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Unusual low-frequency electromagnetic response in anisotropic superconductors: Application to UPt_3

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We calculate the electromagnetic response of an anisotropic superconductor, and show that the temperature dependence of the magnetic penetration depth is strongly suppressed by an ac field even for frequencies very small compared to the gap magnitude. The effect arises as a result of virtual excitations of Bogoliubov quasiparticle-quasihole pairs in the neighborhood of the gap nodes. We propose that this phenomenon is responsible for apparent discrepancies among various recent penetration-depth experiments on superconducting UPt_3. The available data are found to be consistent with a state of E_{1g} symmetry.

Very recently, experiments measuring the inductive skin depth in superconducting UPt_3[1,2] have added to the list of unusual properties of this fascinating heavy-fermion material.[3] It has been thought for some time that thermodynamic and transport measurements strongly suggest the existence of an order parameter vanishing at lines or possibly points of nodes on the Fermi surface.[3] Moreover, longitudinal sound attenuation,[4] torsional oscillator,[5] and recent specific-heat experiments[6] suggest a phase diagram considerably richer than that of an ordinary superconductor. The additional transitions observed have been identified variously as transitions between two bulk *d*-wave superconducting states,[7] vortex lattice[8] and vortex core[9] transitions. For any of these possibilities to be realized, the order parameter in UPt_3 must be unconventional, i.e., the ground state must exhibit broken symmetries in addition to the broken U(1) gauge symmetry characteristic of a BCS superconductor.

Some qualitative features of the phase diagram, as well as the existence of anomalies in measured critical fields and the specific heat may be understood[7,10] by minimizing the Ginzburg-Landau free-energy functional of an order parameter with the symmetry of the two-dimensional representation E_{1g} ("*d* wave") of the hexagonal group D_6 appropriate for UPt_3. Such a representation arises in a group-theoretical classification of possible unconventional superconducting states,[11] and possesses the line of nodes in the basal plane thought to be characteristic of the UPt_3 order parameter.[12,13] The E_{1g} state was first suggested as a possible ground state for the UPt_3 system based on an analysis of sound attenuation data,[12] and provides a qualitatively good account of most transport properties.[12–15] On the other hand, there exist some experimental findings which are difficult to reconcile with a state with this symmetry, as well as other candidate states which fit the data nearly as well.[14] Our knowledge of the microscopic mechanism of superconductivity in these materials is still quite poor, such that a final determination of the symmetry of the order parameter will almost certainly be made only when the overwhelming weight of experimental evidence supports the identification of one particular state. In this context, it is important to resolve the remaining inconsistencies of the phenomenological theory.

One of the most striking puzzles was posed recently by Shivaram, Gannon, and Hinks,[1] who reported measurements of the inductive skin depth in UPt_3 at frequencies 3 orders of magnitude smaller than the critical temperature T_c ($f \simeq 10$ MHz, $T_c/h \simeq 10$ GHz). In addition to an unusual peak just below the transition, these authors reported a low-temperature skin depth $\delta(\omega,T)$ which varied roughly as $[\delta(\omega,T)-\delta_0]/\delta_0 \sim T^4$ at low T, where $\delta_0 = \delta(0,0)$. For a strong type-II superconductor, the zero-frequency limit of the skin depth δ is identical to the London penetration depth Λ. Indeed, the resonant oscillator technique used in Refs. 1 and 2 was similar to that used traditionally to measure $\Lambda(T)$ in ordinary type-II superconductors. It is generally assumed that frequencies in this range give results nearly identical to dc measurements since $\omega \equiv 2\pi f \ll T_c$ or Δ_0, the only relevant energy scale in the BCS case. In superconductors with gap nodes, Bogoliubov quasiparticles are excited at all nonzero frequencies, but at temperatures $\omega \ll T \ll \Delta_0$, one might again expect the difference between a dc and an ac experiment to be insignificant. As Shivaram *et al.* reported,[1] however, their results were in apparent conflict with earlier dc measurements of Gross *et al.*,[16] who reported penetration depths $[\Lambda(T)-\Lambda_0]/\Lambda_0 \sim T^2$ in similar samples of UPt_3. Both experiments reported only relative changes in penetration depths with respect to values measured at some minimum temperature T_{min}. More recently, Shivaram *et al.* have obtained results for all possible orientations of external field **H** and direction of wave propagation **q** with respect to the hexagonal crystal axes.[2] They find a rough T^4 variation for all three orientations (albeit with substantially different prefactors). This is again in conflict with $\omega = 0$ predictions for some simple uniaxial states.[16]

