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Collective excitations of magnetoplasma in truncated metallic superlattices

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We have studied the dispersion characteristics of the bulk and surface magnetoplasmons in a semi-infinite bimetallic superlattice structure. The thicknesses of the constituent layers are assumed to be sufficiently large so that the "quantum-size effects" can be neglected. The superlattice structure is subjected to an applied magnetic field taken to be parallel to the superlattice axis (or perpendicular to the interfaces). The material layers are characterized by the frequency- and magnetic-field-dependent dielectric tensors. The magnetoplasma modes (polaritons) are defined by the electric fields localized at and decaying exponentially away from the interfaces. In spite of the mathematical complexity, we have presented some analytical diagnoses in order to substantiate the asymptotic limits attained by the bulk bands and (some) surface modes, both for zero and nonzero magnetic fields. The numerical results are presented for a number of illustrative cases.

I. INTRODUCTION

Recent advances achieved in vacuum and vapor-deposition techniques, such as molecular-beam epitaxy (MBE) and metalorganic chemical vapor deposition (MOCVD), have made it possible to grow the superlattice structures whose layer thicknesses are in the range of few atomic planes.[1] The presence of these ultrafine layers may affect the motion of electrons and thus lead to the "quantum-size effects" when the physical dimensions of the layers are comparable to the characteristic lengths (e.g., deBroglie wavelength and mean free path) that determine the electron behavior. These microstructures have led to the observation of a variety of deliberately engineered exotic (electronic and optical) properties; some of which, e.g., enhanced electron mobility in modulation-doped semiconductor superlattices, have found application in electron devices. The most important single property of, for example, a semiconductor heterojunction which gives rise to such unusual properties is the band-gap discontinuity of the constituent materials between the conduction and valence bands. While the heterostructure made of III-V compounds are more fully understood from the early studies, the growth and characterization of II-VI and IV-VI materials and, more recently, also of metals and insulators are gaining more and more interest.[2]

The understanding of the heterointerfaces (e.g., interfaces of semiconductors with other semiconductors, metals, and insulators) that control and determine virtually all the properties of solid-state devices has recently stimulated extensive research efforts on the synthetic multilayered structures. The progress in the semiconductor superlattices has inspired investigations on metallic superlattices.[3] The growth and exploration of metallic superlattices is expected to reveal a wide range of modified transport properties resulting from the chemical modulation. For instance, superlattice modification of Fermi-surface topology can be expected. Such expectation has led to several attempts to grow metallic superlattices, culminating into the successful preparation of Nb-Ta superlattices by Durbin *et al.*[4] An ideal metallic superlattice would have sharp interfaces between two constituents, with long-range structural coherence maintained across many layers of the material, as well as in the plane of the layers. The ability to vary the layer thicknesses has made feasible the study of dimensional crossover effects,[5] of thicknesses and proximity effects on superconducting critical temperatures and energy gaps,[6] and of tunneling density of states.[7]

Ever increasing interest in these superlattice systems stems from the existence of entirely novel effects related with the response of the heterointerfaces to the external stimuli. External (electric and magnetic) fields can change the properties of the heterointerfaces significantly, and the consequences of such changes have been systematically explored, particularly in these superlattice structures, in the recent years. Out of all the external probes used to study the response of a system, the magnetostatic field is relatively more interesting. This is because the effect of the magnetic field is more striking and is easily observed in experiments.[8] The important point about the application of the magnetic field is that although the general characteristics of the band structure remain unaltered, the electron energy (in each band) corresponding to the velocity transverse to the magnetic field becomes quantized. It is this quantization which leads to a host of interesting transport phenomena (e.g., de Haas–van Alphen effect, Shubnikov–de Haas effect, cyclotron resonance, etc.).

