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Nonequilibrium in metallic microstructures in the presence of high current density

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Thin narrow films with a cross section of about 10^{-10} cm^2 have been studied under nonequilibrium conditions. Working at liquid-helium temperature, with current densities up to 10^7 A/cm^2, corresponding to electric fields up to 0.3×10^6 V/m, we raised the electron temperature T_e considerably above the phonon temperature T_{ph}. We tried to use weak localization and the Coulomb anomaly of the resistance as thermometers. Weak localization worked very well, but in the case of the Coulomb anomaly we found inconsistencies. We predict that the Coulomb anomaly shows nonohmic behavior. Our experimental data suggest an Eliashberg function $\alpha^2F(\Omega)$ which is linear in Ω. We can give upper limits for the escape time of the phonons from the film into the quartz. The evaluation with our weak-localization thermometer yields consistent results.

I. INTRODUCTION

If an electric current flows through a metal, the external power supply transfers energy into the electron system. One generally assumes that the momenta of the electrons relax towards their equilibrium distribution within the transport lifetime τ_{tr}. Often it is implicitly assumed that the Coulomb heating energy ρj^2 relaxes within the same time. For a metal with a very short mean free path this would mean that the electron system is practically in perfect thermal equilibrium with the bath temperature, i.e., the phonon system. Since the study of weak localization we know, however, that the electrons in a disordered metal maintain their phase coherence (and therefore their energy) over a time τ_i which can be longer than τ_{tr} by 4 or 5 orders of magnitude.

The electron system needs the time $\tau_{e\text{-ph}}$, the electron-phonon relaxation time, to transfer its excess energy into the phonon system of the metal by electron-phonon processes. During this time the electron system accumulates the energy density $\rho j^2\tau_{e\text{-ph}}$, and the electron temperature is raised above the phonon temperature.

Such a temperature difference between the electron and phonon systems has been involuntarily observed in the millikelvin temperature range, where the electron-phonon processes have become so rare that electron heating can hardly be avoided. The first experimental observation was reported by Dolan and Osheroff[1] and has been studied subsequently (see, for example, Ref. 2). It remains the main obstacle to investigating resistance anomalies at very low temperatures.

In the temperature range of liquid helium it is much harder to observe such a nonequilibrium state. Because the electron-phonon relaxation time is much shorter than in the mK range one needs a considerable Joule heating and this tends to heat the whole sample. To avoid the heating of the sample, Gutfeld[3] started to perform pulse experiments more than two decades ago. In such experiments a thin metal film is condensed onto an insulator (for example, sapphire) and a short current or field pulse is applied to the film. Such pulse experiments have the advantage that one can perform time-of-flight experiments with the phonons and obtain information about the phonon emission of the system. However, one never reaches a steady-state situation because, during the experiment, the electron and the phonon temperature rise continuously.

We describe in the present paper steady-state experiments in which we expose the electron films to current densities up to 10^7 A/cm^2. An earlier more qualitative experiment was performed by one of the authors[4] a few years ago, and recently Liu *et al.*[5] published nonequilibrium measurements with a current density of about 10^4 A/cm^2. In this paper we report on recent experiments in which we use thin narrow films with 3-nm thickness and 4-μm width. These films are quench condensed onto a quartz plate which is at helium temperature. We send currents between 3 μA and 1 mA through the narrow films. The higher current corresponds roughly to a current density of 10^7 A/cm^2 and raises the electron temperature considerably above the phonon temperature. Despite the high current density we can still keep the total amount of generated heat relatively small since the total volume of our film is so small.

In a steady-state experiment it is most desirable to be able to measure the electron and phonon temperatures of the system. Fortunately, disordered thin films provide such thermometers in the temperature and field dependence of the electric resistance. These thermometers are the following.

(a) The Coulomb anomaly of the resistance. Its contribution to the resistance depends only on the electron temperature.

(b) Weak localization. The associated magnetoresistance yields the inelastic lifetime of the conduction electrons which is a function of both the electron and phonon temperatures.

The electric resistance in high-resistance films has the following contributions at low temperatures:

$$R(T)=R_0+\Delta R_{\mathrm{WL}}(T)+\Delta R_C(T)+\Delta R_{e\text{-ph}}(T)\ , \tag{1.1}$$

where R_0 is the residual resistance of the film, $\Delta R_{\mathrm{WL}}(T)$ is the contribution of weak localization, $\Delta R_C(T)$ is the contribution of the Coulomb anomaly due to electron-electron interaction, and $\Delta R_{e\text{-ph}}(T)$ is the (usual) thermal part of the electric resistance due to electron-phonon processes.

The Coulomb correction depends only on the electron temperature (see, for example, Refs. 6 and 7)

$$\Delta R_C = R_0^2 \frac{e^2}{2\pi^2\hbar}(1-F')\ln T\ , \tag{1.2}$$

where F' is a screening factor which is much less than one.

The Coulomb anomaly is therefore a suitable electron thermometer if we succeed in making it the dominant temperature-dependent factor in the resistance. For this purpose we use a film with a high resistance per square, because ΔR_C is proportional to the square of the resistance (per square).

On the other hand, weak localization makes it possible to measure the inelastic scattering rate of the conduction electrons $1/\tau_i$ (for reference see, for example, Refs. 8–10). Weak localization is an interference experiment with conduction electrons which are scattered by the impurities or lattice defects.[11] As a consequence, the resistance shows a pronounced dependence on the magnetic field, yielding bell-shaped magnetoresistance curves. The width of the magnetoresistance curves gives the inelastic scattering rate $1/\tau_i$. This rate is closely related to the electron-phonon relaxation time, as we will discuss below, and depends on both the electron and phonon temperatures. Therefore, weak localization is a second (although more indirect) thermometer in our thin films.

Since weak localization also causes a temperature-dependent correction to the resistance, one might suspect that the two thermometers disturb each other. However, since weak localization is very sensitive to a magnetic field, we suppress its contribution to the temperature dependence of the resistance with a magnetic field of 7 T. Therefore, we can decouple the two thermometers. By measuring the temperature dependence and the magnetic-field dependence of the resistance in thermal equilibrium these two thermometers can be calibrated.

The conventional thermal part of the resistance is much smaller than the Coulomb and weak-localization corrections at low temperatures. In addition, we can estimate its contribution: we found in former experiments that $\Delta R_{e\text{-ph}}/R_0 \simeq \tau_0/\tau_i$, where τ_0 is the elastic scattering time and τ_i is the inelastic scattering time as determined by magnetoresistance measurements using the theory of weak localization.

II. EXPERIMENT

Prior to the experiment we prepare the quartz plate at room temperature. First, gold electrodes are evaporated. Then the quartz plate is covered with photoresist. A mask with an undercut is prepared by photolithography to permit us to condense a film with 4-μm width and 80-μm length. At the bottom of the quartz plate, on the opposite side of the film, we glue a heater and an Allen-Bradley resistor to measure the temperature of the quartz plate. The quartz plate is mounted onto a copper block in the vacuum system of the cryostat. We perform the experiment in the following steps.

