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Spin-polarization enhancement of dilute ^{3}He-^{4}He solutions through porous media

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The flow of ^{3}He quasiparticles in a dilute ^{3}He-^{4}He solution through a porous medium is investigated theoretically on the basis of the percolation model. It is pointed out that the chemical potential μ of ^{3}He quasiparticles is related to the percolation density p of the porous medium. This allows us to control the flow rate for polarized ^{3}He quasiparticles through the porous medium by magnetic fields. It is suggested that highly spin-polarized ^{3}He-^{4}He solutions are achieved by this mechanism at millikelvin temperatures.

One of the important problems in the field of low-temperature physics concerns the experimental achievement of spin-polarized quantum systems such as the H↑, D↑, or ^{3}He↑ phase, where completely new phenomena could be expected due to the polarization of nuclear spins. Many ideas to produce these systems have been proposed so far.[1-3] It is, however, difficult to reach the degenerate Fermi-fluid regime in gases, since the condensation to the liquid states generally sets in before the effects of quantum degeneracy become important. The most promising system studying the degenerate case is a dilute ^{3}He↑-^{4}He solution which provides unique insights about many effects common to different types of degenerate Fermi systems,[4] including transport properties.[5] It is sure that dilute solutions can be polarized almost perfectly by the application of a large magnetic field. This brute force method is, however, applicable to the case of very low concentration of ^{3}He in He II of 0.01% because magnetic fields at hand are limited below several ten Tesla. Mullin[6] has suggested that degenerate ^{3}He↑ systems are produced by using a Nuclepore filter. The idea is based on the fact that, in polarized gas, ↑ and ↓ spins have different Fermi velocities so that the dominant spin species diffuse down a channel at a faster rate.

In this Rapid Communication, we propose a novel mechanism utilizing a porous medium by which a highly spin-polarized dilute ^{3}He-^{4}He solution with a *high concentration* of ^{3}He can be achieved. The schematic diagram of our system is given in Fig. 1, in which a dilute ^{3}He-^{4}He solution is partially polarized in one chamber and polarized ^{3}He quasiparticles flow through the porous medium into a second chamber filled by He II. The porous medium is modeled as a percolating network consisting of spherical pores (diameter a) and cylindrical channels (length l and radius d with distribution) connecting pores.[7] The zero-point energy of ^{3}He quasiparticles confined in a porous medium can be expressed as a function of sizes of pores or channels. For instance, the zero-point energy of ^{3}He quasiparticles in a cylindrical channel with the cross section πd^2 becomes $V_{ch} \simeq \pi^2\hbar^2/4m_3^* d^2$, where m_3^* is the effective mass of ^{3}He quasiparticles in a dilute solution. It is crucial that the zero-point energies of ^{3}He quasiparticles in pores are lower than those in channels due to the condition $a \geq 2d$. As a result, when ^{3}He quasiparticles diffuse in the porous medium, they feel the random potential barrier V_{ch} corresponding to the random distribution of channel radii d's. Only ^{3}He quasiparticles with *kinetic energy* $E_{k\sigma}$ (σ is the spin index) higher than the barrier height V_{ch} can diffuse into the adjacent pores (the bond is opened up). This means that the percolation density p for diffusive ^{3}He quasiparticles is related to their energy. We can neglect the tunneling effect of ^{3}He quasiparticles through narrow channels because of its very low tunneling probability. Thus the bond-percolation density $p(E_{k\sigma})$ for ^{3}He quasiparticles with the energy $E_{k\sigma}$ is expressed as

$$p(E_{k\sigma}) = p_{st} \int_0^{E_{k\sigma}} F(V_{ch}) dV_{ch} , \tag{1}$$

where p_{st} is the bond-percolation density for the original porous network. The function $F(V_{ch})$ is the density of the distribution of potential barrier V_{ch}, which we determine from the inverted random void percolation model.[8,9] In this model, the distribution function is expressed by $F(V_{ch}) = 3V_{min}(1 - V_{min}/V_{ch})^{1/2}/2V_{ch}^2$, where V_{min} is the bottom energy of potential wells. We can derive from Eq. (1) that the percolation density $p(E_{k\sigma})$ is expressed by $p(E_{k\sigma}) = p_{st}(1 - V_{min}/E_{k\sigma})^{3/2}$. It is important to note that ^{3}He quasiparticles with kinetic energy lower than some critical energy E_c, defined by $p(E_c) = p_c$, cannot diffuse in the porous medium. Here p_c is the critical percolation density.[10]

