21}^{+}=f^{+},\quad f_{11}^{-}=f^{-},\quad f_{21}^{-}=0\ .$$

On the conditions that $\Delta \ll \Gamma \ll \omega_0$ and $2\hbar\Delta \ll kT$, it can be given from Eqs. (5)

$$\begin{aligned}\theta'&=\frac{\omega_p^2 L\beta_{a1}\omega^2}{4nc\,\omega_0^2[(\omega_0^2-\omega^2)^2+4\omega^2\Gamma^2]}\\ &\quad\times\{[(1-2\beta\hbar\Delta)f^{+}-f^{-}]\omega_0(\omega_0^2-\omega^2)\\ &\qquad-[(1-2\beta\hbar\Delta)f^{+}+f^{-}](\omega_0^2+\omega^2)\Delta\}\ ,\\ \theta''&=\frac{-\omega_p^2 L\beta_{a1}\Gamma\omega}{4nc\,\omega_0^2[(\omega_0^2-\omega^2)^2+4\omega^2\Gamma^2]}\\ &\quad\times\{[(1-2\beta\hbar\Delta)f^{+}-f^{-}]\omega_0(\omega_0^2+\omega^2)\\ &\qquad+[(1-2\beta\hbar\Delta)f^{+}+f^{-}](\omega_0^2-\omega^2)\Delta\}\ ,\end{aligned} \tag{7}$$

where $e^{-2\beta\hbar\Delta}\cong 1-2\beta\hbar\Delta$.

With Eqs. (7) there are four cases to be discussed as follows.

(1) If $f^{+}=f^{-}=f_0$, $\beta_{a1}\neq\beta_{a2}$, then the probabilities of electron transitions excited, respectively, by the left and right circular polarized light are equal but the occupation probabilities are not the same. Correspondingly, if $T>T_c$, but not $T\gg T_c$, then it can be shown from Eqs. (7)

$$\begin{aligned}\theta'&=-\frac{\omega_p^2 L\beta_{a1}f_0\Delta\omega^2}{2nc\,\omega_0^2[(\omega_0^2-\omega^2)^2+4\omega^2\Gamma^2]}\\ &\quad\times[\beta\hbar\omega_0(\omega_0^2-\omega^2)+(\omega_0^2+\omega^2)]\ ,\\ \theta''&=-\frac{\omega_p^2 L\beta_{a1}f_0\Delta\omega}{2nc\,\omega_0^2[(\omega_0^2-\omega^2)^2+4\omega^2\Gamma^2]}\\ &\quad\times[(\omega_0^2-\omega^2)-\beta\hbar\omega_0(\omega_0^2+\omega^2)]\ .\end{aligned} \tag{8}$$

(a) Far from the transition region where $\omega \ll \omega_0$ and $\hbar\omega_0/kT \gg 1$, from Eqs. (8) then

$$\begin{aligned}\theta'&=-\frac{\omega_p^2\beta_{a1}f_0L\omega^2}{4nck\,\omega_0^3}\frac{2\hbar\Delta}{T}\ ,\\ \theta''&=\frac{\omega_p^2\beta_{a1}f_0L\omega\Gamma}{4nck\,\omega_0^3}\frac{2\hbar\Delta}{T}\ .\end{aligned} \tag{9}$$

The ground level splitting ΔE due to H_v and H_e is

$$\Delta E=2\hbar\Delta=mg\mu_B\mu_0H_i\ , \tag{10}$$

where m is the magnetic quantum number, g is Landé factor, μ_B is the Bohr magneton, μ_0 is the magnetic permeability in vacuum, and the effective field H_i is equal to (H_e+H_v).

The effective exchange field (molecular field) H_v in the paramagnetic media which is transformed from the ferromagnetic media at $T>T_c$ is equal to vM. For the paramagnetic media transformed from the ferrimagnetic and antiferromagnetic media, supposing that there are l-

number sublattices in every medium, we have the expression of the effective exchange field on every sublattice

$$\mathbf{H}_{\nu_1}=\nu_{11}\mathbf{M}_1+\nu_{12}\mathbf{M}_2+\cdots+\nu_{1l}\mathbf{M}_l,$$
$$\mathbf{H}_{\nu_2}=\nu_{21}\mathbf{M}_1+\nu_{22}\mathbf{M}_2+\cdots+\nu_{2l}\mathbf{M}_l\ ,$$
$$\cdots \tag{11}$$
$$\mathbf{H}_{\nu_l}=\nu_{l1}\mathbf{M}_1+\nu_{l2}\mathbf{M}_2+\cdots+\nu_{ll}\mathbf{M}_l\ ,$$

where M_i ($i=1,2,\ldots,l$) is the magnetization of the ith sublattice. At any site in the medium H_ν is