(a) Cryostat and quartz plate are cooled to helium temperature. An Au film is quenched condensed. This film is annealed to 50 K using a maximum current of about 1 mA. Afterwards the resistance per square is about 1000 Ω. The film itself is always in vacuum. This allows us to vary the equilibrium temperature of the film.

(b) We measure the resistance of the film as a function of temperature in thermal equilibrium. This is achieved by sending a small current (3 μA at 4 K and 30 μA above 20 K) through the film and heating the quartz plate with the heater on its back. (In the following, this is called "equilibrium heating.") This is done for about ten temperatures between 4.3 and 40 K.

(c) Then we vary the current at each equilbrium temperature between 3 and about 1000 μA in 10–20 steps. For each heating and current we measure the temperature of the quartz plate and the resistance of the film. Such a plot is shown in Fig. 1, which we discuss below.

(d) For most combinations of equilibrium heating and current we measure the magnetoresistance in the field range between -7 and +7 T.

In Fig. 1 the dependence of the resistance per square of an Au film is plotted versus the temperature of the quartz plate. The dashed curve gives the resistance in thermal

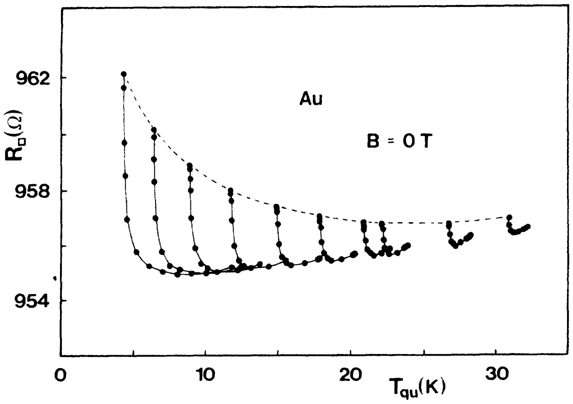

FIG. 1. The resistance per square of a Au film in zero magnetic field is plotted vs the temperature of the quartz plate. The dashed curve gives the resistance in thermal equilibrium (for sufficiently small current). From each equilibrium point a solid curve branches off which is obtained by increasing the current. The values of the current are 3, 10, 30, 50, 100, 200, 300, 400, 500, 700, and 900 μA.

equilibrium (for sufficiently small current). From each equilibrium point a solid curve, which is obtained by increasing the current, branches off. The values of the current are 3, 10, 30, 50, 100, 200, 300, 400, 500, 700, and 900 μA.

The equilibrium curve has a resistance minimum of about 956.8 Ω at 24 K. The nonequilibrium curves with higher currents yield considerably lower minima in the resistance. If the electron and phonon systems were in equilibrium, then the minimum of the resistance would be independent of the current. We conclude that the current density is high enough to heat the electron temperature considerably above that of the phonon system. (A large current increases the electron temperature far above the phonon temperature and reduces the resistance contribution of ΔR_C. Because of the strong spin-orbit scattering in Au, weak localization reduces the resistance at low temperature, i.e., increases it at high temperature. If, however, the phonon temperature remains low, then the contribution of weak localization increases more slowly with increasing current, and we obtain a lower-resistance minimum.)

To use the resistance as an electron thermometer we suppress the temperature dependence of weak localization with a magnetic field of 7 T. In Fig. 2 we have plotted the resistance for another film versus the temperature of the quartz plate for different equilibrium heatings and different currents. The plot is quite analogous to Fig. 1. The main difference is that we apply a magnetic field of 7 T perpendicular to the film and suppress weak localization. For each current through the film we determine the electron temperature by means of the resistance. As the dotted line demonstrates for a current of 100 μA and an equilibrium temperature of 4.4 K, we move horizontally to the equilibrium curve and then vertically down to the corresponding temperature (which yields, in this case, 10.9 K). In the present evaluation we apply this method only to temperatures up to about 15 K and leave the higher temperatures for a later, more sophisticated, evaluation.

In Fig. 3 we have plotted the equilibrium conductance L/L_{00} $[L_{00}=e^2/(2\pi^2\hbar)]$ of the film in the external field $B=7$ T as a function of $\ln(T)$. The slope of the straight line is 1.05. The logarithmic behavior is fulfilled up to about 15 K.

In Fig. 4 we present the measurement of Fig. 2 in a different plot. The resistance (in 7 T) is plotted versus the logarithm of the current for different equilibrium heatings. Since the resistance is proportional to $\ln T$ and the Joule-heating power $P=I^2R$ is proportional to the square of the current, this corresponds to a log-log plot of power versus electron temperature. One recognizes that for small currents the resistance is essentially constant and equal to the equilibrium value. With increasing Joule heating the curve bends to lower resistance. If we start with a low equilibrium temperature then we find a linear relation between $\ln P$ and $\ln T_e$.

For most experimental points in Fig. 2 we have measured the magnetoresistance. In Fig. 5 we have plotted several magnetoresistance curves in thermal equilibrium (dashed curves). The temperature on the left-hand side gives the equilibrium temperature. In the same figure we have plotted several nonequilibrium curves. These solid curves were obtained with zero heating of the quartz plate but increasing current through the film. The

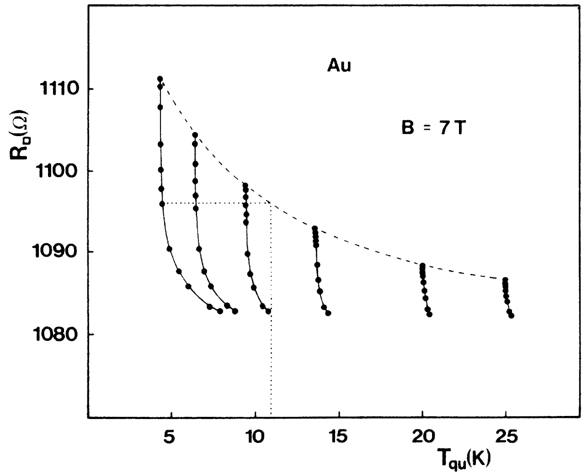

FIG. 2. The resistance per square of a Au film in a magnetic field of 7 T perpendicular to the film is plotted vs the temperature of the quartz plate. The dashed curve gives the resistance in thermal equilibrium (for sufficiently small current). From each equilibrium point a solid curve branches off which is obtained by increasing the current. The values of the current are 5, 10, 20, 40, 60, 80, 100, 200, 300, 400, 600, and 700 μA.