The diffusion constant of ^{3}He quasiparticles with the energy $E_{k\sigma}$ is given in the following general form:[8,11]

$$D(E_{k\sigma}) \sim \frac{l^2}{\bar{\tau}(E_{k\sigma})} [p(E_{k\sigma}) - p_c]^{t-\beta} , \tag{2}$$

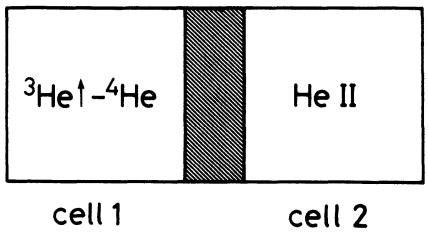

FIG. 1. Schematic diagram of our system. Cells 1 and 2 are separated by a thin porous medium with thickness L, where the cells 1 and 2 are filled initially with a partially polarized ^{3}He-^{4}He solution and He II, respectively.

where t and β are the exponents for the conductivity and for the order parameter (the probability belong to the infinite cluster) of the percolating network, respectively.[12] The channel length is given by l. The averaged hopping time of diffusive particle with $E_{k\sigma}$ is given by $1/\bar{\tau}(E_{k\sigma}) \sim E_{k\sigma}^{1/2}$.[13]

The diffusion constant for polarized ^{3}He quasiparticles in a dilute ^{3}He-^{4}He solution with the chemical potential μ under a magnetic field B is defined by $\mathbf{J}_\sigma = -D_\sigma(\mu, B, T)\nabla n_\sigma$, where $\mathbf{J}_\sigma$ and n_σ are the flow density and the number density of ^{3}He quasiparticles with the spin σ, respectively. The diffusion constant $D_\sigma(\mu,B,T)$ is obtained by averaging the diffusion constant $D(E_{k\sigma})$ for a diffusive ^{3}He quasiparticle with $E_{k\sigma}$ as, by using the number density $n(E_{k\sigma})$ of ^{3}He quasiparticles,

$$D_\sigma(\mu,B,T) = \frac{\int_{E_c}^{\infty} D(E_{k\sigma}) n(E_{k\sigma}) dE_{k\sigma}}{\int_{E_c}^{\infty} n(E_{k\sigma}) dE_{k\sigma}}, \tag{3}$$

where the number density $n(E_{k\sigma})$ is given by the product of the density of states (DOS) for ^{3}He quasiparticles and the Fermi distribution function expressed by $f(E_{k\sigma}) = 1/\{\exp[-(E_{k\sigma} - \gamma B\sigma - \mu)/k_B T] + 1\}$. Here γ is ^{3}He nuclear magnetic moment (0.778 mK/T) and $\sigma = \pm 1$. The density of states $\rho(E_{k\sigma})$ for polarized ^{3}He quasiparticles has a continuous spectrum for $E_{k\sigma} > E_c$ and a discrete one for $E_{k\sigma} < E_c$. This is due to the fact that ^{3}He quasiparticles for $E_{k\sigma} < E_c$ are localized in the porous medium. The DOS per unit volume for the continuous spectrum for $E_{k\sigma} > E_c$ is given by

$$\rho(E_{k\sigma}) = \frac{m_3^{*3/2}}{\sqrt{2}\pi^2\hbar^3}\sqrt{E_{k\sigma}}. \tag{4}$$

Using Eqs. (2)–(4), we find that the diffusion constant for zero magnetic field shows the universal critical behavior in the vicinity of $\mu = E_c$ as a function of the chemical potential μ as $D \sim (\mu - E_c)^{t-\beta}$. Under a magnetic field B, the diffusion constant for ^{3}He quasiparticles behaves as $D_\sigma \sim (\mu + \gamma B\sigma - E_c)^{t-\beta}$. This indicates that the mobility edge E_σ^m for ^{3}He quasiparticles is different for ↑ and ↓ spins under a magnetic field, where the mobility edge is defined as the minimum energy at which ^{3}He quasiparticles can flow, i.e., $E_\sigma^m = E_c - \gamma B\sigma$. Hereafter, ^{3}He quasiparticles with parallel spin to the magnetic field and with anti-parallel spin are denoted by ^{3}He↑ and ^{3}He↓, respectively. In the case of $\mu = E_c$, we see that ^{3}He↑ quasiparticles diffuse dominantly according to the power law on the magnetic field B as

$$D_\uparrow \sim B^{t-\beta}, \tag{5}$$

but ^{3}He↓ quasiparticles do not diffuse through the porous medium (see Fig. 2).

