Spectral and magnetic transitions in the quantum Ising model

Indubala I. Satija
Department of Physics, George Mason University, Fairfax, Virginia 22030
(Received 13 October 1989)

The quantum Ising model in a transverse field is studied where the exchange interaction is a modulating function of sites with many harmonics. The magnetically disordered phase is found to exhibit a "mixed" spectrum containing both critical and localized states, while the magnetically ordered phase has a pure spectrum with all states localized. Therefore, the spectral transition, although broadened, occurs simultaneously with the magnetic transition, and the magnetic disorder is accompanied by spectral disorder.

Following an extensive study of quasiperiodic (QP) tight-binding models (TBM) for electrons,[1] QP spin models are currently being investigated.[2,3] The main purpose for such a study is to investigate the effect of the lack of translational order on the magnetic and spectral properties of the system. The one-dimensional QP quantum Ising model in a transverse field has turned out to be an ideal candidate for such a study, as the periodic model is exactly solvable.[4] The system is described by the following Hamiltonian:

$$H=-\sum_n J_n\sigma_n^x\sigma_{n+1}^x-h_n\sigma_n^z\,, \qquad (1)$$

where the σ_n^j are Pauli matrices associated with the site n. Without loss of generality, we will set the transverse magnetic field h_n equal to unity. The model is made QP by choosing the nearest-neighbor exchange interaction J_n to be QP. The Ising models, by Jordan-Wigner transformations,[4] can be mapped to fermion models, quadratic in fermion degrees of freedom,

$$H=\sum_{n,m}[c_n^\dagger A_{nm}c_m+\tfrac{1}{2}(c_nB_{nm}c_m+\text{H.c.})]\,. \qquad (2)$$

Here, the c_n are anticommuting fermion operators. The matrices A and B are, respectively, symmetric and antisymmetric and are defined in Ref. 2. The second term in Eq. (2) is responsible for the long-range order in the spin systems and hence makes the study of the spin problem very different from all the previous studies[1] of tight-binding models describing an electron in an external potential.

A unitary transformation reduces the model to the following generalized tight-binding model

$$J_{n-1}\psi_{n-1}+(1+J_n^2)\psi_n+J_{n+1}\psi_{n+1}=(E^2/4)\psi_n\,. \qquad (3)$$

Here, ψ_n is the wave function of the electron at sight n. This generalized tight-binding model associated with QP spin models exhibits a rich energy spectrum with either critical or localized states depending upon the type of quasiperiodicity.[2,3]

In the previous studies of the QP models, two types of exchange interaction have been studied: (A) J_n takes two values λ_i and λ_2 in a Fibonacci sequence.[2] (B) $J_n=\lambda\sin(2\pi\sigma n)$, where σ is the golden mean.[3] The above interactions will, respectively, be referred to as type-(A) and type-(B) interactions. Both models were found to exhibit a magnetic transition to long-range order (LRO). In model (A), the energy spectrum was found to be a cantor set and the states were critical in both magnetically disordered and ordered phases. However, in model (B) the states are critical in the disordered phase and are localized in the LRO phase; hence, the magnetic transition is found to be accompanied by a spectral transition: In the disordered phase, all the states were critical, while in the magnetically ordered phase, all the states were localized. However, the common feature of both the models is the fact that the spectrum is always pure: i.e., all states are either singular continuous (critical) or dense point (localized).

The electronic TBM with QP potential is

$$\psi_{n+1}+\psi_{n-1}+J_n\psi_n=E\psi_n\,. \qquad (4)$$

This model, where the potential J is given by the above two types (A) and (B), also exhibits a pure spectrum. In the type-(A) case, all states are critical. On the other hand, in the type-(B) case, states are extended for $\lambda<2$ and are localized for $\lambda>2$. At $\lambda=2$, the Andre-Aubry transition takes place and the states are critical. The corresponding spin model exhibits critical states for $\lambda<2$ and the spectral transition at $\lambda=2$ is accompanied by the transition to LRO. Some recent studies[5] of the electronic TBM involving a more general type of potential have pointed out that the QP potentials of types (A) and (B) are very special and a pure spectrum is not a general feature of the QP systems. Models with more general potentials containing higher harmonics exhibit a mixed spectrum with both extended and localized states resulting in mobility edges.

