Confluence of extended and localized states: Implications on the Mott-Cohen-Fritzsche-Ovshinsky model

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The possibility and consequences of the presence of the poles of the Green's function (in the Anderson model) in the Riemann sheets of a cut Z plane have been discussed. Such poles lack the pure-point nature and correspond to states formed by a "confluence" of degenerate extended and localized states. The character of these states is different from the "pure" extended and localized states both in the mathematical and in the physical sense. They form a new regime of "slow" diffusion in the energy spectrum, which indicates that there should exist another critical energy, like the mobility edge, that separates the new regime of the spectrum from the "absolutely continuous" part.

I. INTRODUCTION

The electron localization (delocalization) in a medium of random potential is mathematically defined in terms of the convergence (divergence) of a renormalized perturbation expansion of the self-energy.[1] This definition can equivalently be put in terms of the Green's function having poles or branch cut along the real energy axis:[1,2] the poles correspond to the discrete set of localized states and the branch cut corresponds to the continuum of the extended states. According to the Mott-Cohen-Fritzsche-Ovshinsky model (hereafter referred to as MCFO model)[3,4] the regime of localized states and that of the extended states do not overlap and are separated sharply from each other by a pair of energies called "mobility edges." This may indicate that the poles lie on either sides of the cut and that the poles and the cut do not overlap as shown schematically in Fig. 1(a). To test if the MCFO model means that the poles must stay clear off the branchcut as shown in the Fig. 1(a) we propose to study the situation depicted in Fig. 1(b) where the regime of poles is assumed to overlap with the branch cut near the mobility edges E_c and E_c'.

We ask the question: can the poles appear inside the cut, and if so what will be their nature in comparison with the isolated poles? The answer to the first question is that, in principle, such a possibility does exist. If we view the branch cut as Riemann manifold, then some poles may be found on the so-called "unphysical" sheets lying below the top (or the physical) sheet. Then the overlapping regions of Fig. 1(b) can correspond to such a situation. The second question cannot be answered straightforwardly and is the subject of study here.

We show that the poles inside the cut do not have the pure-point nature and represent a resonance (or virtual-bound-state) -like situation. We call them "confluence states" supposedly formed by a confluence of localized and extended states. Should a set of such states exist in the vicinity of the mobility edges, it may indicate the possibility of existence of a pair of "pseudomobility edges" separating them from the (pole-free) absolutely continuous spectrum while the mobility edges of the MCFO model separate them from the pure-point spectrum.

"Can localized and extended states coexist at the same energy and at the same time maintain their respective localized and extended characters (?)" might have been the very first question bothering Mott when he generalized Anderson's ideas[1] to include the mobility edges.[3] This question has since been discussed several times (see, e.g., Ref. 5) and always the agreeable answer given has been that whenever such a situation arises the two states will "sooner or later" mix and will result into an extended state. That this indeed is the right answer is proved here for the first time through a systematic mathematical analysis. A localized state degenerate with an extended state has been represented by a pole of the Green's function embedded inside the branch cut, and Anderson's "stay-put probability"[1,2] has been calculated and found to be zero. In the process of doing this we find an additional information about the confluence states—besides being extended in the true Anderson sense—we find that

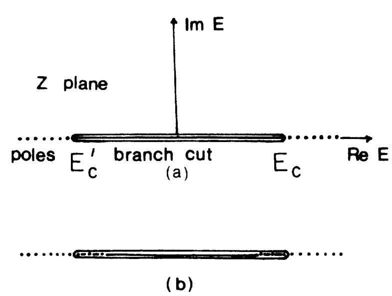

FIG. 1. Branch cut and poles of the disordered Green's function in the complex energy (Z) plane representing the absolutely continuous and the discrete point spectra: (a) if they are isolated and (b) if they can overlap near the mobility edges.

the stay-put probability for them goes to zero at $t \to \infty$ so slowly so that its time integral *diverges*. This is a nontrivial finding as it shows that the confluence states share properties of both—the localized and the extended—states. This calls for a finer characterization of the states in terms of two parameters, namely, the stay-put probability as well as its time integral. On the physical side it sheds a fresh light on Mott's later views on conductivity near the mobility edge and on the mathematical side it clarifies a subtle point about the definition of localization in terms of the Green's function.

In Sec. II we recapitulate the definitions of extended and localized states, in particular, their meaning in terms of cut and poles of the disordered Green's function. The novel possibility of their sharing a certain range of energy is analyzed in Sec. III. The physical nature of the confluence states is discussed in Sec. IV. It is proposed in Sec. V that a finer characterization of the energy spectrum of a disordered system is possible than that done in the MCFO model. The definition of localization (delocalization) in terms of the reality (complexity) of the Green's function is reexamined in the Sec. VI and some observations of basic interest are made. Main findings and certain questions of interest are summarized in the Sec. VII.