By taking the suitable values for our percolating system, the pore diameter $a = 50$ Å, the channel length $l = 50$ Å, and the percolation density of the porous medium $p_{st} = 0.5$, the mobility edge at zero magnetic field is calculated as $E_c = 70$ mK. Using these values and the known values of the exponents t and β for the $d = 3$ percolating network, the diffusion constants for ↑ spins at $\mu = E_c$ are computed in Fig. 3 from Eq. (3) as a function of magnetic field B at various temperatures. We find that the diffusion constant shows the power-law dependence of Eq. (5) in the region $B \lesssim 10$ T.

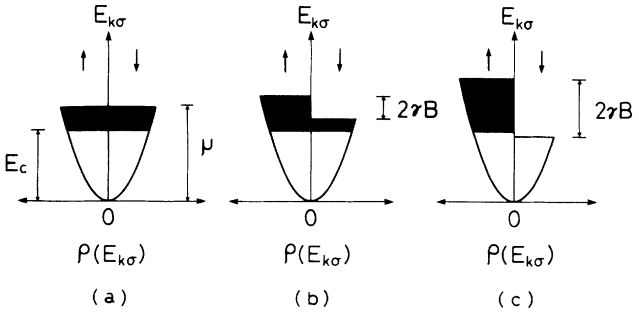

FIG. 2. Schematic illustrations of the density of states (DOS) under various magnetic fields. The DOS's are shown as a function of the kinetic energy $E_{k\sigma}$ of ^{3}He quasiparticles with spin σ in a dilute ^{3}He-^{4}He solution with chemical potential μ (a) under zero magnetic field and (b),(c) magnetic field B. The magnetic field for (c) is higher than that of the case (b). The shaded regions indicate that ^{3}He quasiparticles can diffuse into porous media, i.e., $E_{k\sigma} > E_c$. Figures (b) and (c) correspond to the case I $[n_{1\uparrow}(0) > n_{1\downarrow}(0) > n_c]$ and the case II $[n_{1\uparrow}(0) > n_c > n_{1\downarrow}(0)]$, respectively.

Let us consider the situation in which ^{3}He quasiparticles in a dilute ^{3}He-^{4}He solution in one cell can flow through a porous medium into the second cell filled by superfluid ^{4}He. As argued above, the magnitude of the diffusion constant becomes different among ↑ and ↓ spins because these particles have different kinetic energies under high magnetic fields, i.e., the dominant spin species

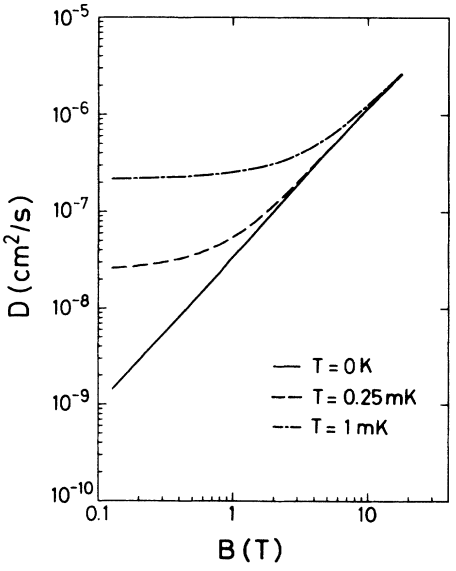

FIG. 3. The diffusion constants $D_\uparrow$ of ^{3}He quasiparticles in dilute ^{3}He-^{4}He solution with $\mu = E_c$ are calculated as a function of magnetic fields from Eq. (3) for various temperatures. We have used the following values of the parameter: the pore diameter $a = 50$ Å, the channel length $l = 50$ Å, and the percolation density of the porous medium $p_{st} = 0.5$, respectively. The critical energy is obtained as $E_c = 70$ mK and the known exponents for percolating nets, $\beta = 0.4$ and $t = 2.0$, are used.

diffuses faster. This implies that the spin polarization in the second cell will be enhanced. In the next paragraph we will discuss the time development of the spin polarization in each cell.