$$\begin{aligned}\mathbf{H}_\nu&=\alpha_1\mathbf{H}_{\nu_1}+\alpha_2\mathbf{H}_{\nu_2}+\cdots+\alpha_l\mathbf{H}_{\nu_l}\\&=\sum_{i=1}^{l}\alpha_i\nu_{i1}\mathbf{M}_1+\sum_{i=1}^{l}\alpha_i\nu_{i2}\mathbf{M}_2+\cdots+\sum_{i=1}^{l}\alpha_i\nu_{il}\mathbf{M}_l\\&=\nu_1\mathbf{M}_1+\nu_2\mathbf{M}_2+\cdots+\nu_l\mathbf{M}_l\ .\end{aligned} \tag{12}$$

When $T>T_c$, $\mathbf{M}_1,\mathbf{M}_2,\ldots,\mathbf{M}_l$ are all along the direction of $\mathbf{H}_e$, and their values, like M (the total magnetization in medium), are proportional to H_e, respectively, so $\mathbf{H}_\nu$ can be written as

$$\mathbf{H}_\nu=\nu_1\beta_1\mathbf{M}+\nu_2\beta_2\mathbf{M}+\cdots+\nu_l\beta_l\mathbf{M}=\nu\mathbf{M}\ . \tag{13}$$

Based on the preceding analyses, it could be seen that $\mathbf{H}_\nu$ is equal to $\nu\mathbf{M}$ in all paramagnetic media which are respectively transformed from ferromagnetic, ferrimagnetic, and antiferromagnetic media at $T>T_c$, and the only difference is that, for the former, ν is the molecular field coefficient, and, for the latter, ν are the coefficients related to the molecular field coefficient but not the molecular field coefficient.

By substituting Eq. (10) into Eqs. (9), then

$$\theta'=-\frac{\mu_0 mg\mu_B\omega_p^2\beta_{a1}f_0\omega^2}{4nc\omega_0^3kT}LH_i\ ,$$
$$\theta''=\frac{\mu_0 mg\mu_B\omega_p^2\beta_{a1}f_0\omega\Gamma}{4nc\omega_0^3kT}LH_i\ . \tag{14}$$

Here H_i can be expressed in another form

$$H_i=(1+\nu x)H_e\ , \tag{15}$$

where x is the magnetic susceptibility. Combining Eqs. (14) and (15) we can find

$$\theta'=V_{11}LH_e\ ,$$
$$\theta''=V_{12}LH_e\ , \tag{16}$$

where

$$V_{11}=-\frac{\mu_0 mg\mu_B\omega_p^2\beta_{a1}f_0\omega^2}{4nc\omega_0^3kT}(1+\nu x)\ ,$$
$$V_{12}=-\frac{\Gamma}{\omega}V_{11}\ .$$

Since the magnetic susceptibility obeys the Curie-Weiss law $x=C/(T-T_p)$ (C is the Curie constant and T_p is the paramagnetic Curie point), then it can be rewritten as

$$V_{11}/x=A_{11}(1+B_{11}/T)\ ,$$
$$V_{12}/x=A_{12}(1+B_{12}/T)\ , \tag{17}$$

where

$$A_{11}=-\frac{\mu_0 mg\mu_B\omega_p^2\beta_{a1}f_0\omega^2}{4nc^2\omega_0^3k}\ ,$$
$$B_{11}=\nu C-T_p\ ,$$
$$A_{12}=-\frac{\Gamma}{\omega}A_{11},B_{12}=B_{11}\ .$$

(b) The electron transition region $\omega=\omega_0$. From Eqs. (8) and (10)

$$\theta'=-\frac{\mu_0 mg\mu_b\omega_p^2\beta_{a1}f_0}{8nc\hbar\Gamma^2}LH_i=V_{21}LH_e\ ,$$
$$\theta''=\frac{\mu_0 mg\mu_B\omega_p^2\beta_{a1}f_0}{8nc\Gamma kT}LH_i=V_{22}LH_e\ , \tag{18}$$

where

$$V_{21}=-\frac{\mu_0 mg\mu_B\omega_p^2\beta_{a1}f_0}{8nc\hbar\Gamma^2}(1+\nu x)\ ,$$
$$V_{22}=-\frac{\hbar\Gamma}{kT}V_{21}\ .$$

This can be rewritten as

$$V_{21}/x=A_{21}(1+B_{21}T)\ ,$$
$$V_{22}/x=A_{22}(1+B_{22}/T)\ , \tag{19}$$

where

$$A_{21}=-\frac{\mu_0 mg\mu_B\omega_p^2\beta_{a1}f_0}{8nc^2\hbar\Gamma^2}(\nu C-T_p)\ ,$$
$$B_{21}=\frac{1}{\nu C-T_p}\ ,$$
$$A_{22}=-\frac{\hbar\Gamma}{k(\nu C-T_p)}A_{21}\ ,$$
$$B_{22}=1/B_{21}\ .$$