In this work, we propose a simple explanation for the apparent discrepancy between Refs. 1, 2, and 16 at low temperatures. There is in fact an additional energy scale in the system, given by particle-hole excitations of the Bogoliubov quasiparticle gas of energy $\omega_{p\text{-}h}(\mathbf{k},\mathbf{q}) = E_{\mathbf{k}+\mathbf{q}} - E_{\mathbf{k}} \simeq (\nabla_{\mathbf{k}} E_{\mathbf{k}} \cdot \mathbf{q})$. (Here we use the term particle-hole excitation in analogy with a similar process in a normal Fermi system, although the excitation does not span a well-defined Fermi surface.) In an anisotropic superconductor at $T \ll T_c$, where substantial numbers of quasiparticles are concentrated only in the neighborhood of the gap nodes, the typical excitation energies $\omega_{\text{p-h}}$ can be substantially lower than the temperature T and comparable to the frequency ω used in the experiment of Refs. 1 and 2.

The above process may be viewed as the reactive counterpart of Landau damping in a normal Fermi system. For $\omega \simeq \omega_{\text{p-h}}(\mathbf{k},\mathbf{q})$, quasiparticles with group velocity $\nabla_{\mathbf{k}} E_{\mathbf{k}}$ move in phase with the electromagnetic wave fronts and are virtually excited with high probability. Those excited quasiparticles with wave vector $\mathbf{k}$ along the vector potential $\mathbf{A}$ enhance the reactive part of the current. As the form of $\omega_{\text{p-h}}$ shows, the effect is particularly pronounced for experimental orientations with $\mathbf{q}$ oriented perpendicular to gap nodes.

Below we present a simple model calculation to illustrate the described effect, and argue that the results are consistent with a state of E_{1g} symmetry, provided resonant impurity scattering is assumed. We further argue that dissipation and order-parameter collective modes may be ruled out as alternative explanations for the low-T effect. Finally, we discuss possible origins of the peak in $\delta(T)$ near T_c.

The electromagnetic response of a superconductor to a transverse electromagnetic wave is given by

$$\mathbf{j}_s(\mathbf{q},\omega) = \overleftrightarrow{\mathrm{K}}(\mathbf{q},\omega)\cdot\mathbf{A}(\mathbf{q},\omega)\,, \tag{1}$$

where we have chosen the gauge $\mathbf{q}\cdot\mathbf{A}=0$, $\phi=0$, valid in the absence of coupling to density fluctuations. In general, the calculation of $\overleftrightarrow{\mathrm{K}}(\mathbf{q},\omega)$ in an anisotropic system is a difficult problem, especially if one wishes to include the effects of impurity scattering. Even without impurities, the problem is considerably more complicated than the BCS case because of the coupling to order-parameter collective modes.[17,18] Fortunately, in the present case the relevant frequencies are so low that collective modes may be safely neglected, as discussed below. We therefore begin by examining the case of a pure anisotropic superconductor, postponing for the moment a treatment of the more realistic case including impurities. In this case the response is given by

$$\overleftrightarrow{\mathrm{K}}(\mathbf{q},\omega) = \frac{ne^2}{mc}\left[1+3\int\frac{d\Omega}{4\pi}\hat{\mathbf{k}}{:}\hat{\mathbf{k}}\frac{\eta^2}{\omega^2-\eta^2}[1-\lambda_{\mathbf{k}}(\mathbf{q},\omega)]\right], \tag{2}$$

where $\eta = \mathbf{v}_{\mathbf{k}}\cdot q$ and $|\mathbf{v}_{\mathbf{k}}| = v_F$ is the Fermi velocity. The function $\lambda_{\mathbf{k}}$ is the Tsuneto pair-correlation function given by $(q \ll k_F)$

$$\lambda_{\mathbf{k}}(\mathbf{q},\omega) = -4|\Delta_k|^2\int d\xi_k\frac{\omega^2\theta_k+\eta^2\xi_k\theta_k'}{\omega^2(\omega^2-4E_k^2)-\eta^2(\omega^2-4\xi_k^2)}\,, \tag{3}$$

where $\theta_k \equiv (1/2E_k)\tanh E_k/2T$, $\theta_k' \equiv d\theta_k/d\xi_k$, and $E_k \equiv (\xi_k^2+|\Delta_k|^2)^{1/2}$. For the order-parameter Δ_k we assume the form appropriate for a state with E_{1g} symmetry in a system with a spherical Fermi surface, $\Delta_k = 2\Delta_0\hat{\mathbf{k}}_z(\hat{\mathbf{k}}_x + i\hat{\mathbf{k}}_y)$. Note that Δ_0 represents the maximum of the gap over the Fermi sphere, and that Δ_k has a line of zeros at the equator and point nodes at the poles. The manifold of nodes is thus a combination of those characteristic of the p-wave "polar" and "axial" states, suggesting the name "hybrid" state.[14]