Consider the case that two cells of each volume V_1 and V_2 are separated by a thin porous medium with the thickness L ($z=0$ and $z=L$). The current density of ^{3}He quasiparticles with spin σ is written by

$$J_\sigma = -D_\sigma \frac{\partial n_\sigma}{\partial z}, \tag{6}$$

where the diffusion constant D_σ is defined by Eq. (3), and n_σ is the number density of ^{3}He quasiparticles with spin σ in the porous medium. The time derivative of the density $n_{1\sigma}$ in cell 1 is proportional to the current density J_σ as

$$\frac{dn_{1\sigma}}{dt} = -\frac{A}{V_1} J_\sigma(0), \tag{7a}$$

where A is the effective cross section of the porous medium at $z=0$. The identical equation holds for the density $n_{2\sigma}$ in the other cell,

$$\frac{dn_{2\sigma}}{dt} = \frac{A}{V_2} J_\sigma(L). \tag{7b}$$

The current density of ^{3}He quasiparticles in the porous medium can be determined by Eq. (6) and the continuity condition for the density of the form $\partial n_\sigma/\partial t = \partial J_\sigma/\partial z$. It is obvious that, as long as the total void space in the porous medium is much smaller than that of each cell, the density reaches the steady state faster ($\partial n_\sigma/\partial t = 0$) in the porous medium. Thus the current density is obtained by integrating Eq. (6) with respect to z from 0 to L,

$$J_\sigma = -\frac{1}{L}\int_{n_{1\sigma}}^{n_{2\sigma}} D_\sigma \, dn_\sigma, \tag{8}$$

where $n_{1\sigma}$ and $n_{2\sigma}$ indicate the boundary conditions for n_σ at $z=0$ and $z=L$, respectively. The time derivative of the density for ^{3}He quasiparticles with spin σ in the cell i is obtained from Eqs. (7) and (8),

$$\frac{dn_{i\sigma}}{dt} = (-1)^{i+1}\frac{A}{LV_i}\int_{n_{1\sigma}}^{n_{2\sigma}} D_\sigma \, dn_\sigma \quad (i=1,2). \tag{9}$$

We have iteratively computed these equations at $T=2$ mK for the following initial conditions (see also the caption of Fig. 4): the case I in which $n_{1\uparrow}(t=0) > n_{1\downarrow}(t=0) > n_c$, and the case II in which $n_{1\uparrow}(t=0) > n_c > n_{1\downarrow}(t=0)$. Here the critical density n_c, where ^{3}He quasiparticles in the cell cannot diffuse into the porous medium, is given by[10]

$$n_c = \frac{(2m_3^* E_c)^{3/2}}{6\pi^2\hbar^3}. \tag{10}$$

The density in cell 2 is zero for both types. The time dependence of the polarization in the cell i is defined by

$$M_i(t) = \frac{n_{i\uparrow}(t) - n_{i\downarrow}(t)}{n_{i\uparrow}(t) + n_{i\downarrow}(t)}. \tag{11}$$

Let us consider, at first, the case I. Since the relevant velocities of ^{3}He quasiparticles are different for $\uparrow$ and $\downarrow$ spin, ^{3}He$\uparrow$ quasiparticles diffuse faster into cell 2. As a next step, ^{3}He$\downarrow$ quasiparticles flow into cell 2. The time development of the polarization ratio in cell 2 is calculated in Fig. 4, as well as the time dependence of ^{3}He concentration. The mechanism of polarization enhancement of this type is analogous to that suggested by Mullin.[6] The case II is unique where ^{3}He$\uparrow$ quasiparticles flow dominantly into cell 2. This is because ^{3}He$\downarrow$ quasiparticles do not diffuse through the porous medium. As a result, the polarization in cell 2 becomes almost unity and in cell 1 decreases from the initial one. One sees from the calculated curves for the case II in Fig. 4 that the concentration of ^{3}He quasiparticles in cell 2 increases monotonically as a function of time. This implies that an almost complete spin-polarized ^{3}He$\uparrow$ system with a high concentration of ^{3}He quasiparticles can be produced. It should be emphasized that the mechanism of spin-polarization enhancement of the type II is unique compared with that of the type I because we utilize the fact that the mobility edge exists for the system. It should be noted that the spin-relaxation time due to the interaction with a wall coated by ^{4}He is much larger than the diffusion time $\tau_D \sim L^2/D$ (L is the size of sample) of polarized ^{3}He quasiparticles through the porous medium (for the case of thickness 1 μm, τ_D is of the order of 1 s).[14]