To the understanding of the electronic and optical properties of superlattices, the knowledge of elementary collective excitations in these systems is of fundamental importance. The collective excitations in (semiconductor) superlattices have been investigated by several research groups. In those studies essentially two different situations have been envisaged: (i) "quantum limit" (when the layer thicknesses are assumed to be much smaller than the electron mean free path and the material layers cannot be characterized by macroscopic dielectric functions), and (ii) "classical limit" (when the layer thicknesses are assumed to be sufficiently large so that the material layers are characterized by macroscopic dielectric functions). The latter situation has been followed by using either a hydrodynamical model or Maxwell's equations with proper electromagnetic boundary conditions.[9-19] The former situation is followed by a self-consistent field with the random-phase approximation.[20-27] In the absence of any magnetic field, the approach in the latter situation has significant advantages owing to its simplicity. This picture simplifies the analysis and also retains the essential feature of the more complicated treatments in the former situation. Also, this model yields analytic results which are found to be readily conclusive.[12]

The present work is concerned with the investigation of the effect of an applied magnetic field (oriented parallel to the superlattice axis) on the collective (bulk and surface) excitations in the classical limit. Since we employ a theory which is not fundamentally different from the one presented in Ref. 19, we will present only a brief outline of the theory. In what follows, we will quote a number of equations directly from Ref. 16 (hereinafter referred to as I) and Ref. 19 (hereinafter referred to as II) and specify them, respectively, as (I.n) and (II.n), where n stands for the equation number in the corresponding papers.

After a brief outline of the theory (Sec. II), we will apply it to a bimetallic superlattice and present the numerical examples, both with and without an applied magnetic field (Sec. III). In Sec. IV we will present some analytic diagnosis in the nonretardation limit to substantiate the asymptotic limits attained by the bulk bands and some surface modes. Finally, Sec. V is devoted to the concluding remarks.

II. THEORY: AN OUTLINE

In I we had derived the dispersion relations for bulk and surface plasmon polaritons for a three-component semiconductor heterostructure. The analytic results in Eqs. (I.22) and (I.30), respectively, for the bulk and surface (collective) excitations were used to obtain the dispersion relations for a two-component heterostructure. Subsequently, in II we investigated the effect of an applied magnetic field on the collective (both bulk and surface) excitations in a two-component semiconductor superlattice. As we have pointed out in I and II, the model theories are applicable to any choice of material parameters. Since we are interested in the present work to study the bimetallic superlattices, the same general formalisms for plasmons (in I) and magnetoplasmons (in II) are applied, just by replacing the material background dielectric constant ϵ_L by unity; the validity of the latter case requires essentially the identical field configuration.

We consider metallic superlattice structure as shown in Fig. 1. The growth direction will be taken as the z axis, which is termed as the superlattice axis. The host materials, hereafter labeled as A and B, are the two different metallic layers. The magnetic field is assumed to be

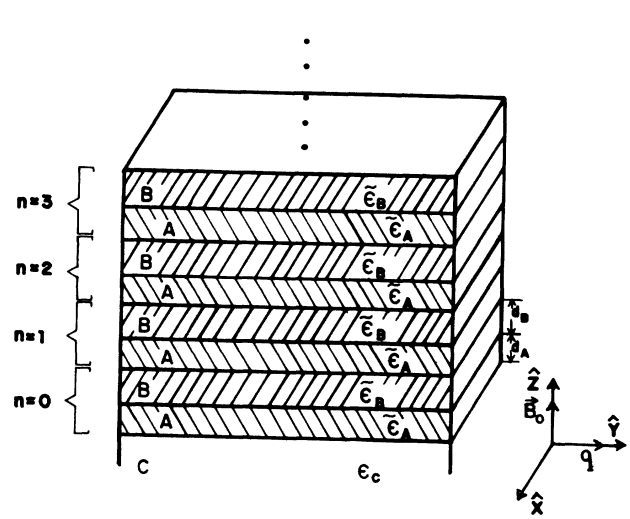

FIG. 1. Schematics of a periodic heterostructure consisting of two different types of metallic slab.

parallel to the superlattice axis and perpendicular to the direction of propagation. Since we are interested in the nonradiative electromagnetic (polariton) modes, the electric fields are taken to be localized at each interface and expressed just as in (II.5) and (II.6) in the respective regions. The retardation is included, but collisional damping, and spatial dispersion are neglected. We use Maxwell's equations and standard electromagnetic boundary conditions to derive the exact dispersion relations for the collective magnetoplasma excitations in the classical limit. Furthermore, since the (superlattice) structure is periodic, its elementary excitations are determined in part by imposing Bloch's theorem.