In this paper, I study the spin model (1) where J_n is a modulating function of sites with three or more harmonics.[6] The main purpose of such a study is twofold: to understand the nature of spectral transitions in the QP Ising model where the exchange interaction has a more general form, and to study the relationship between the magnetic and spectral transition. In particular, in models with a more general type of exchange interaction, I address the following questions: (i) Do the magnetic and spectral transitions occur simultaneously? (ii) How does the mixed spectrum, which I will refer to as spectral disorder, affect magnetic order? (iii) Which states are localized first as the parameters of the model are changed? In the electronic TBM with two harmonics, it appears that states at the edges are localized first.[7] However, a recent study on the electronic TBM involving a more general type of potential has reported that localization could begin at the center.[5]

I now summarize the results of this paper: In a phase with no LRO, the energy spectrum is mixed containing both critical and localized states, while in the LRO phase the spectrum is pure and all the states are localized. Therefore, magnetic disorder is found to be accompanied by spectral disorder. The spectral transition, although somewhat broadened, still seems to occur at the same point as the magnetic transition. On the other hand, the nature of magnetic transition is unaffected by this. This study with various QP spin models indicates that both types of localizations are possible; i.e., either the states at the band edges are localized first or the states at the center are localized first.

In this paper, three types of QP exchange interactions containing higher harmonics are investigated and the results are compared with the QP model containing a single harmonic. The exchange interaction is chosen to be of the form $J_n = \lambda f(2\pi\sigma n)$, where the function $f(x)$ has the following forms:

$$f(x) = \lambda[\cos(x) + \alpha\cos(3x) + \beta\cos(5x)] , \qquad (5)$$

$$f(x) = \lambda\frac{\sqrt{(1+\epsilon)}}{2\pi}\arcsin\left[\frac{\sin(x)}{\sqrt{(1+\epsilon)}}\right] , \qquad (6)$$

$$f(x) = \lambda\left[\frac{\tanh[a\cos(x)]}{\tanh(a)}\right] . \qquad (7)$$

The above types of interactions have recently been investigated in various contexts in the study of QP systems. Models (5) and (6) have been studied in the context of understanding the effect of higher harmonics in area preserving maps.[8] Model (6), in the limit as ϵ goes to infinity, reduces to model (B), while in the limit ϵ equal to zero reduces to the piecewise-linear potential. Model (7) was recently investigated[5] in the TBM to understand the nature of the energy spectrum for a more general type of potential. It should be noted that this model reduces a single harmonic model as a goes to zero, and reduces to the QP model with type-(A) interaction as a goes to infinity. This model was studied to demonstrate the existence of a mixed spectrum where the localization begins at the center for some values of the parameters. This is unlike the previously studied case involving two harmonics[7] where the states near the band edges were found to be localized first.

The numerical study of the QP model is along the lines of previous studies.[3] Instead of studying the QP model, I study a sequence of periodic models which approach the QP model in the limiting case. The above-described QP modulating interactions were studied as the parameter λ was varied keeping the other parameters fixed. The model with the QP interactions as given above are found to exhibit a transition to LRO signaled by the vanishing of the gap and nonzero long-range correlation. The critical point corresponding to the onset of LRO depends upon the parameters in the interaction. Figures 1 and 2, respectively, show the magnetization and the total Lebesgue measure or the total bandwidth (TBW) of the spectrum for the single- and three-harmonic cases. In the model with a single harmonic (Fig. 1), TBW increases monotonically as the system approaches the critical point from both the disordered and ordered phases. This is intuitively expected as the system tries to behave as close as possible to a periodic system at the onset of LRO where the correlation length is infinity. However, in models involving higher harmonics, TBW is not a monotonic function of λ, but instead exhibits hills and valleys. [Analogous plots are obtained for models (6) and (7).] By studying the individual bands, this is found to be due to localization of some of the

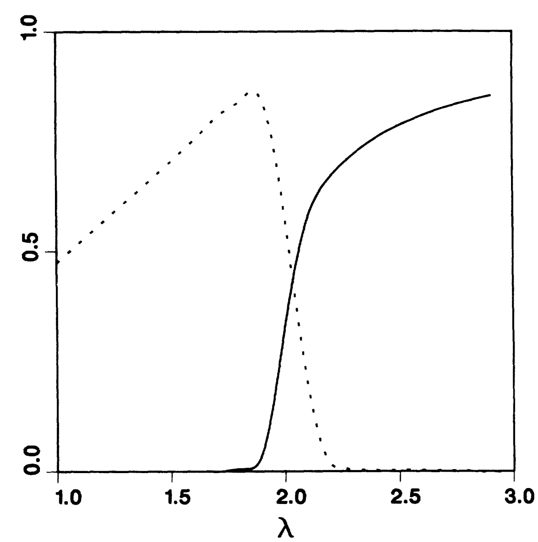

FIG. 1. The TBW (scaled up by a factor of 9) (dotted line) and the magnetization (solid line) for model (2) containing single harmonics for 89 sites.