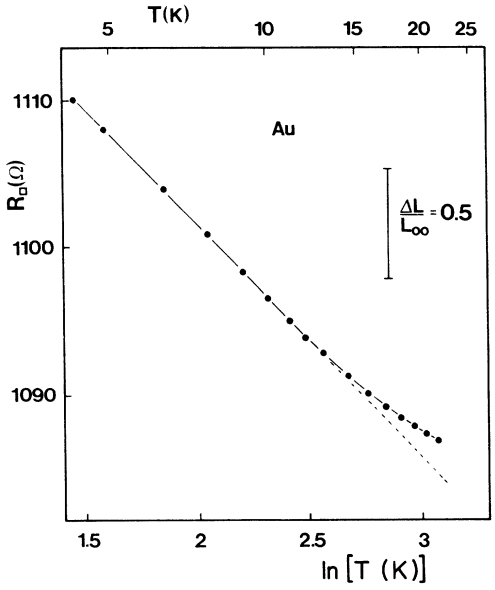

FIG. 3. The conductance correction in units of $L_{00}=e^2/2\pi^2\hbar$ for the same Au film as in Fig. 2 is plotted vs the logarithm of the temperature. The current for the measurement is 5 μA and the electrons and phonons are in thermal equilibrium. The corresponding scales for the square resistance are shown on the left-hand side, and the temperatures are shown on the top.

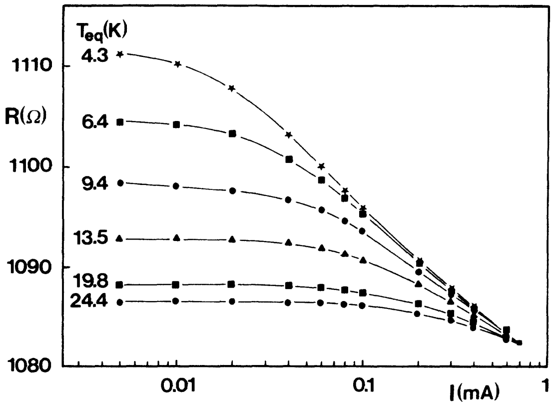

FIG. 4. The square resistance (in 7 T) is plotted vs the logarithm of the current for different equilibrium heatings. The temperatures for the equilibrium heating with small currents are labeled to the left of the data.

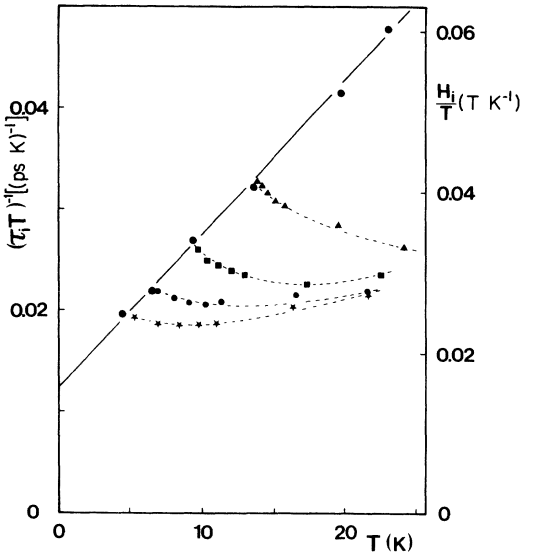

FIG. 6. The inelastic rate divided by the temperature $1/(\tau_i T)$ is plotted vs the temperature. The solid circles represent the equilibrium magnetoresistance measurement. They fulfill a straight line with an intersection at the vertical axis $A_C=0.0133(\mathrm{ps\,K})^{-1}$ and a slope $B_{e\text{-ph}}=0.001\,42(\mathrm{ps\,K}^2)^{-1}$. All the other data connected by dashed lines are from the nonequilibrium magnetoresistance measurement with higher currents. The temperatures for the nonequilibrium data are determined by means of the Coulomb anomaly of the resistance. The scale on the right-hand side is related to the scale on the left-hand side with Eq. (2.1).

currents are given on the right-hand side of the curves in mA. Both sets of curves can be described by essentially one parameter, the inelastic scattering rate of the conduction electrons. It does not matter whether the curves are obtained for equilibrium or nonequilibrium.

The magnetoresistance curves have been analyzed with the theory of Hikami *et al.*[12] and each curve yields a characteristic inelastic field H_i. From H_i we obtain the inelastic rate $1/\tau_i$ by the relation

$$1/\tau_i=H_i 4/(\hbar e\rho 2N_0)\ , \tag{2.1}$$

where ρ is the resistivity of the film and N_0 is the density of states per spin (i.e., $2N_0$ is the total density of states).

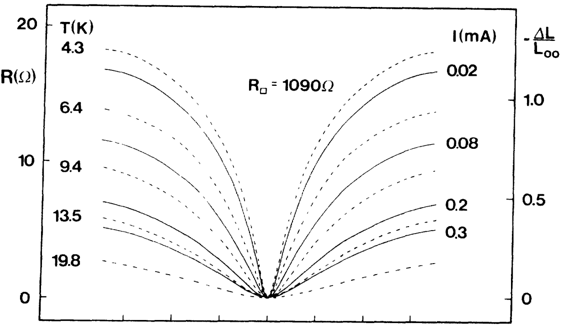

FIG. 5. Magnetoresistance for a Au film measured at different heatings. The dashed curves are obtained in thermal equilibrium, measured with only 5 μA. The temperatures on the left-hand side give the equilibrium temperatures. The solid curves are measured with zero heating of the substrate, but with increasing current through the film. The currents are given on the right-hand side of the curves in *m*A. The scale on the left-hand side is the correction of resistance with respect to the resistance at zero field. The scale on the right is the correction of conductance in units of $L_{00}=e^2/2\pi^2\hbar$, with respect to the conductance at zero field. The scale for the magnetic field is given at the bottom.

In Fig. 6 we have plotted the inelastic rate divided by the temperature $1/(\tau_i T)$ versus the temperature. The solid curve gives the equilibrium measurement. We recognize that the inelastic rate follows a temperature dependence given by

$$1/\tau_i=A_C T+B_{e\text{-ph}}T^2\ . \tag{2.2}$$

The experimental results for the coefficients are $A_C=0.0133/\mathrm{ps\,K}$ and $B_{e\text{-ph}}=0.001\,42/\mathrm{ps\,K}^2$. The linear temperature dependence is caused by the electron-electron interaction. Its coefficient is given by Eq. (3.17). In films with high resistance this contribution is much larger than in low-resistance films. Our experimental value of A_C agrees very well with the theoretical value of $0.012/\mathrm{ps\,K}$ given by Altshuler and Aronov.[13] The quadratic contribution is due to the electron-phonon interaction.

For each nonequilibrium point we obtain an inelastic scattering rate $1/\tau_i$. By means of Fig. 6 we determine the temperature with the same equilibrium inelastic lifetime. We denote this temperature as the weak localization temperature T_{WL}. Although this is only a formal temperature, its definition is helpful because T_{WL} lies between the electron and the phonon temperature:

$T_{\rm ph} < T_{\rm WL} < T_e$. In Fig. 8, which will be discussed below, $T_{\rm WL}$ is plotted versus the current through the film for the lowest equilibrium heating.

III. THEORY

In this section we assume that the phonon system can be described by a phonon temperature $T_{\rm ph}$. This permits us to perform analytic calculations. The calculation for distorted phonon occupation has to be left to future investigation.