(2) If $f^+=f^-=f_0$, $\beta_{a1}=\beta_{a2}=\frac{1}{2}$, the electron transition probabilities are equal and the occupation probabilities are the same also. Correspondingly, $T\gg T_c$, and $\hbar\omega_0/kT\ll1$. Then from Eqs. (7) and (10),

$$\theta'=-\frac{\mu_0 mg\mu_B\omega_p^2f_0(\omega_0^2+\omega^2)\omega^2}{8nc\hbar\omega_0^2[(\omega_0^2-\omega^2)^2+4\omega^2\Gamma^2]}LH_i=V_{31}LH_e\ ,$$
$$\theta''=-\frac{\mu_0 mg\mu_B\omega_p^2f_0\Gamma(\omega_0^2-\omega^2)\omega}{8nc\hbar\omega_0^2[(\omega_0^2-\omega^2)^2+4\omega^2\Gamma^2]}LH_i=V_{32}LH_e\ , \tag{20}$$

where

$$V_{31}=-\frac{\mu_0 mg\mu_B\omega_p^2f_0(\omega_0^2+\omega^2)\omega^2}{8nc\hbar\omega_0^2[(\omega_0^2-\omega^2)^2+4\omega^2\Gamma^2]}(1+\nu x)\ ,$$
$$V_{32}=\frac{\Gamma(\omega_0^2-\omega^2)}{\omega(\omega_0^2+\omega^2)}V_{31}\ .$$

This can also be rewritten as

$$V_{31}/x = A_{31}(1+B_{31}T) ,$$
$$V_{32}/x = A_{32}(1+B_{32}T) , \tag{21}$$

where

$$A_{31} = -\frac{\mu_0 m g \mu_B \omega_p^2 f_0 (\omega_0^2+\omega^2)\omega^2}{8nc^2\hbar\omega_0^2[(\omega_0^2-\omega^2)^2+4\omega^2\Gamma^2]}(\nu C - T_p) ,$$

$$B_{31} = \frac{1}{\nu C - T_p} ,$$

$$A_{32} = \frac{\Gamma(\omega_0^2-\omega^2)}{\omega(\omega_0^2+\omega^2)} A_{31} ,$$

$$B_{32} = B_{31} .$$

(3) If $f^+ \neq f^-$, $\beta_{a1}=\beta_{a2}=\frac{1}{2}$, the electron transition probabilities are not equal but the occupation probabilities are the same. With $T \gg T_c$, then from Eqs. (7) and (10)

$$\theta' = \frac{\omega_p^2 L \omega^2}{8nc\omega_0^2[(\omega_0^2-\omega^2)^2+4\omega^2\Gamma^2]} \times \left[(f^+ - f^-)\omega_0(\omega_0^2-\omega^2) - \frac{\mu_0 m g \mu_B (f^+ + f^-)(\omega_0^2+\omega^2)}{2\hbar} H_i \right]$$
$$= V_{01}L + V_{41}LH_e$$
$$\theta' = -\frac{\omega_p^2 L \omega \Gamma}{8nc\omega_0^2[(\omega_0^2-\omega^2)^2+4\omega^2\Gamma^2]} \times \left[(f^+ - f^-)\omega_0(\omega_0^2+\omega^2) + \frac{\mu_0 m g \mu_B (f^+ + f^-)(\omega_0^2-\omega^2)}{2\hbar} H_i \right]$$
$$= V_{02}L + V_{42}LH_e , \tag{22}$$

where

$$V_{01} = \frac{\omega_p^2 (f^+ - f^-)(\omega_0^2-\omega^2)\omega^2}{8nc\omega_0[(\omega_0^2-\omega^2)^2+4\omega^2\Gamma^2]} ,$$

$$V_{41} = -\frac{\mu_0 m g \mu_B (f^+ + f^-)(\omega_0^2+\omega^2)\omega_p^2\omega^2}{16nc\omega_0^2\hbar[(\omega_0^2-\omega^2)^2+4\omega^2\Gamma^2]}(1+\nu x) ,$$

$$V_{02} = -\frac{\Gamma(\omega_0^2+\omega^2)}{\omega(\omega_0^2-\omega^2)} V_{01} ,$$

$$V_{42} = \frac{\Gamma(\omega_0^2-\omega^2)}{\omega(\omega_0^2+\omega^2)} V_{41} ,$$