To now calculate the inductive skin depth, we solve the boundary-value problem for an electromagnetic wave incident on a half-space containing a superconductor with a specularly reflecting surface. This leads to the definition[19]

$$\delta(\omega,T) = \frac{2}{\pi}\int_0^\infty dq\,\mathrm{Re}\frac{1}{q^2+(4\pi/c)K(\mathbf{q},\omega)}\,, \tag{4}$$

where $K \equiv \hat{\mathbf{n}}\cdot\overleftrightarrow{\mathrm{K}}\cdot\hat{\mathbf{n}}$ and $\hat{\mathbf{n}}$ is a unit vector in the direction of $\mathbf{A}$. In the absence of smaller length scales introduced by massive collective modes,[18] the skin depth δ is determined by Fourier components of $\overleftrightarrow{\mathrm{K}}$ ranging from $q=0$ to roughly $q=1/\Lambda$. If $\omega=0$, there are no other length scales in the problem (type-II superconductor), and δ reduces to $\Lambda \equiv K(0,0,T)^{-1/2}$. At finite frequencies, the additional length $av_F/\omega \equiv q_0^{-1}$ is introduced, but plays no role until q_0^{-1} becomes comparable to Λ. Here a is a dimensionless constant which may, however, be a strong function of temperature.

In the static homogeneous limit $\omega \to 0$, $q \to 0$, the response $\overleftrightarrow{\mathrm{K}}(\mathbf{q},\omega)$ given by Eqs. (2) and (3) must approach $K(T) \equiv (ne^2/mc)\overleftrightarrow{\mathrm{n}}^s \equiv (ne^2/mc)(1-\overleftrightarrow{\mathrm{n}}^p)$, where $\overleftrightarrow{\mathrm{n}}^s$ and $\overleftrightarrow{\mathrm{n}}^p$ are the superfluid and paramagnetic (normal fluid) densities.[17] At low temperatures, $T \ll \Delta_0$, the two eigenvalues $n_\parallel^p$ and $n_\perp^p$ for the hybrid state may be estimated to vary as $1.63T/\Delta_0$ and $2.47(T/\Delta_0)^2$, respectively. These determine the temperature dependence of the static penetration depth in configurations with $\mathbf{A}$ parallel and perpendicular to the gap axis $\hat{\mathbf{z}}$.

The dynamic response $\overleftrightarrow{\mathrm{K}}(\mathbf{q},\omega)$ takes on a particularly simple form in the so-called "macroscopic" limit $\omega \ll T$, which will be sufficient for our purpose. Defining $\overleftrightarrow{\mathrm{K}} \equiv \overleftrightarrow{\mathrm{K}}_0(T)+\delta\overleftrightarrow{\mathrm{K}}(\mathbf{q},\omega;T)$, we find

$$\delta\overleftrightarrow{\mathrm{K}}(\mathbf{q},\omega;T) = \frac{ne^2}{mc}3\int\frac{d\Omega}{4\pi}\hat{\mathbf{k}}{:}\hat{\mathbf{k}}\int d\xi_k\frac{\omega^2}{\omega^2-(\mathbf{q}\cdot\mathbf{v}_k^\beta)^2}\times\left(\frac{-\partial f}{\partial E_k}\right), \tag{5}$$

where $\mathbf{v}_k^\beta \equiv \nabla_k E_k$ is the Bogoliubov quasiparticle velocity and $f(E)$ is the Fermi function. The denominator under the integral is seen to describe the resonant virtual particle-hole processes alluded to above. Note that in the limit $\omega \gg |\mathbf{q}\cdot\mathbf{v}_k^\beta|$, Eq. (5) reduces to the paramagnetic density.[17]