II. DEFINITIONS

The basic definition of localization is given in terms of the so called stay-put probability denoted here by $P_a(t)$. It is the probability of rediscovering a particle at an arbitrary site a at time t if initially (at $t=0$) it was there with probability 1. In the limit, $t \to \infty$, if $P_a(t)$ stays nonzero then the particle is said to be localized in the surroundings of site a. A rigorous definition in terms of the Green's function can be deduced from it by recalling that $P_a(t)$ can be written in terms of the product of the diagonal parts of the Greens functions, $G_{aa}(E \pm is)$,[1,2,5] as

$$\lim_{t \to \infty} P_a(t) = \lim_{s \to 0} \left[\frac{s}{\pi} \int_{-\infty}^{\infty} G_{aa}(E+is) G_{aa}(E-is) dE \right] .$$

In order to concentrate on the states corresponding to a single eigenenergy we should consider a vast region in space instead of a site and look at the probability to stay put in this region[6a] as $t \to \infty$.[6(b)] Denoting the "region" by "0" we can write $P_0(t)$ as

$$\lim_{t \to \infty} P_0(t) = \lim_{s \to 0} \left[\frac{s}{\pi} G_{00}(E+is) G_{00}(E-is) \right] . \tag{1}$$

In the tight-binding model[1] of a disordered system the G_{00} is written as

$$G_{00}(E \pm is) = [E \pm is - e_0 - \Sigma_0(E \pm is)]^{-1} , \tag{2}$$

where Σ_0 is the self-energy and e_0 is the potential offered by the region "0." By substituting (2) into (1) one can check that the stay-put probability will be nonzero in the limit $t = \infty$, if

$$\lim_{s \to 0} [\mathrm{Im}\Sigma_0(E \pm is)] = 0 , \tag{3a}$$

such that

$$\lim_{s \to 0} (\mathrm{Im}\Sigma_0 / s) \neq 0 , \tag{3b}$$

where Im denotes the imaginary part. It is important that the criterion (3) is examined probabilistically for a given eigenstate of E rather than considering the Σ_0 in a configurationally averaged system. This criterion can be translated in terms of G_{00} as

$$\lim_{s \to 0} [\mathrm{Im}G_{00}(E \pm is)] = 0 \text{ almost everywhere} , \tag{4a}$$

such that

$$\lim_{s \to 0} \left[\int dE \, \mathrm{Im}G_{00} \right] > 0 . \tag{4b}$$

Equation (4b) ensures that the density of states in the localized regime is nonzero and Eq. (4a) tells that it consists of a dense set of δ-function spikes.

Thus, we see that in the energy range where G_{00} has a branch cut which gives a continuum of states, $\lim_{t \to \infty} P_0(t)$ vanishes and so the states are extended in space. On the other hand, in the energy range where it has poles, the energy spectrum is discrete and the states are spatially localized owing to the nonvanishing of $\lim_{t \to \infty} P_0(t)$.

III. POLES IN THE RIEMANN MANIFOLD

A new situation, which is the subject of discussion here, arises if the Green's function (GF), in the region where it has the branch cut, is analytically continued inside the cut Z plane (the complex E plane) into a manifold $\mathcal{R}$, the Riemann surface, and if it happens to have a pole in one of the sheets of $\mathcal{R}$ underneath the top sheet (the so-called physical sheet). The G_{00}, which was nonanalytic along the portion of the real E axis where it had a cut, can now be replaced by a single-valued *meromorphic* function,

$$\mathcal{G}_{00}(E) = [E - e_0 - S_0(E)]^{-1} . \tag{5}$$

$\mathcal{G}_{00}$ is analytic on $\mathcal{R}$ and its only singularities are poles. The poles of physical interest will be located along those lines in $\mathcal{R}$ that fall under the real E axis. We will denote these lines by $\bar{E}$ and the individual branches by $\bar{E}_{n-1,n}$. The upper lip of the cut in the nth sheet joins the lower lip of the cut in the $(n-1)$th sheet along the line $\bar{E}_{n-1,n}$ (see Fig. 2). The $\mathcal{G}_{00}$ and S_0 are related to G_{00} and Σ_0 in the following manner for a cut of order m:

$$\mathcal{G}_{00}(\bar{E}_{0,1}) = \lim_{s \to 0} [G_{00}^{(1)}(E+is)] ,$$

$$\mathcal{G}_{00}(\bar{E}_{1,2}) = \lim_{s \to 0} [G_{00}^{(1)}(E+is)] + 2\pi i \mathcal{D}(E)$$

$$\equiv \lim_{s \to 0} [G_{00}^{(1)}(E-is)] ,$$

$$\ldots ,$$

$$\mathcal{G}_{00}(\bar{E}_{n-1,n}) = \lim_{s \to 0} [G_{00}^{(n-1)}(E+is)] + 2\pi i \mathcal{D}(E)$$

$$\equiv \lim_{s \to 0} [G_{00}^{(n)}(E-is)] ,$$

$$\ldots ,$$

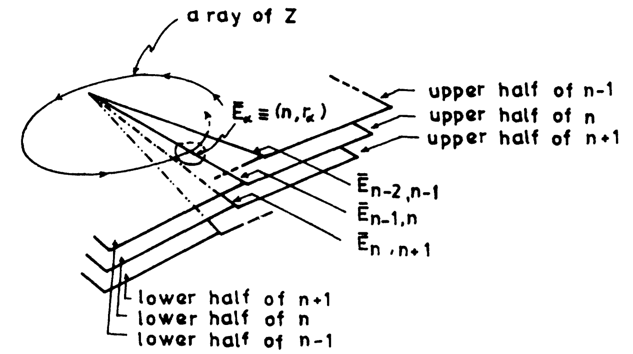

FIG. 2. Drawing of a part of the manifold of the Riemann surface. Analytic continuation is carried along the rays of Z, the complex energy, one of which is shown to meet a pole at $\bar{E}_\alpha$.

$$\mathcal{G}_{00}(\bar{E}_{m,0})=\lim_{s\to 0}[G_{00}^{(1)}(E+is)]\ .$$

Similarly,

$$S_0(\bar{E}_{0,1})=\lim_{s\to 0}[\Sigma_0^{(1)}(E+is)]\ ,$$

$$S_0(\bar{E}_{1,2})=\lim_{s\to 0}[\Sigma_0^{(1)}(E+is)]+\lim_{s\to 0}[2\pi i\mathcal{D}(E)/(G_{00}^{(1)+}+G_{00}^{(1)-})]\ ,$$

$$\ldots,$$

$$S_0(\bar{E}_{m,0})=\lim_{s\to 0}[\Sigma_0^{(1)}(E+is)]\ .$$

Here

$$\mathcal{D}(E)=\lim_{s\to 0}(G_{00}^{-}-G_{00}^{+})/2\pi i$$

denotes the density of states; the superscripts denote the different branches of G_{00} and Σ_0 and the superscript $\pm$ denotes the $\pm$ in the argument of the function.

In order to take into account the contributions made by the poles along $\bar{E}_{n-1,n}$ in the $\mathcal{R}$ we must go from G_{00} to $\mathcal{G}_{00}$. This cannot be done within the framework of G_{00} alone. The poles of $\mathcal{G}_{00}$ are somewhat different in character from those of $\mathcal{G}_{00}$ which occur in the top (or the physical) sheet. Before we go into this analysis we should understand that, in general, the cuts can be arrayed arbitrarily, but the poles cannot be moved; however, in the present case the cut must be kept along the real E axis in order to yield real and positive density of states and also because the poles of physical interest lie along the real E axis.

We will now understand the analytic properties of G_{00}, $\mathcal{G}_{00}$, S_0, and Σ_0. The G_{00} is analytic everywhere in the Z plane except along those portions where it has cuts or poles. The analytic properties of Σ_0 then directly follow from (2) and are listed as follows. (i) It is nonanalytic along a portion of the real E axis where it has a cut *that coincides with the cut in* G_{00}; (ii) at places where G_{00} has poles, Σ_0 is analytic; and (iii) poles of Σ_0 coincide with the zeros of G_{00} and are, therefore, of no interest.

The $\mathcal{G}_{00}$ has already been stated to be analytic throughout $\mathcal{R}$ except where it has poles. The S_0 is also *generally* analytic throughout $\mathcal{R}$ except on the points where it has poles which are of no interest. *But unlike the property (ii) for* Σ_0 *we cannot take for granted the analyticity of* S_0 *at the points where* $\mathcal{G}_{00}$ *has poles.* We shall investigate this in the following. Essentially what we find is that S_0 is continuous across a pole in $\mathcal{R}$ but not *necessarily* differentiable.