A. Phenomenological overview

An electric current heats the electron system. The energy input per volume is $\sigma E^2=\rho j^2$. By electron-phonon relaxation processes the Joule heating is transferred into the phonon system while the electron system tries to relax towards the phonon temperature. Let the electron-phonon relaxation time be $\tau_{e\text{-ph}}$. During this time the electron system accumulates the energy density $\sigma E^2\tau_{e\text{-ph}}$ from the Joule heating. Its own internal energy (above the zero-point energy) is $(\pi^2/6)(2N_0)(k_BT_{\rm ph})^2$. Therefore, the total internal energy density corresponds to an effective electron temperature of

$$(k_BT_e)^2=(k_BT_{\rm ph})^2+(6/\pi^2)(eE)^2D\tau_{e\text{-ph}} \, , \tag{3.1}$$

where D is the diffusion constant.

In a recent paper one of the authors demonstrated[14] that the presence of a large electric field yields an energy distribution of the electrons which is different from the equilibrium distribution with the same internal energy. However, for practical purposes, the effect of this difference is minor.

The Joule-heating power σE^2 which is pumped into the electron system is transferred after the time $\tau_{e\text{-ph}}$ into the phonon system of the film. From the phonon system it passes into the heat bath, i.e., the quartz plate. Let τ_b be the time the phonons need to escape from the phonon system of the thin film into the phonon system of the underlying quartz plate. During this time they increase the internal energy of the phonon system of the film in analogy to the electron system. This increase is $\sigma E^2\tau_b$ per volume. The internal energy of the phonon system at the temperature $T_{\rm qu}$ is

$$U_{\rm ph}(T_{\rm qu})=C_{Dz}T^{z+1} \, , \tag{3.2}$$

where z is the dimension of the film. C_{Dz} is given in three dimensions for longitudinal and transverse phonons by

$$C_{D3}=\sum_{\lambda}\frac{\pi^2}{30}\frac{k_B^4}{(\hbar c_\lambda)^3} \, , \tag{3.3}$$

where λ is the polarization of phonons and c_λ the corresponding sound velocity. This expression for C_{D3} has to be modified if we have to consider the film a quasi-two-dimensional phonon system.

The effective temperature of the relevant phonons in the film is given by

$$T_{\rm ph}^{z+1}=T_{\rm qu}^{z+1}+\frac{1}{C_{Dz}}(eE)^2\tau_b \, . \tag{3.4}$$

As we will discuss below, the occupation factor $n_{\rm ph}(\Omega)$ of the phonon spectrum is far from the equilibrium occupation $n(\Omega,T_{\rm ph})$. Therefore, $T_{\rm ph}$ has here only the meaning of an averaged internal energy.

Next we consider the phonon escape time τ_b from the film. We discuss first a lower limit of τ_b in the perfectly matched quartz-film model. Here we assume that the masses and elastic constants of the metal and the quartz are identical so that the phonons can propagate from the metal to the quartz and vice versa without distortion. The phonons are generated in the metal film with wave number q and they propagate afterwards in all directions. Let U_f be the energy density in the film. Then the film radiates its energy through the interface with the quartz by a process similar to the black-body radiation. The energy that escapes through the interface is given by

$$\frac{dU_f}{dt}=\frac{c}{4d}U_f, \quad \tau_b=\frac{4d}{c} \, . \tag{3.5}$$

This yields a phonon escape time which is temperature independent and very short for thin films. In our films we get about $\tau_b=3\times10^{-12}$ s if we take the longitudinal sound velocity of Au which is $c=3200$ m/s. But even during such a short time the phonon system accumulates an energy density which raises its temperature. Little[15] included an acoustical mismatch between the two media and calculated the transparency of the interface. We return to this point in the discussion section.

B. Electron-phonon energy relaxation

Kaganov, Lifshitz, and Tanatarov[16] calculated the energy transfer from the electron to the phonon system, and recently Allen[17] expressed the energy relaxation in terms of $\alpha^2F(\Omega)$, the Eliashberg function, which gives the probability for electron-phonon processes. The energy transfer from the electron to the phonon system is, according to Ref. 17,

$$dE_e/dt=2\pi N_0\int d\Omega\,\alpha^2F(\Omega)(\hbar\Omega)^2[n(\Omega,T_{\rm ph})-n(\Omega,T_e)], \tag{3.6}$$

where $n(\Omega,T)$ is the Bose distribution function and N_0 is the density of states per spin at the Fermi energy. For low temperatures we approximate $\alpha^2F(\Omega)$ by the lowest power in Ω,

$$\alpha^2F(\Omega)\simeq C_\zeta\Omega^\zeta \, . \tag{3.7}$$

Then we obtain for Eq. (3.6)

$$\frac{dE_e}{dt}=-\sigma E^2 =2\pi N_0\frac{k_B^{\zeta+3}}{\hbar^{\zeta+1}}C_\zeta(T_{\rm ph}^{\zeta+3}-T_e^{\zeta+3})\int dx\frac{x^{\zeta+2}}{e^x-1} \, . \tag{3.8}$$

Together with Eq. (3.1) we obtain for the electron-phonon relaxation time

$$\frac{1}{\tau_{e\text{-ph}}}=\frac{6}{\pi}\left[\frac{k_B}{\hbar}\right]^{\zeta+1}C_\zeta\frac{T_e^{\zeta+3}-T_{\text{ph}}^{\zeta+3}}{T_e^2-T_{\text{ph}}^2}\int dx\frac{x^{\zeta+2}}{e^x-1}\ . \quad (3.9)$$

For small temperature differences one obtains for $\tau_{e\text{-ph}}$

$$\frac{1}{\tau_{e\text{-ph}}}=\frac{3}{\pi}\int d\Omega\,\alpha^2F(\Omega)\left[\frac{\hbar\Omega}{k_BT}\right]^2T\frac{d}{dT}n(\Omega,T)\ . \quad (3.10)$$

For small temperatures this yields

$$\frac{1}{\tau_{e\text{-ph}}}=\frac{3}{\pi}(k_BT/\hbar)^{\zeta+1}(\zeta+3)C_\zeta\int dx\frac{x^{\zeta+2}}{e^x-1}\ . \quad (3.11)$$

From the temperature dependence of $\tau_{e\text{-ph}}$ one can derive the Ω dependence of $\alpha^2F(\Omega)$.