To make further progress, we consider the special case $\mathbf{q}\parallel\hat{\mathbf{z}}$, $\mathbf{A}\parallel$ basal plane. Then for temperatures $\omega \ll T \ll \Delta_0$, $\mathrm{Re}K_\perp(\mathbf{q},\omega)$ is found to have the limiting forms $\mathrm{Re}K_\perp \simeq (ne^2/mc)$ for $\gamma D \gg 1$ and $\mathrm{Re}K_\perp \simeq (ne^2/mc)(1-n_\perp^p+3\gamma^2 D/\pi)$ for $\gamma D \ll 1$, where $\gamma \equiv \omega/v_F q$ and $D \equiv \Delta_0/T$. The essential point here is that the superconductor at finite temperatures possesses a perfect diamagnetic response for all wave vectors from 0 to roughly $q_0 \equiv \omega/T\xi_0$, where $\xi_0 \equiv v_F/\pi\Delta_0$ is the coherence length. Note that in this geometry, q_0 is enhanced at low T by the phase-space factor $a = \Delta_0/T$ contributed by the line node in the basal plane. The reduction of the Meissner current by thermally excited quasiparticles may thus be largely *suppressed* if q_0 is of the order of $1/\Lambda$, which sets the scale in Eq. (4). Clearly, the low-temperature skin depth $\delta(\omega,T)$ will always be smaller than the penetration depth $\Lambda(T)$ by an amount determined essentially by the size of q_0. Note, however, that at $T=0$ both quantities reduce to Λ_0, so that these effects are important only in an intermediate temperature range. We remark further that it is the size of the ratio $\kappa \equiv \Lambda_0/\xi_0$ (thought to be in the range 10^2-10^3 in UPt_3)[2,20,21] which renders the effect so dramatic in a heavy-fermion system.

In Fig. 1, we display a numerical evaluation of $\delta(\omega,T)$ using an approximate form of Eq. (5) valid for $\omega \ll T \ll \Delta_0$ (solid lines). Since $\delta-\delta_0$ scales with the ratio $\alpha \equiv \kappa\omega/\Delta_0$, and the experimental frequencies are roughly $\omega/\Delta_0 \simeq 10^{-3}$, the curves $\alpha=0.1$ and 1.0 correspond to values of $\kappa=10^2$ and 10^3, respectively. For $\alpha \ll 1$, the skin depth plotted reduces to the approximate penetration depth $\Lambda_0/(1-1.63T/\Delta_0)^{1/2}$. For larger α, $\delta-\delta_0$ is no longer expressible as a simple power series in T, and eventually exhibits a very shallow minimum.

While we have shown that drastic changes in the temperature dependence of the skin depth are possible at extremely small frequencies, our $\omega \rightarrow 0$ result for the E_{1g} state is still not comparable with the dc measurement of Gross *et al.*,[16] who found a T^2 dependence for $\Lambda-\Lambda_0$. Such a result would be recovered if one had point nodes in the basal plane.[17] However, point nodes should be much less sensitive to the effect described in this work. In addition, the only such states in group theoretical classifications correspond to one-dimensional representations of D_6, and are therefore unable to account for the complex phase diagram observed in UPt_3 in a magnetic field. We therefore discuss the possibility that impurity scattering is sufficiently strong to give the behavior observed in Ref. 16. As noted in Refs. 17, 22, and 23, resonant scattering leads to a T^2 law for $\Lambda-\Lambda_0$ regardless of the superconducting state. This effect can be crudely modeled by accounting for the broadening of line nodes into strips in the presence of impurities. We may assume, for example, that the order parameter takes the form $|\Delta_k|^2 = 4\Delta_0^2(k_z^2-x_0^2) \times (1-k_z^2)$, where $2x_0$ is the angular width of the strip. This model may be justified by an examination of the angle-resolved density of states in a full self-consistent treatment of impurity scattering; x_0 is then found to be of order Γ/Δ_0, where Γ is the scattering rate.[19,22] However, we emphasize that this approach is only a crude way of obtaining "quick and dirty" results for the impure case. While it does reproduce the correct T^2 behavior in the resonant "gapless" limit, a proper treatment of the scattering is necessary before definitive comparison with experiment can be made.

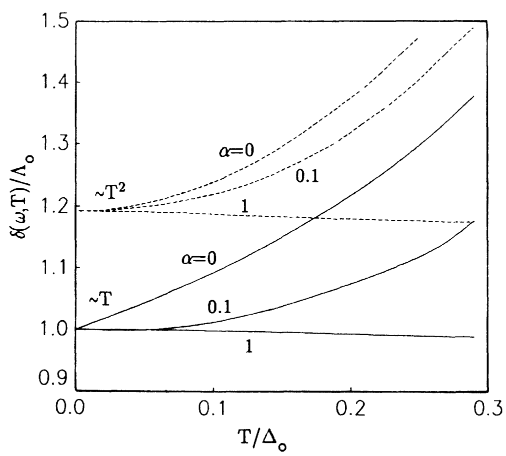

FIG. 1. Normalized skin depth $\delta(\omega,T)/\Lambda_0$ vs T/Δ_0 for $\alpha \equiv \kappa\omega/\Delta_0 = 0, 0.1, 1.0$. Solid lines: pure E_{1g} state. Dashed lines: impurity scattering parameter $x_0 = 0.2$.