Consider a pole in $\mathcal{G}_{00}$ at $\bar{E}_\alpha\equiv(n,r_\alpha)$, i.e., at $r=r_\alpha$ on the line $E_{n-1,n}$. Unlike Σ_0, which becomes real whenever G_{00} has a pole on the real E axis, S_0 stays complex at the pole position of $\mathcal{G}_{00}$ because in $\mathcal{R}$ we are off the real E axis, though infinitesimally. Take a small deviation σ from the point E_α on the side of the $(n-1)$th sheet, so that S_0 develops an additional imaginary part and we can write

$$\begin{aligned}\mathcal{G}_{00}(\bar{E}_\alpha-i\sigma)&=[\bar{E}_\alpha-i\sigma-e_0-S_0(\bar{E}_\alpha-i\sigma)]^{-1}\\&=\{[\bar{E}_\alpha-e_0-\mathcal{S}_0(\bar{E}_\alpha)]\\&\quad-i[\sigma+d_0(\bar{E}_\alpha)]\}^{-1}\ ,\end{aligned}\tag{6}$$

where $S_0(\bar{E}_\alpha-i\sigma)=\mathcal{S}_0(\bar{E}_\alpha)+id_0(\bar{E}_\alpha)$. The d_0, developed due to σ, must approach zero as $\sigma\to 0$ for the pole to appear at $\bar{E}_\alpha=e_0+\mathcal{S}_0(\bar{E}_\alpha)$ (note that $\mathcal{S}_0$ is complex). To study the behavior of $\mathcal{G}_{00}$ and S_0 we will adopt the standard method of considering a small deviation from $\bar{E}_\alpha$ on the side of the nth sheet also and then approaching the pole from both the sides.

It is crucial for our purpose to note that the analytic continuation along a ray of $Z\,(\equiv E\pm is)$ into the nth sheet and beyond shall not be possible through the point $\bar{E}_\alpha$;[7] analytic continuations along other rays that pass through its neighboring points, however close, shall not be forbidden. Up to the nth sheet the process of analytic continuation along the ray of Z works well and the point $\bar{E}_\alpha$ can be reached *from above* starting from the top sheet of $\mathcal{R}$. Since $\mathcal{G}_{00}$ can always be expressed by a convergent power series in any arbitrary vicinity of the point $\bar{E}_\alpha$ inside a semidisc on the $(n-1)$th sheet (hereafter called *upper* semidisc), we can say that $\mathcal{G}_{00}$ diverges in a *continuous* manner inside the *upper* semidisc. The situation is, however, different inside the *lower* semidisc around $\bar{E}_\alpha$ on the nth sheet. Since the process of analytic continuation terminates as soon as E_α is reached from above, the point $\bar{E}_\alpha$ behaves like a normal isolated pole when approached from below. Consequently $\mathcal{G}_{00}$ diverges *discontinuously* in the lower semidisc situated on the nth sheet. This means that in any arbitrarily close vicinity in $\bar{E}_\alpha$, on the side of nth sheet, the $\mathcal{G}_{00}$ remains finite (though very large) and jumps to an infinite value at $\bar{E}=\bar{E}_\alpha$. Thus, $\mathcal{G}_{00}$ behaves *asymmetrically* in the two semidiscs so that its values at $(\bar{E}_\alpha-i\sigma)$ and $(\bar{E}_\alpha+i\sigma)$ shall be different for *any* σ, however small. We should study this "difference" for σ approaching zero.

Denoting the GF in the lower semidisc by $\tilde{\mathcal{G}}_{00}$ and the corresponding self-energy by $\tilde{S}_0$ we can write

$$\mathcal{G}_{00}(\bar{E}_\alpha - i\sigma) - \tilde{\mathcal{G}}_{00}(\bar{E}_\alpha + i\sigma) = \{1 + [S_0(\bar{E}_\alpha - i\sigma) - \tilde{S}_0(\bar{E}_\alpha + i\sigma)]/2i\sigma\}[2i\sigma\mathcal{G}_{00}(\bar{E}_\alpha - i\sigma)\tilde{\mathcal{G}}_{00}(\bar{E}_\alpha + i\sigma)] \ . \tag{7}$$