and this can also be rewritten as

$$V_{41}/x = A_{41}(1+B_{41}T) ,$$
$$V_{41}/x = A_{42}(1+B_{42}T) , \tag{23}$$

where

$$A_{41} = -\frac{\mu_0 m g \mu_B (f^+ + f^-)(\omega_0^2+\omega^2)\omega_p^2\omega^2}{16nc^2\omega_0^2\hbar[(\omega_0^2-\omega^2)^2+4\omega^2\Gamma^2]}(\nu C - T_p) ,$$

$$B_{41} = \frac{1}{\nu C - T_p} ,$$

$$A_{42} = \frac{\Gamma(\omega_0^2-\omega^2)}{\omega(\omega_0^2+\omega^2)} A_{41} ,$$

$$B_{42} = B_{41} .$$

(4) If $f^+ = f_0$, $f^- = 0$, $\beta_{a1} \to 1$, $\beta_{a2} \to 0$. This is the case in which the ground level is degenerate or the temperature is very low and the effective field is quite large. Then the above approximations are no longer suitable, and it must be derived from Eqs. (5). With $\omega_0 \gg \Gamma$, Eqs. (5) give for this case,

$$\theta' = \frac{\omega_p^2 \omega^2 (\omega_0^2-\omega^2) f_0}{4nc\omega_0[(\omega_0^2-\omega^2)^2+4\omega^2\Gamma^2]} L ,$$
$$\theta'' = -\frac{\omega_p^2 \omega (\omega_0^2+\omega^2) f_0 \Gamma}{4nc\omega_0[(\omega_0^2-\omega^2)^2+4\omega^2\Gamma^2]} L . \tag{24}$$

Now, θ' and θ'' are not related to the temperature.

IV. DISCUSSIONS

It is shown by the preceding theoretical analysis that the Faraday effect in paramagnetic media, in which the relationship of the susceptibility x and temperature T obeys the Curie-Weiss law, have the following characteristics.

(1) By affecting both the effective exchange field νM and the applied field H_e in this medium, the ground level will be split. On the other hand, because the thermal motion energy is larger than the exchange-interaction energy in the medium, the electron probability distributions can exist on two or even more split ground levels when H_e is not too large, so the magneto-optical effects have various temperature behaviors.

(2) It is shown that both the real part θ' and the imaginary part θ'' of the Faraday rotation generally are proportional to the effective field H_i. As $H_i = (1+\nu x)H_e$, both θ' and θ'' can formally be expressed as proportional to H_e. This conforms with the classical theory discussed in Ref. 4.

(3) In the nonabsorption region, θ' and θ'' have similar temperature behaviors, and near the absorption region they may be different.

(4) If T_0 is the temperature where the electron distribution probabilities on two split ground levels are equal, then for $T_c < T < T_0$, $f^+ = f^- = f_0$, $\omega_0 \gg \omega$, and if H_e is not too large, the ratio of the Verdet constant to magnetic susceptibility V/x will vary with $1/T$ [as shown in Eqs. (17)]. It was experimentally shown by Leycuras *et al.*[3] that in CeF_3

$$V/x = -13706(1+75.9/T) .$$

This is consistent with our results.

(5) When $\omega=\omega_0$, i.e., in the absorption region, and if the other conditions are the same as in (4), and V/x for θ' is proportional to T [as shown in Eqs. (19)]. This means that the temperature behavior of the Faraday effect in the absorption region may be different than that in the nonabsorption region.

(6) When $T>T_0$, i.e., where $T\gg T_c$, and H_e is not too large, whether the electron transition probabilities induced, respectively, by the left and right circular polarized light are equal or not, V/x is always proportional to T [as shown in Eqs. (21) and (23)]. It is shown by experiments in Ref. 3 that, with $T>77$ K in NdF_3 and with $T>40$ K in PrF_3, $V/x=A(1+BT)$. This is perfectly consistent with the theoretical analysis in our paper. It is also worth mentioning that this paper shows that the coefficient A is closely related to the light wavelength λ (or the light frequency ω) but the coefficient B is not related to λ. This has also been shown by the experimental work in Ref. 3.

(7) When the ground level is degenerate, or the temperature is very low and the effective field (involving the applied field) is very large, θ' and θ'' are not related to the temperature T. This type of rotation is usually called a "paramagnetic rotation" with a "paramagnetic line shape."

(8) In different temperature regions there are different temperature behaviors of the Faraday effect in the same kind of paramagnetic media. For example, if $T_c<T<T_0$, $V/x=A(1+B/T)$, but if $T>T_0$, this may change into $V/x=A(1+BT)$.

It should be indicated in the end that the range of the effective field is very wide. There are the effective fields related to the magnetic crystal anisotropy, stress, and crystal field besides the effective exchange field discussed in this paper. These effective fields being taken into account, the anisotropy of θ' and θ'', etc., may be explained. In addition, the thesis that the ground level for a double transition exists in some paramagnetic media is also suitable for the Kerr effect. This is planned to be discussed in another paper.

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