As seen in Fig. 1, the depression of the temperature dependence of $\delta-\delta_0$ with increasing α is qualitatively similar when impurities are included in this phenomenological way (dashed lines). Thus, the E_{1g} state is consistent with a variation of the penetration depth between a T^2 behavior in the dc case[16] and the nearly temperature-independent behavior observed at $f \simeq 10$ MHz, provided the ratio κ is $\gtrsim 3\times 10^2$. Given a value of $\xi_0 \simeq 100$ Å,[20] this suggests $\Lambda_0 \gtrsim 3$ μm, compared with the value 0.8 μm, found by Gross *et al.*[21]

We comment briefly on other experimental configurations. The original geometry employed by Shivaram *et al.*[1] ($\mathbf{q} \parallel \hat{\mathbf{x}}$, $\mathbf{A} \parallel \hat{\mathbf{y}}$) may be expected to show the weakest depression of $\delta(\omega,T)$ with α; in fact a low T estimate clearly shows that, if one accounts only for the basal-plane line node contribution, δK in Eq. (5) has the same T dependence as $n^p(T)$ itself. Only in the E_{1g} state, with additional point nodes along the c axis, is a further suppression to the observed rough T^4 power law possible. Preliminary estimates of δ for this and the third geometry, $\mathbf{A} \parallel \hat{\mathbf{z}}$, $\mathbf{q} \parallel \hat{\mathbf{y}}$ indicate that the anisotropy in δ is qualitatively accounted for by the present theory for the E_{1g} state. A detailed numerical evaluation is in progress.

It is natural to ask if there are other small energy scales which may play a role in an unconventional superconductor. The first is the scattering rate: even at low frequencies the response has a dissipative part which could in principle be important. However, it is easy to check that $\mathrm{Im}K \ll \omega\tau \mathrm{Re}K$ in such a system at low temperatures and frequencies. For the present experiment, $\omega\tau \simeq 10^{-3}$,[1,2] so that $\mathrm{Im}K$ is small compared to $\hat{\mathbf{n}}\cdot\overleftrightarrow{n}^p\cdot\hat{\mathbf{n}}$ at about $T/\Delta_0 \simeq 0.1$. A second possibility in an unconventional superconductor is that an order-parameter collective mode enhances the current at low frequencies.[18] The only low-frequency mode in the E_{1g} state, however, is the phase mode, which is pushed up to the plasma frequency whenever it couples to the electromagnetic field. An interesting possibility is that a new mode may appear as a result of the small symmetry-breaking field thought to be responsible for the second superconducting transition in UPt_3.[24] All such modes will, however, be overdamped when $\omega\tau \ll 1$.[17]

Finally, we note that, while a peak in $\delta(T)$ near T_c similar to that reported by Shivaram *et al.*[1,2] has been seen and explained[25] in ordinary superconductors as a crossover between the divergent penetration depth $\Lambda(T)$ and the finite skin depth δ_N at T_c, such peaks are to be expected[25] only when the mean free path $l \ll \xi_0 \ll \Lambda_0$ *or* $\Lambda_0 \ll \xi_0$, neither of which is appropriate for UPt_3. It is possible that the much larger dissipative response of an anisotropic superconductor may give rise to such effects, however. An investigation of these questions and a more detailed quantitative study of the low-temperature response are in progress.

In summary, we have shown that strong type-II superconductors with gap nodes on the Fermi surface display a remarkable effect when irradiated by an electromagnetic field: the vanishing of the temperature dependence of the penetration depth even for frequencies much smaller than the gap. This effect, which occurs when Bogoliubov quasiparticle-quasihole excitations amplify the shielding current, may be responsible for the discrepancy between dc and ac penetration-depth measurements in UPt_3. The technique may thus potentially be used as a sensitive probe of the gap structure in heavy fermion and HTC superconductors. Preliminary analysis indicates that an E_{1g} or hybrid state is consistent with existing measurements.

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