One obtains the same relation for $\tau_{e\text{-ph}}$ for small temperature differences by using

$$dE_e/dt=(dE_e/dT_e)(dT_e/dt)\ ,$$

$$dE_e/dT_e=\pi^2k_B^2 2N_0T_e/3\ ,$$

and

$$dT_e/dT=-(T_e-T_{\text{ph}})/\tau_{e\text{-ph}}\ .$$

C. The inelastic lifetime

In addition, we determine the dephasing time τ_i by measuring the magnetoresistance. The dephasing is caused by electron-phonon and electron-electron processes. According to our present understanding,[18] the electron-phonon part of $1/\tau_i$ is essentially given by

$$\begin{aligned}\frac{1}{\tau_i^{e\text{-ph}}}=2\pi\int d\epsilon\left[-\frac{\partial f}{\partial\epsilon}\right]&\\ \times\int d\Omega\,\alpha^2F(\Omega)\{[1-f(\epsilon+\Omega)]n(\Omega)&\\ +[1-f(\epsilon-\Omega)]&\\ \times[n(\Omega)+1]\}\ .&\quad (3.12)\end{aligned}$$

In thermal equilibrium between the electron and the phonon system this yields for the inelastic lifetime

$$\frac{1}{\tau_i^{e\text{-ph}}}=2\pi\int d\Omega\,\alpha^2F(\Omega)\frac{(\hbar\Omega/k_BT)}{[2\sinh(\hbar\Omega/2k_BT)]^2}\ . \quad (3.13)$$

To our surprise this formula has not been derived previously. In the literature the appropriate averaging over the energy range k_BT has been forgotten. This causes a deviation which can easily cause a difference of several tenths of a percent. For a linear energy dependence of $\alpha^2F(\Omega)$ Eq. (3.13) yields values for $1/\tau_i^{e\text{-ph}}$ which are by a factor 0.33 larger than the values obtained from previous formulas.

In nonequilibrium between the electron and phonon systems we obtain

$$\begin{aligned}\frac{1}{\tau_i^{e\text{-ph}}}=2\pi\int d\Omega\,\alpha^2F(\Omega)\Bigg[n(\Omega,T_{\text{ph}})&\\ +\left[T_e\frac{d}{dT_e}-1\right]n(\Omega,T_e)\Bigg]\ .&\\ &(3.14)\end{aligned}$$

For low temperatures where $\alpha^2F(\Omega)$ can be approximated by $C_\zeta\Omega^\zeta$ one finds

$$\frac{1}{\tau_i^{e\text{-ph}}}=2\pi(k_B/\hbar)^{\zeta+1}C_\zeta(\zeta T_e^{\zeta+1}+T_{\text{ph}}^{\zeta+1})\int dx\frac{x^\zeta}{(e^x-1)}\ . \quad (3.15)$$

Close to the equilibrium state the ratio between the inelastic rate and the electron-phonon rate is given by

$$\frac{1/\tau_i^{e\text{-ph}}}{1/\tau_{e\text{-ph}}}=\frac{2\pi^2(\zeta+1)}{3(\zeta+3)}\frac{I_\zeta}{I_{\zeta+2}},\quad I_\nu=\int dx\frac{x^\nu}{e^x-1}\ . \quad (3.16)$$

The same relation is valid for arbitrary electron and phonon temperatures if $\zeta=1$. By numerical calculation we find for $I_1\simeq1.645$, $I_2\simeq2.404$, $I_3\simeq6.494$, $I_4\simeq24.886$, and $I_5=122.08$. Therefore, we obtain for $\zeta=1$ the ratio 0.833.

In addition to the electron-phonon contribution, $1/\tau_i$ has also a contribution from the enhanced electron-electron interaction in disordered two-dimensional electron systems. Altshuler and Aronov[13] derived the following implicit relation for $1/\tau_i^{ee}$:

$$\frac{\hbar}{k_BT\tau_i^{ee}}=\frac{e^2}{2\pi\hbar}R_0\ln\left[\frac{\hbar}{k_BT\tau_i^{ee}}\right]\ . \quad (3.17)$$

D. The phonon system

In a first evaluation we describe the phonon system by a phonon temperature and therefore the phonon occupation number by the Bose-Einstein occupation number. However, the phonons emitted by the electrons have a finite escape time τ_b from the film into the quartz plate. This situation has been theoretically studied by Perrin and Budd[19,20] who found strong deviations between the actual phonon occupation and the equilibrium occupation for a temperature with the same total energy. We use a slightly different description here which corresponds to Allen's calculation instead. The nonequilibrium phonon occupation number of the phonon system we denote by $n_{\text{ph}}(\Omega)$. The rate of phonons with the energy $\hbar\Omega$ which are emitted and adsorbed from the electron system is proportional

$$\alpha^2F(\Omega)\int d\epsilon\{f(\epsilon)[1-f(\epsilon-\hbar\Omega)](n_{\text{ph}}+1)-f(\epsilon)[1-f(\epsilon+\hbar\Omega)]n_{\text{ph}}\}=\alpha^2F(\Omega)\hbar\Omega[n(\Omega,T_e)-n_{\text{ph}}]\ .$$

If one assumes a constant escape time for all phonon frequencies then the rate for the occupation of the phonon of frequency Ω is given by

$$F(\Omega)\frac{dn_{\mathrm{ph}}(\Omega)}{dt}=\frac{2\pi}{\hbar}2N_0\alpha^2F(\Omega)\hbar\omega[n(\Omega,T_e)-n_{\mathrm{ph}}(\Omega)] -\frac{n_{\mathrm{ph}}(\Omega)-n(T_{\mathrm{qu}},\Omega)}{\tau_b}F(\Omega)\ . \qquad (3.18)$$

Here $F(\Omega)$ is the phonon density of states. Its form depends on the coupling of the film to the quartz plate, i.e., whether the phonon states lie continuously inside of the Debye sphere or whether they occupy only planes within the Debye sphere. We set

$$G(\Omega)=\frac{2\pi}{\hbar}2N_0\frac{\alpha^2F(\Omega)\hbar\Omega}{F(\Omega)}\ . \qquad (3.19)$$

For the steady state we obtain

$$n_{\mathrm{ph}}(\Omega)=\frac{n(T_{\mathrm{qu}},\Omega)+n(T_e,\Omega)G(\Omega)\tau_b}{1+G(\Omega)\tau_b}\ . \qquad (3.20)$$

In the limit of small τ_b this yields

$$n_{\mathrm{ph}}(\Omega)=n(\Omega,T_{\mathrm{qu}})+[n(\Omega,T_e)-n(\Omega,T_{\mathrm{qu}})]G(\Omega)\tau_b +O(\tau_b^2)\ . \qquad (3.21)$$

Transitions between the phonon states, for example, conversions of longitudinal phonons into transverse ones, complicate the treatment further. Only if these transitions are so frequent that they restore thermal equilibrium would the situation again become simple.

IV. EVALUATION

As we found in Sec. II the inelastic scattering rate $1/\tau_i$ is a sum of a linear and a quadratic term. The linear term agrees very well with the theoretical prediction by Altshuler and Aronov[13] resulting from the electron-electron interaction. We expect that the remaining quadratic term is due to electron-phonon interaction. According to Eq. (3.15) in the theoretical part a quadratic temperature dependence of $1/\tau_i$ corresponds to an Eliashberg function linear in Ω, i.e., $\alpha^2F(\Omega)\simeq\Omega$. This frequency dependence is a puzzling problem on which we comment later.