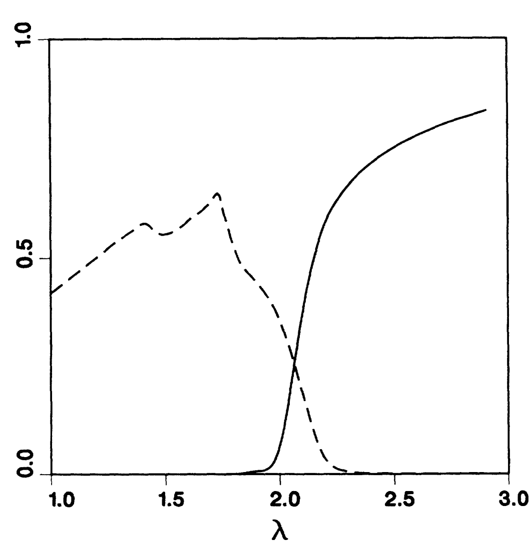

FIG. 2. The TBW (dotted line) and magnetization (solid line) for model (5) for a lattice with 89 sites. $\alpha = 0.05$ and $\beta = 0.01$.

states. As λ is increased toward its critical value, more and more states change their character from critical to localized. Therefore, the system is subjected to two competing effects: the increase in the TBW as the system approaches LRO and decrease in the TBW as the various states undergo transitions to localization. Note that unlike the single-harmonic case where all the states become localized simultaneously, the localization transition is spread throughout the disordered phase. However, the maximum drop in the TBW still occurs at the critical value of λ corresponding to the onset of LRO where all of the states become localized. As the λ is increased beyond the critical values, all the states remain localized resulting in a pure spectrum in the ordered phase. It is rather interesting that the spectral disorder is present in the magnetically disordered phase only. This fact further emphasizes a close relationship between the spectral and magnetic transitions in QP spin systems where the exchange interaction varies smoothly.

Further study of the individual bands revealed that in models (5) and (6) the localization starts from the edges and eventually spreads throughout. This behavior is analogous to that of electronic models involving two harmonics[7] and is identical to Anderson localization[9] in three dimensions. However, model (7) was found to exhibit different behavior: Analogous to the corresponding electronic problem, but in contrast with the Anderson localization, the localization was found to start from the center for some values of a.

The work described here along with that of Ref. 3 clearly demonstrates the fact that in models exhibiting a transition to LRO and a transition to localization, the two transitions are closely related. They not only occur simultaneously, but the magnetic disorder is also accompanied by spectral disorder.

This research was supported by National Science Foundation Grant No. DMR-8819850. I would like to acknowledge the hospitality of the National Institute of Standards and Technology (formerly know as National Bureau of Standards), where part of this work was done.

[1]M. Kohmoto, L. P. Kadanoff, and C. Tang, Phys. Rev. Lett. **50**, 1870 (1983); F. Nori and J. P. Rodriguez, Phys. Rev. B **34**, 2207 (1986).

[2]M. Doria and I. Satija, Phys. Rev. Lett. **60**, 444 (1988); I. Satija and M. Doria, Phys. Rev. B **38**, 5174 (1988).

[3]I. I. Satija and M. Doria, Phys. Rev. B **39**, 9757 (1989).

[4]E. Lieb, T. Schultz, and D. Mattis, Ann. Phys. (N.Y.) **16**, 407 (1961).

[5]H. Hiramoto and M. Kohmoto, Phys. Rev. Lett. **62**, 2714 (1989).

[6]The model with two harmonics was studied before (Ref. 3). Such a model has a rather undesirable feature that the spin interaction vanishes at various sites, resulting in a breakup of the chains into various pieces.

[7]C. Soukoulis and E. N. Economou, Phys. Rev. Lett. **48**, 1043 (1982).

[8]T. Scheidsteger and H.J. Schellenhuber (unpublished); J. Wilbrink, Physica **26D**, 358 (1987); I. Satija and J. Wilbrink, Phys. Rev. B **37**, 3832 (1988); R. Black and I. Satija, Phys. Rev. A **40**, 2864 (1989).

[9]For a review, see P. A. Lee and T. V. Ramakrishnan, Rev. Mod. Phys. **57**, 287 (1985).