As $\sigma \to 0$, the left-hand side (lhs) remains nonzero in any vicinity of $\bar{E}_\alpha$, but eventually at $\sigma = 0$, it must yield a *complex zero.* The lhs, when it is nonzero, will be positive because $\mathcal{G}_{00}$ will always *have to be* greater than $\tilde{\mathcal{G}}_{00}$ for a given σ. The reason being that the rate of divergence of $\mathcal{G}_{00}$ should be greater than that of $\tilde{\mathcal{G}}_{00}$ so that it can cover up for the discontinuous manner of divergence of $\tilde{\mathcal{G}}_{00}$. We can, in fact, think in terms of a sequence $\{\eta_i\}$ of very small positive numbers obtained from the lhs of (7) for a series of values of σ approaching zero. In the case of a normal pole this sequence will consist of zeros due to the symmetry on either side of the pole, but in the present situation the sequence will approach zero as $\sigma \to 0$ ($\sigma \to 0 \Longrightarrow i$ increases). The rate at which η_i approaches zero for increasing values of i is important for us and depends upon the argument of $\mathcal{G}_{00}$ and the multiplicity of the pole (e.g., higher multiplicity will lead to a slower decay to zero). In general, owing to the asymmetric behavior of $\mathcal{G}_{00}$ and $\tilde{\mathcal{G}}_{00}$ and the discontinuous divergence of $\tilde{\mathcal{G}}_{00}$, the sequence $\{\eta_i\}$ will approach zero at a rate *slower* than that of $\sigma \to 0$ except when $\mathcal{G}_{00}$ and $\tilde{\mathcal{G}}_{00}$ join each other smoothly at $\bar{E}_\alpha$. In the latter situation the $\mathcal{G}_{00}$ will look similar to a parabola or a higher-order contact curve at $\bar{E}_\alpha$, whereas, in general, the two branches, $\mathcal{G}_{00}$ and $\tilde{\mathcal{G}}_{00}$, join each other in the form of a cusp at $\bar{E}_\alpha$. In other words $\mathcal{G}_{00}$ and $\tilde{\mathcal{G}}_{00}$ may either be *angularly pivoted* or *continuously pivoted at* $\bar{E}_\alpha$. These lead to two physically different situations as shown later. Hereafter, we shall denote the two situations, namely, $\{\eta_i\} \to 0$ *slower* than or *faster* than (including *as fast as*) $\sigma \to 0$, by Sl and Fa, respectively.

Turning our attention to the right-hand side (rhs) of Eq. (7) we see that since the lhs must become zero at $\bar{E} = \bar{E}_\alpha$, the product $\mathcal{G}_{00}\tilde{\mathcal{G}}_{00}$ on the rhs must diverge as $\sigma \to 0$ at a rate *slower* than that of $\sigma \to 0$. At the same time since S_0 must equal $\tilde{S}_0$ when $\mathcal{G}_{00} = \tilde{\mathcal{G}}_{00}$ the difference $(S_0 - \tilde{S}_0)$ will approach zero as $s \to 0$ at the same rate at which $\{\eta_i\} \to 0$ for $\sigma \to 0$. So the first term on the rhs will diverge in the situation Sl but it will remain finite in the situation Fa. In either case, however, the vanishing of the second term on the rhs ensures the vanishing of the rhs when $\sigma = 0$.

The behavior of $(S_0 - \tilde{S}_0)$ for $\sigma \to 0$ as deduced above is of significance to us. We find that the asymmetry of $\mathcal{G}_{00}$ and $\tilde{\mathcal{G}}_{00}$ about the point $\bar{E}_\alpha$ should be reflected in the self-energy S_0. This will make S_0 nonanalytic across $\bar{E}_\alpha$ in the situation Sl—S_0 will be continuous, but not differentiable. In the situation Fa, however, it will stay analytic similar to how the self-energy behaves in the case of a normal pole. The possibility of S_0 becoming nondifferentiable at a pole is peculiar to the present situation under study and is the source of the special behavior exhibited by the pole at $\bar{E}_\alpha$ inside $\mathcal{R}$. With particular reference to localization, the implications of the above on the stay-put probability and the density of states are of importance and should be checked explicitly. Let us take up the stay-put probability first. Its definition (1) gets modified as G_{00} is now replaced by $\mathcal{G}_{00}$. Considering the product $\mathcal{G}_{00}(\bar{E} - i\sigma)\tilde{\mathcal{G}}_{00}(\bar{E} + i\sigma)$ in place of $G_{00}(E + is)G_{00}(E - is)$ we get

$$\lim_{t \to \infty} P_0(t) = \lim_{\sigma \to 0} \frac{[\mathcal{G}_{00}(\bar{E} - i\sigma) - \tilde{\mathcal{G}}_{00}(\bar{E} + i\sigma)]}{1 + [S_0(\bar{E} - i\sigma) - S_0(\bar{E} + i\sigma)]/2\sigma} \ . \tag{8}$$

It gets contributions only from the poles along $\bar{E}$ such as the one at $\bar{E}_\alpha$. The above discussion shows that in the situation Sl the stay-put probability of (8) will vanish, but it stays nonzero in the situation Fa. Thus, the poles in $\mathcal{R}$, in general, yield a vanishing stay-put probability (except in the special situation represented by Fa). We have called such states formed by the coupling of poles and the cut as the confluence states.