For a quadratic temperature dependence of $1/\tau_i$, i.e., a linear dependence of $\alpha^2F(\Omega)$ on the phonon frequency we predict according to Eqs. (3.9) and (3.15) that the electron-phonon relaxation rate $1/\tau_{e\text{-ph}}$ should show the same temperature dependence and be identical to $1/\tau_i^{e\text{-ph}}$ except for a factor $\frac{1}{0.83}$. This applies for both the equilibrium and nonequilibrium cases.

In our first approach to evaluate the electron-phonon relaxation rate we approximate the phonon temperature T_{ph} by the temperature of the quartz plate T_{qu}. For the electron temperature we take T_C from the evaluation of the Coulomb anomaly. With Eq. (3.1) we obtain the electron-phonon relaxation rate. In Fig. 7 we have plotted the inelastic rate due to electron-phonon interaction $1/\tau_i^{e\text{-ph}}$ versus the electron-phonon relaxation rate $1/\tau_{e\text{-ph}}$. Theoretically we expect a linear relation between the two parameters with the slope of 0.83. While the points for the two lowest equilibrium heatings appear to fulfill a linear relation, there are considerable deviations for the higher equilibrium heating. In addition, the slope for the lower curves is 1.2. The electron-phonon relaxation rate has the right order of magnitude but there are clear deviations from a linear dependence between $1/\tau_i^{e\text{-ph}}$ and $1/\tau_{e\text{-ph}}$.

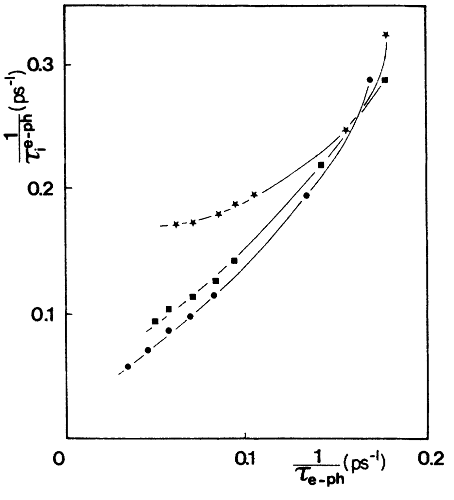

FIG. 7. The inelastic rate due to electron-phonon interaction $1/\tau_i^{e\text{-ph}}$ is plotted vs the electron-phonon relaxation rate $1/\tau_{e\text{-ph}}$ for three different substrate heatings. The inelastic rate due to electron-phonon interaction is the quadratic part of the temperature dependence of the inelastic scattering rate. The electron-phonon relaxation time is determined with Eq. (3.1), taking the electron temperature from Coulomb anomaly and approximating the phonon temperature by the quartz temperature.

One might conclude that the approximation of the phonon temperature by T_{qu}, the temperature of the quartz substrate is the origin of the observed deviation between $1/\tau_{e\text{-ph}}$ and $1/\tau_i^{e\text{-ph}}$. However, the reason is more complicated. This is demonstrated in Fig. 6 where we also plot for the nonequilibrium data $1/(\tau_iT)$ versus T. Here we use T_C for the electron temperature. For a linear $\alpha^2F(\Omega)$ the nonequilibrium curves should always lie above the value for the equilibrium point, i.e., the dashed curves should start with a horizontal tangent and bend upwards. Experimentally this is not observed. There can be several reasons for these deviations. The following ones come to mind.

(a) The conclusion that $\alpha^2F(\Omega)$ is linear in Ω is not correct.

(b) The Coulomb anomaly is not a reliable thermometer.

Our conclusion at the present time is that the resistance in 7 T is not a trustworthy thermometer. For the argumentation we use the evaluation of the inelastic rate $1/\tau_i$.

The inelastic rate is more than a thermometer in our investigation. The electron-phonon part of the inelastic rate is (besides a factor of the order of 1) identical with the electron-phonon relaxation rate. The latter determines the temperature difference between the electron and the phonon system and therefore the inelastic rate contains essentially all the relevant information which we need in our investigation. If one knows the frequency dependence of $\alpha^2F(\Omega)$ then one obtains a full analysis of the experimental data just from $1/\tau_i$. For a linear dependence of the Eliashberg function $\alpha^2F(\Omega)$ on the frequency Ω one obtains for $1/\tau_i$

$$1/\tau_\iota^{e\text{-ph}}=1/\tau_\iota-A_CT_e=B_{e\text{-ph}}/2(T_e^2+T_{\text{ph}}^2)\ , \tag{4.1a}$$

$$1/\tau_{e\text{-ph}}=(6/\pi^2)D(eE/k_B)^2/(T_e^2-T_{\text{ph}}^2)\ , \tag{4.1b}$$

$$1/\tau_\iota^{e\text{-ph}}=0.83/\tau_{e\text{-ph}}\ . \tag{4.1c}$$

Combining these equations yields an equation for the electron temperature T_e

$$\frac{1}{\tau_i}+\frac{B_{e\text{-ph}}\sigma E^2}{2N_0k_B^2}\frac{I_1}{I_3}\frac{1}{1/\tau_i-A_CT_e}-B_{e\text{-ph}}T_e^2-A_CT_e=0\ , \tag{4.2}$$

where I_ν are the integrals in Eq. (3.16) and A_C and $B_{e\text{-ph}}$ are the linear and quadratic coefficients of the temperature T in Eq. (2.2).

Before we evaluate Eq. (4.2) we have to recall that the inelastic rate $1/\tau_i$ is obtained from the inelastic field H_ι of the magnetoresistance curves. The determination of $1/\tau_i$ is performed with the help of Eq. (2.1) which requires the knowledge of the resistivity ρ and the density of states $2N_0$ of the film. The resistivity requires the thickness of the film which has an accuracy of about 15%. The density of states is approximated by the free-electron value. Furthermore, nobody has yet considered a possible influence of mass enhancement, i.e., a factor of $(1+\lambda)$ in Eq. (2.1). Therefore, we have to permit an uncertainty in the absolute value of $1/\tau_\iota$ which is of the order of a few 10%.

Keeping this uncertainty in mind we have plotted in Fig. 8 a detailed evaluation of the lowest nonequilibrium curve (i.e., for no equilibrium heating). Figure 8 shows essentially the different measured and evaluated temperatures as a function of the current through the film for the lowest curve in Fig. 2, i.e., the lowest equilibrium heating. The scale on the top of the figure gives the corresponding electric fields. T_{qu} is the measured temperature of the quartz plate. T_C is the electron temperature which we obtained from the Coulomb anomaly, i.e., the (logarithmic) temperature dependence of the resistance in 7 T. The evaluation of T_{WL}, the weak-localization temperature, has been described in Sec. II and corresponds to the equilibrium temperature which shows the same inelastic scattering rate as in the considered nonequilibrium state.