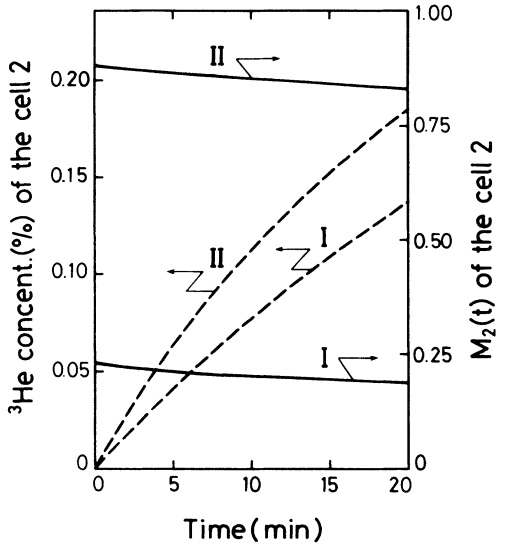

FIG. 4. The time development of the polarization ratio $M_2(t)$ (solid lines) and ^{3}He concentration (dashed lines) in cell 2 for two cases. These two I and II are realized for the cases that 0.59% ^{3}He-^{4}He solution ($E_F=80$ mK) in cell 1 is polarized under magnetic fields $B=2$ and 12 T at $T=2$ mK; i.e., the initial conditions are taken as $n_c=5.15\times10^{19}/\text{cm}^3$, $n_{1\uparrow}(t=0)=6.40\times10^{19}/\text{cm}^3$, $n_{1\downarrow}(0)=6.00\times10^{19}/\text{cm}^3$ for I, and $n_{1\uparrow}(0)=7.29\times10^{19}/\text{cm}^3$, $n_{1\downarrow}(0)=5.11\times10^{19}/\text{cm}^3$ for II, respectively. These conditions produce the initial polarization ratio $M_1(0)=0.03$ and 0.18, respectively. We have used the following values of parameters: the volume of cells 1 and 2 are $V_1=4V_2=4$ cm^3, the thickness of the porous medium $L=1$ μm, the effective area $A=1$ cm^2, and other parameters the same as those in Fig. 3.

In summary, we have studied the mechanism of spin-polarization enhancement for ^{3}He quasiparticles in dilute

^{3}He-^{4}He solutions through a porous medium. It is crucial that the mobility edge for ^{3}He quasiparticles is different for ↑ and ↓ spins under high magnetic fields. Using this magnetic technique, it is possible to produce the degenerate Fermi system with a very high polarization ratio. In particular, an almost completely polarized system with a high concentration of ^{3}He quasiparticles can be achieved for the case that the number density of one spin species (↑ or ↓) under magnetic fields is smaller than the critical density n_c.

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[10]The diffusion of ^{3}He quasiparticles is treated as a problem of percolation in a classical point of view from the following reason. A wave packet of ^{3}He quasiparticles has a spatial extent over $\delta x \sim \hbar/\delta p$, where the thickness of the energy shell at T K is $\delta_p \simeq 2k_B T/v_F$. This leads us to take the spatial extent as $\xi_0 \simeq 0.5/T(\text{Å})$ by using the Fermi velocity $v_F = 1.3\times 10^3$ cm/sec for 0.5% solution, i.e., a wave packet of ^{3}He quasiparticle extends over a few hundred angstrom in the millikelvin temperature region (e.g., $\xi_0 \simeq 100$ Å at 5 mK). Since we consider the situation where the average distance between pores is of the order of 100 Å, the localization effect due to the quantum interference of ^{3}He quasiparticles is irrelevant.

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$$\bar{\tau}(E_{k\sigma}) = \frac{6a}{[2(E_{k\sigma} - V_{min})/m_3^*]^{1/2}} \left\{ \ln \frac{1 + (1 - V_{min}/E_{k\sigma})^{1/2}}{1 - (1 - V_{min}/E_{k\sigma})^{1/2}} - 2(1 - V_{min}/E_{k\sigma})^{1/2} \right\}.$$

From this we see that the dominant $E_{k\sigma}$ dependence for $\bar{\tau}(E_{k\sigma})$ becomes $\bar{\tau}(E_{k\sigma}) \sim E_{k\sigma}^{-1/2}$ because of $E_{k\sigma} \gg V_{min}$.

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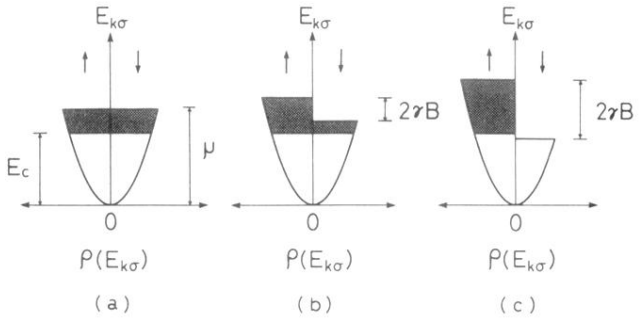

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