Turning attention to the density of states, we note that the numerator in Eq. (8) represents the contribution to it from the poles on E. While for a normal pole on the real E axis a numerator of the type in Eq. (8) would give rise to a δ-function spike, the situation here is modified by the fact that $(\mathcal{G}_{00} - \tilde{\mathcal{G}}_{00})$ becomes a complex zero for $\sigma \to 0$. In the process of going round the branch point at $Z = 0$, i.e., in moving from $\bar{E}_{n-2,n-1}$ to $\bar{E}_{n-1,n}$, the $\mathcal{G}_{00}$ acquires an imaginary part equal to the continuum density of states which is further modified at the points like $\bar{E}_\alpha$ by the contributions that look like *broadened* δ functions and can be calculated from Eq. (6) by splitting $\mathcal{S}_0$ into real and imaginary parts.

IV. PHYSICAL NATURE OF THE "CONFLUENCE STATES"

It is clear from the above that the point $\bar{E}_\alpha$, in general, does not bear the "pure-point" nature, consequently the corresponding state does not retain its intrinsic localizing character. This is because it is coupled to a state in the continuum. Thouless[8] has put forward elegant arguments to show how electrons in the localized states occupying isolated clusters of sites can be strongly coupled to the extended states very close to them in energy and located on infinite clusters of sites. That the localized and the extended states cannot coexist at the same energy without the localized states losing their localizing character has always been understood in this manner. This has been the reason why in the MCFO model a pair of mobility edges is taken to isolate sharply the regimes of localized and extended states. In agreement with this the discussion in the preceding section shows in a rigorous manner that if a localized state (represented by a pole) happens to be degenerate with the extended states (represented by the cut) then the localized state will lose its pure-point nature and such a confluence of localized and extended states will eventually (in the limit $t \to \infty$) evolve into a conducting extended state which is confirmed by the fact that the stay-put probability for such a state will approach zero for $t \to \infty$.

The formation of confluence states in the manner described above indicates that their wave function should have a ramified shape with isolated and very pronounced bumps of amplitude joined to a main-stream envelope function spreading from one end of the system to another. Consequently an electron in a confluence state spends a significant length of time in these isolated regions, but eventually relaxes into the main stream to conduct through the system. This will lead to a significant slowing down in the rate of diffusion. An understanding of the dispersive transport in certain amorphous systems[9] was suggested along these lines by Srivastava and Chaturvedi.[10]

The situation denoted by Fa is rather strange in the light of the above and deserves special mention. It pertains to the circumstance where a localized state is degenerate to an extended state yet, somehow, the two are prohibited from interacting with each other. An example of such a situation can apparently be found in the "quantum-percolation problem,"[11,12] where in a strongly disordered binary system certain configurations are known to give rise to very strongly localized states at special energies throughout the continuum. This is not a localization in the Anderson's sense, it is instead caused by an interesting "hop-scotch" type of exchange mechanism. That they could be attributed to certain poles in $\mathcal{R}$ is being suggested here for the first time and puts them on firmer basis.

More insight is gained into the nature of the confluence states if we calculate the time-integral $\mathcal{J}$ of $P_0(t)$:

$$\mathcal{J} \equiv \int_0^\infty P_0(t)dt \ , \tag{9}$$

which will depend on the rate at which $P_0(t) \to 0$ as $t \to \infty$. For $P_0(t)$ given by Eq. (8) we can write $\mathcal{J}$ as

$$\mathcal{J} = \lim_{\sigma \to 0} \left[\frac{\mathcal{G}_{00}(\bar{E} - i\sigma) - \tilde{\mathcal{G}}_{00}(\bar{E} + i\sigma)}{2\sigma + [S_0(\bar{E} - i\sigma) - \tilde{S}_0(\bar{E} + i\sigma)]} \right] . \tag{10}$$

With the numerator possessing a property of the type (4) and that described after (7), the integral diverges for $\sigma \to 0$ irrespective of the rate at which $(S_0 - \tilde{S}_0)$ approaches zero, i.e., in both the situations referred to as Sl and Fa. Thus, the continuity of S_0 across the pole guarantees the divergence of $\mathcal{J}$ *even if* S_0 is nonanalytic.