Using the inelastic rate for each nonequilibrium point (which is represented by T_{WL}) we can calculate by means of Eq. (4.2) the expected electron and phonon temperatures T_e' and T_{ph}'. The corresponding data are represented by dashed curves. T_e' lies considerably below T_C, but the main objection is that T_{ph}' lies partially below the quartz temperature T_{qu}. Although this is impossible, it does not disqualify this evaluation. As we pointed out above, the absolute value of $1/\tau_i$ is only determined within some 10% accuracy. Our evaluation shows that τ_i (and therefore $\tau_{e\text{-ph}}$) must be shorter to reduce the temperature difference between the electron and the phonon system. For the dashed-dotted curves we reduced τ_i by 30%. (The curves are denoted by T_e and T_{ph}.) T_{ph} lies reasonably above the quartz temperature.

The new values of the electron temperature T_e lie still further below T_C, the value obtained with the Coulomb anomaly. This difference between T_C and T_e represents the third discrepancy when using T_C for the electron temperature.

We put the difference between T_e and T_C aside for a moment. Then we conclude: After the reasonable adjustment of 30% of the time scale of τ_ι the evaluation with weak localization yields a satisfying and consistent description of the experimental data.

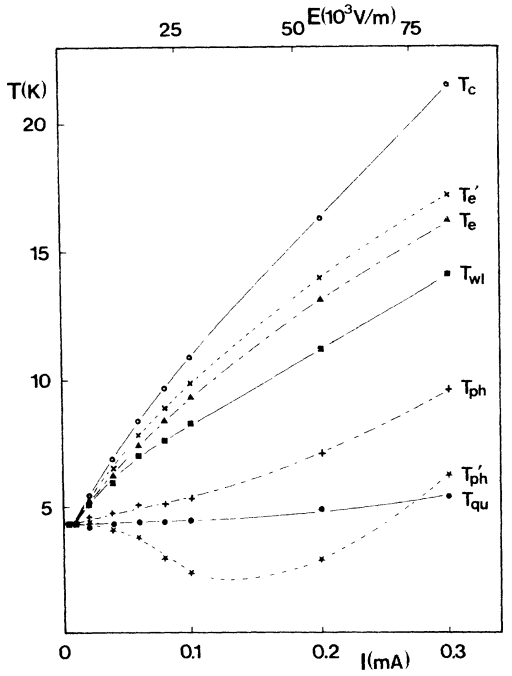

FIG. 8. The electron and phonon temperatures determined with different methods are plotted vs the current. T_{WL} is a formal temperature from the inelastic lifetime determined with weak localization. T_C (circles) is determined from the Coulomb anomaly of the resistance. T_{qu} is the temperature of the quartz substrate (measured by a carbon resistor on the back of the quartz). T_e' and T_{ph}' are calculated with Eqs. (4.1) and (4.2), taking the inelastic lifetime τ_ι as the input, where τ_ι is obtained from weak localization which yields the inelastic field H_ι using the relation $H_\iota\tau_\iota=e\hbar\rho2N_0/4$. T_e and T_{ph} are obtained by reducing $H_\iota\tau_\iota$ by a factor of 0.7.

V. DISCUSSION

During the evaluation of our experiments we realized that we know amazingly little about the interplay of electrons and phonons in thin metal films. What is the dimension of the phonon system? Which phonons are created by the electron-phonon interaction? How do the phonons escape the film? What is the character of the electron-phonon transition? Obviously our evaluation could only be a first step in understanding and evaluating the experimental data. In this section we discuss briefly some of the problems we have to address in the future. For some questions such as the phonon spectrum or the phonon escape time we already have a qualitative picture. Others like the nonequilibrium phonon distribution function require additional knowledge about the electron-phonon interaction.

A. The phonon spectrum in the thin film

The sound velocities of longitudinal and transverse phonons are considerably larger in quartz than in Au. In addition, the density of Au is almost larger by a factor of 10 than in quartz. As a consequence the Au-quartz interface has a critical angle of total reflection for longitudinal phonons of $\vartheta=33°$ (from the Au side). This means that only phonons with $q_z > q_{xy}/\tan(\vartheta)$ can cross the interface from Au to quartz. In the following we call this part of the q space the q cone of transparency. If the z component q_z is less than $q_{xy}/\tan(\vartheta)$ then the phonon is totally reflected. The form of the pressure p as a function of the distance from the interface is

$$p=[\exp(ik_z z)+\exp(-ik_z z+i\Phi)] ,$$

where Φ is the phase shift at the interface. Φ varies between 0 and π if the incident angle varies between the critical angle of total reflection and grazing incidence. Therefore, one obtains a quantization of k_z in the range of total reflection and the boundary conditions vary as a function of the incident angle. For the angle of total reflection the quantization yields $q_z=\nu\pi/d$ and for grazing incidence we have $q_z=(2\nu+1)\pi/2d$. As a consequence, the phonon spectrum of the film on top of the quartz plate consists of two very different contributions.

(a) For $q_z > q_{xy}/\tan(\vartheta)$ the phonons extend from the film into the quartz plate, are not restricted in their q_z values, and behave three dimensionally. However, the phase space of the three-dimensional phonons is reduced, i.e., we have a reduced density of phonon modes.

(b) For $q_z < q_{xy}/\tan(\vartheta)$ the phonons are confined to the thin Au film (decaying exponentially in the quartz) and their q_z values are quantized.

The total reflection at interface for $q_z < q_{xy}/\tan(\vartheta)$ is the reason why the escape chance of phonons from the film is strongly reduced. The formula by Little[15] yields only an escape chance of 20% (corresponding to a $\Gamma=0.1$ in Little's formula). The density of the three-dimensional phonon modes in Au is reduced by roughly the same factor.

The above considerations apply when one has an ideal interface between the Au film and the quartz plate. One cannot exclude that the interface between a quartz plate and a disordered Au film behaves different. One way to check this question is to reduce the Au thickness to a few atomic layers. Then the two-dimensional part of the spectrum should be sufficiently high in frequency so that it is not occupied at helium temperature. We plan to extend our investigation to such thin films so that the two-dimensional modes are completely frozen out.

Finally we want to mention the Rayleigh waves at the interface between Au and quartz. They might as well be activated by the electron-phonon transitions. We plan to study their influence in the future as well.

B. The phonon escape time τ_b

By means of electron-phonon processes the Joule heating is transferred from the electron system to the phonon system. Finally the phonons escape from the thin film into the quartz. The longer this escape time is, the more energy is accumulated in the phonon modes, i.e., the higher the phonon occupation number will be for a given q and Ω. From the above discussion we easily realize that the escape time is a strong function of the direction of q.

It is quite suggestive to assume that the quantized phonons, i.e., those which experience total reflection, do not escape the film and therefore are in thermal equilibrium with the electron system, i.e., have a thermal occupation $n(\Omega,T_e)$. On the other hand, the three-dimensional phonons within the q cone of transmission escape roughly during the time $\tau_b=d/c_\lambda$. In this case, the small fraction of escaping phonons contributes the heat transfer from the electron system into the phonon system, i.e., to the electron-phonon relaxation time. The quantized phonons, since they are trapped, would assume the electron temperature and therefore do not contribute to the electron-phonon relaxation rate. But they would fully contribute to the inelastic rate (as long as their energy is not too high and they are thermally activated).