The divergence of $\mathcal{J}$ in a situation where $P_0(t)$ vanishes at $t = \infty$ can physically be understood as an electron diffusing very slowly, so that $P_0(t)$ stays nonzero inside the region 0 for very very long times. For the confluence states such a situation can arise in the manner discussed above.

V. RECHARACTERIZATION OF SPECTRUM

The interesting thing that comes out of the above analysis is that the divergence of $\mathcal{J}$, which is necessarily an attribute of the localized states, can also happen, under special circumstances (pole in situation Sl), when the stay-put probability vanishes. $\mathcal{J} = \infty$ together with $P_0(t = \infty) = 0$ indicates that $P_0(t)$ approaches zero *sufficiently* slowly. The divergence (or otherwise) of $\mathcal{J}$ can thus be taken as a measure of rate of $\lim_{t \to \infty} P_0(t) \to 0$ which should be related to the rate of diffusion of an electron away from an origin, and so we can utilize it, in conjunction with $P_0(t)$, to make a finer characterization of the energy spectrum of a disordered system. We suggest the following.

	$\lim_{t \to \infty} P_0(t)$	$\mathcal{J}$		Nature of state
(a)	0	finite	$\Longrightarrow$	extended state
(b)	0	∞	$\Longrightarrow$	confluence state
(c)	>0	∞	$\Longrightarrow$	localized state

The regimes (a) and (c) are separated by the mobility edge in the MCFO model, but the introduction of the new regime (b) in between (a) and (c) modifies the picture and suggests that the mobility edge of the MCFO model should separate the regimes (b) and (c), and that the regimes (a) and (b) should be separated by another critical energy which we may tentatively call a pseudomobility edge.

On a transition over the pseudomobility edge the diffusion gets slowed down significantly making the electron *quasilocalized* which may have important implications in actual experiments done on finite systems over finite lengths of time. Figure 3 shows the possible behavior of the conductivity (at zero temperature) as the Fermi level moves through the mobility edge E_c. The possibility I corresponds to Mott's prediction of the minimum metallic conductivity which is now believed to hold only in the presence of magnetic field.[13] Most of the newer experiments indicate behaviors II and III.[13] The reduction in the conductivity compared to I can be attributed to (a) the quantum interference effects leading to weak localization,[14] and (b) the presence of the confluence states described here. The kink in the behavior II may mark the pseudomobility edge. The behavior III, on the other hand, may indicate that the number and strength of the poles in $\mathcal{R}$ increases steadily as E approaches E_c from the extended-states side.

We may point out that the quantity $\mathcal{J}$ does not merely produce an artifact in the form of the pseudomobility edge that distinguishes between "fast" and "slow" conduction, it has a fundamental physical importance too.

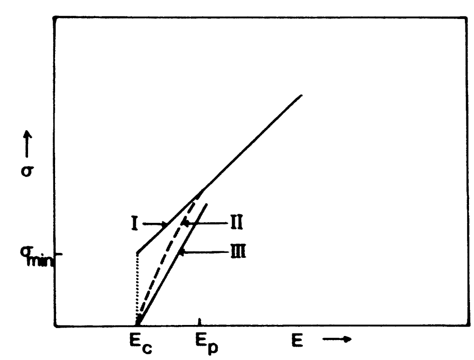

FIG. 3. Schematic depiction of the possible behavior of the conductivity as a function of energy. $\sigma_{\min}$ represents the minimum metallic conductivity. E_c and E_p are the mobility edge and the pseudomobility edge, respectively.

We show in the following that the quantity $\mathcal{J}$ has an important bearing on to the definition of localization in terms of the GF.

VI. DEFINITION OF LOCALIZATION

In Sec. II we essentially showed that

$$\lim_{t\to\infty}[P_0(t)]>0 \Longrightarrow \left\{\begin{array}{l}(\mathrm{a})\ \lim\limits_{s\to 0}[\mathrm{Im}\Sigma_0(E\pm is)]=0\\ (\mathrm{b})\ \lim\limits_{s\to 0}(\mathrm{Im}\Sigma_0/s)=\text{finite}\end{array}\right.$$

$$\Longrightarrow \left\{\begin{array}{l}(\mathrm{a})\ \lim\limits_{s\to 0}[\mathrm{Im}G_0(E\pm is)]=0\\ (\mathrm{b})\ \lim\limits_{s\to 0}\left[\int \mathrm{Im}G_0 dE\right]>0\end{array}\right.$$