If, however, the quantized phonons scatter from the quasi-two-dimensional states into the cone of three-dimensional phonons then they can escape into the quartz. As a consequence their occupation number is no longer given by the electron temperature and then they contribute to the energy transfer from the electron system to the phonon system and therefore to the electron-phonon relaxation rate. It does not make sense to evaluate our experimental data in all possible scenarios. First we need a better understanding of the underlying processes. This question can be experimentally investigated by using, instead of the heavy Au, the light metal Mg where the q cone of transparence is much larger.

C. Non-Ohmic behavior of the Coulomb anomaly

In the last section we found that the values of the electron temperature T_e (evaluated with weak localization) lie clearly below T_C (the value obtained with the Coulomb anomaly). This difference between T_C and T_e, as well as the negative slope in the $1/(\tau_i T)$ plot, raised suspicion that the Coulomb anomaly is not a reliable thermometer. Therefore, we considered the field dependence of the electric resistance carefully. One of the authors[21] translated and explained the Hartree part of the Kubo diagram for the Coulomb anomaly in physical terms. It was concluded that one electron diffusing along a closed (impurity) loop generates, by self-interference, a charge hologram. The second electron which diffuses along the same loop is scattered by this charge hologram, thereby readjusting its phase. Finally it performs a self-interference at its starting point, modifying its effective mean free path. The essential point for our present investigation is that the resulting mean free path and the correction to the resistance should be electric-field dependent. The reason is that the two electrons start from different positions r_1 and r_2 in real space. This means that their chemical potential is shifted with respect to each other by $\Delta\mu = eE(r_1 - r_2)$. As a consequence, the electric field introduces an additional phase shift in the corresponding Kubo diagram and therefore reduces its contribution. The same applies for the Fock term. Therefore, we conclude that the quantum correction to the resistance due to the Coulomb interaction violates Ohm's law. From this qualitative interpretation we expect an electric field dependence which is of the order of

$$\Delta R_C / R^2 = \frac{e^2}{4\pi^2\hbar} \ln[(k_B T)^2 + \alpha(eE)^2 D\hbar / k_B T] \ , \qquad (5.1)$$

where α is a factor of the order of 1. We are in the progress of calculating this correction quantitatively.

Stimulated by this idea we also considered the field dependence of the residual resistance. The elastic scattering centers in a disordered metal are generally impurities or lattice defects. The potential of these scattering centers is screened. Therefore, we have to check whether this screening might be affected by a current density as in our experiments or by the temperature of the electron gas. We plan to look into this question in more detail. Such new effects might be discovered since our experiment represents the first one in which a high electric field is combined with a very accurate measurements of the resistance. This is not possible in pulse experiments.

It should be emphasized, however, that weak localization does not suffer from an electric-field dependence. Such a (destructive) effect of the electric field was originally controversial and predicted by some authors.[22,23] One of the authors[24] confirmed the prediction by Altshuler and Aronov[25] who claimed that weak localization is not affected by an electric field. Therefore, we can trust weak localization as a thermometer.

D. The electron-phonon interaction

In our evaluation of the inelastic scattering rate we obtained an Eliashberg function $\alpha^2F(\Omega)$ which was linear in Ω. In the final evaluation in Fig. 8 we used this linear frequency dependence. Such a linear dependence is theoretically rather surprising. It was experimentally observed 20 years ago in tunneling experiments[26,27] but the low-energy results were not taken that seriously. Schmid[28] and Keck and Schmid[29] concluded that such a linear dependence does not occur for $ql < 1$ if one includes vertex corrections. Belitz and Sarma[18] interpreted the experimental temperature dependence of $1/\tau_i$ as a superposition of a linear term from the electron-electron interaction and a nonquadratic contribution from the electron-phonon interaction. We do not believe that this interpretation applies to our experiments since we can clearly separate the linear contribution from the electron-electron interaction. Therefore, the quadratic temperature dependence of $1/\tau_i$ remains a theoretical challenge.

During the last 10 years we have improved our skill to treat disordered electron systems if small q vectors and energies are involved. We know that the matrix elements of e^{iqr} between eigenfunctions of the disordered system are peaked for energy differences of the order $\hbar Dq^2$, since this is the width of the "diffusion pole." Such electron transitions generate (in an average) charge-density waves in the film which couple to the phonons but which are purely diffusive. (We expect that the z component is quantized as $q_z = \nu\pi/d$.)

The above model presents a well-defined problem which is treatable. It should particularly address the question of electron-phonon processes with transverse phonons. Perrin *et al.* recently suggested[30] that only longitudinal phonons are generated in the film which might decay into transverse phonons.

VI. CONCLUSIONS

We investigated thin, narrow Au films, quench condensed onto a quartz plate, in the presence of high current densities, i.e., high electric fields. By means of weak localization we determined the inelastic scattering rate of the conduction electrons in equilibrium and nonequilibrium. We found a very good agreement between the linear experimental contribution to the inelastic scattering rate and the theoretical prediction due to the electron-electron interaction. The second contribution was quadratic in the temperature and caused by electron-phonon interaction, corresponding to a linear frequency dependence of the Eliashberg function $\alpha^2F(\Omega)$. For the interpretation of our experimental data we derived the expression for the inelastic rate in nonequilibrium (and equilibrium which was not properly averaged over the thermal energy distribution in the literature). By a slight adjustment of the conversion factor between the characteristic field H_i and the rate (which we calculated so far only in the free-electron approximation, neglecting possible mass enhancement effects), we succeeded in interpreting the experimental data consistently.

Originally we intended to use two "thermometers," weak localization and the Coulomb anomaly. It turned out that the Coulomb anomaly does not work properly as a thermometer. We believe that this is in itself an interesting effect. We suggest that the Coulomb anomaly shows non-Ohmic behavior of the resistance. Other non-Ohmic contributions due to a change of screening might be possible.

Our experiments raise a number of interesting questions.

(a) What kind of phonons are present in a very thin (quasi-two-dimensional) film on top of an insulator like quartz?

(b) What is the electron-phonon interaction in such a thin film? In particular, do the transverse phonons participate in the interaction?

(c) What is the electron distribution function in the presence of a large electric field? We answered this question during the present investigation for thermal phonons with the temperature T_{ph}. For a nonthermal distribution of the phonons this question still must be solved.

(d) Is there a non-Ohmic behavior of the resistance? We suggest that the Coulomb anomaly is non-Ohmic for large electric fields. This is not the trivial effect of the electric field to alter the distribution function of the electrons but on top of that. In addition, we find it worthwhile to check the screening of impurities in the presence of a large electric current. Such a current might alter the scattering potential of the impurity and as a consequence the residual resistance.

The steady-state investigation of electrons in a high current density is a new and very promising area which will yield considerable new insight into the properties of quasi-two-dimensional electron-phonon systems. Experimentally we have many parameters which we can alter to explore the system and gain new information. A combination of the steady-state and the pulse experiments would be of particular interest.

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