In an attempt to reverse the direction of the sign "$\Longrightarrow$" Ishii[15] found that

$$\left.\begin{array}{l}(\mathrm{a})\ \lim\limits_{s\to \infty}[\mathrm{Im}G_0(E\pm is)]=0\\ (\mathrm{b})\ \lim\limits_{s\to 0}\left[\int \mathrm{Im}G_0 dE\right]>0\end{array}\right\} \Longrightarrow \int_0^\infty P_0(t)dt\equiv\mathcal{J}=\infty\ .$$

What we have seen in Sec. III is that $\mathcal{J}$ can *also* diverge when the states are extended, i.e., $\mathrm{Im}G_0(E\pm is)$ stays nonzero for $s\to 0$ *and* $P_0(t)$ goes to zero in the limit $t\to\infty$ sufficiently slower. To synthesize the whole thing, we have

$$\begin{array}{ll}
(\mathrm{i})\ \left.\begin{array}{l}\lim\limits_{s\to 0}[\mathrm{Im}G_0(E\pm is)]=0\\ \lim\limits_{s\to 0}\left[\int \mathrm{Im}G_0 dE\right]>0\end{array}\right\} & \Longleftrightarrow \lim\limits_{t\to\infty}[P_0(t)]>0 \Longrightarrow \text{localization}\\
 & \searrow \qquad \Downarrow\ (\textit{always})\\
 & \int_0^\infty P_0(t)dt\equiv\mathcal{J}=\infty\\
 & \nearrow \qquad \Uparrow\ [\text{if } P_0(t)\to 0\ \textit{slowly}]\\
(\mathrm{ii})\ \left.\begin{array}{l}\lim\limits_{s\to 0}[\mathrm{Im}G_0(E\pm is)]>0\\ \lim\limits_{s\to 0}\left[\int \mathrm{Im}G_0 dE\right]>0\end{array}\right\} & \Longleftrightarrow \lim\limits_{t\to\infty}P_0(t)=0 \Longrightarrow \text{delocalization}\ .
\end{array}$$

There are two main outcomes: (a) the divergence of $\mathcal{J}$ is a property of a pole and (b) that the GF has a pole is implied not only by (i), but *also* by (ii). These conclusions dilute the accepted notion that the poles of the GF correspond to localized states. One also gets the impression that the reality of GF (and that of the self-energy) may only be a *sufficient* condition for localization and *not a necessary* one.

Consequently, it becomes necessary that one must distinguish between the two kinds of poles implied by (i) and (ii), as well as between the spectral regions with and without (any kind of) poles. This is possible only if the intermediate factor $\mathcal{J}$ is included in the definitions of the different regions of the spectrum as is done in the preceding section.

In fact, there exists a subtle difference even in the manner in which the $\mathcal{J}$ diverges for (i) and for (ii). It is simple to check that

$$\lim_{t\to\infty}[P_0(t)]=\lim_{s\to 0}[s\Gamma_0(s)]=\lim_{s\to 0}\left[s\int_0^\infty P_0(t)dt\right]\equiv\lim_{s\to 0}(s\mathcal{J})\ . \tag{11}$$

If the rate of divergence of $\mathcal{J}$ is *slower* than that of $s\to 0$ then $P_0(t)$ will vanish at $t=\infty$ which will correspond to (ii); on the other hand if $\mathcal{J}$ diverges *at least* as fast as $s\to 0$ then $P_0(t)$ will stay nonzero at $t=\infty$ and we will have the case (i).

VII. CONCLUSION

A pole of the GF retains its pure-point nature only if it is isolated from the branch cut of the GF and then it represents a localized state in the Anderson's sense. A pole can also appear into the cut, but then it loses its pure-point nature. In spite of this, such a pole cannot be taken to represent an extended state in the strict sense because it still shares a property with the pole representing a localized state—namely, the divergence of the integral $\mathcal{J}$ of Eq. (9). Such a localized state diluted in its character by the coupling with an extended state can cause the "absence of diffusion" only in a weak sense which should be of importance in actual experimental conditions. In principle, there exists the possibility that a pole inside the cut may retain its pure-point nature, implying thereby that a pole in a cut may not necessarily be coupled to the cut.

It may be worthwhile examining the following questions.

(i) Is the new regime, where the poles and the branch cut of the GF are coexisting in the same energy range, the same as the so-called "singularly continuous" regime of the spectrum?[16]

(ii) What does the extreme situation, where a large number of poles accumulate on a particular branch of E inside $\mathcal{R}$ to form a dense distribution (the "natural boundary") correspond to?

(iii) Is there a relationship between the confluence states and the "power-law states" (see, e.g., Ref. 13)?

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