We shall not attempt to repeat any mathematical expressions, from I and/or II. Therefore, it is noteworthy to state that we are basically concerned with Eqs. (I.23) and (I.31), respectively for the bulk and surface excitations, with $\epsilon_L = 1$ in the case of zero magnetic field. [For the sake of consistency in the formalism for the zero and nonzero magnetic field cases, we would presumably replace the subscript D in the definition of n, in (I.29), by C.] In the case of the nonzero magnetic field we are concerned with (II.21) and (II.34), respectively, for the bulk (in an infinite superlattice) and surface (in a truncated superlattice) excitations.

III. NUMERICAL RESULTS

Here we present some numerical studies of the dispersion relations for the collective excitations in semi-infinite bimetallic superlattice structure. For this purpose, we have taken the background dielectric constant ϵ_L $(=\epsilon_{LA}=\epsilon_{LB})=1.0$. This implies that our numerical results are appropriate for the metallic superlattices. In all the cases, we have performed the calculations in the situation where both material layers A and B are conductors and are characterized by the frequency- and magnetic-field-dependent dielectric functions in the local theory incorporating the retardation effect. We plot our numerical results in terms of the dimensionless frequency $\xi=\omega/\omega_{pA}$, the dimensionless wave vector $\zeta=cq_y/\omega_{pA}$, and the dimensionless layer thicknesses $\delta_i=d_i\omega_{pA}$; $i\equiv A,B$. We introduce a dielectric layer truncating the periodic superlattice system in the region $z\leq 0$ with a dielectric constant $\epsilon_C=1.0$. In order to provide the comparison, we have *also* presented the results in the *absence* of an applied magnetic field in the respective situations.

Since the precedent for this paper comes from II, the interested reader is referred to that work for the details of the classification of the modes propagating in the superlattice system in the presence of a transverse magnetostatic field. As stated above, we are interested in the collective magnetoplasma excitations localized at and decaying exponentially away from the interfaces. As such, the SP (surface-polariton) and the GC (generalized-complex) modes are of particular interest than the bulk and PS (pseudosurface) modes. Remember that this designation of the propagation modes as bulk, SP, GC, etc., requires essentially the identical nature of the decay constants $\alpha_{i\pm}$ in the two material layers (see II). This is, however, not always the situation encountered in the frequency (ω) wave vector (q) space, even though the modes are the bonafide ones. Interestingly, this happens when the decay constants $\alpha_{A\pm}$ are real and $\alpha_{B\pm}$ are complex conjugates of each other (with $\mathrm{Re}\alpha_{B\pm}\gg\mathrm{Im}\alpha_{B\pm}$), or *vice versa*. We designate these modes here as the hybrid surface-polariton–generalized-complex (HSPGC) modes. Their occurrence is thus possible only in a superlattice structure whose unit cell consists of two different conducting layers.

We have carried out the computation with the choice of the following material parameters: ϵ_L $(=\epsilon_{LA}=\epsilon_{LB})=1.0$; $\epsilon_C=1.0$; $\omega_{pB}=2.0\omega_{pA}$; ω_c $(=\omega_{cA}=\omega_{cB})=0.5\omega_{pA},0.9\omega_{pA}$; $\delta_A=1.0$; $\delta_B=0.5,1.0,2.0$. This implies that we are working with a metallic superlattice where the two constituent layers contain different carrier concentrations, but almost the same electron effective mass. We study the dispersion of the magnetoplasma excitations [(II.21) and (II.34)] as a function of the magnitude of magnetic field and the relative thicknesses of the layers.

Although the mathematical complexity prevents us from doing the exact analytical diagnosis consistent with the numerical results, we will later discuss some approximate analysis in the nonretardation limit ($c\to\infty$) to find the consistency with the asymptotic limits attained by the bulk bands and certain surface modes, both in the absence and in the presence of an applied magnetic field ($\mathbf{B}_0$).

A. $\delta_A=1.0$ and $\delta_B=0.5$

In Figs. 2–4 are shown the dispersion curves for $\mathbf{B}_0=0$, $\omega_c=0.5\omega_{pA}$, and $\omega_c=0.9\omega_{pA}$, respectively. In the case of the zero magnetic field, the lowest bona fide mode is the pure SP mode originating from the zero of the ω-q plane, rising toward the right of the light line (in vacuum), and approaching the asymptotic limit at higher

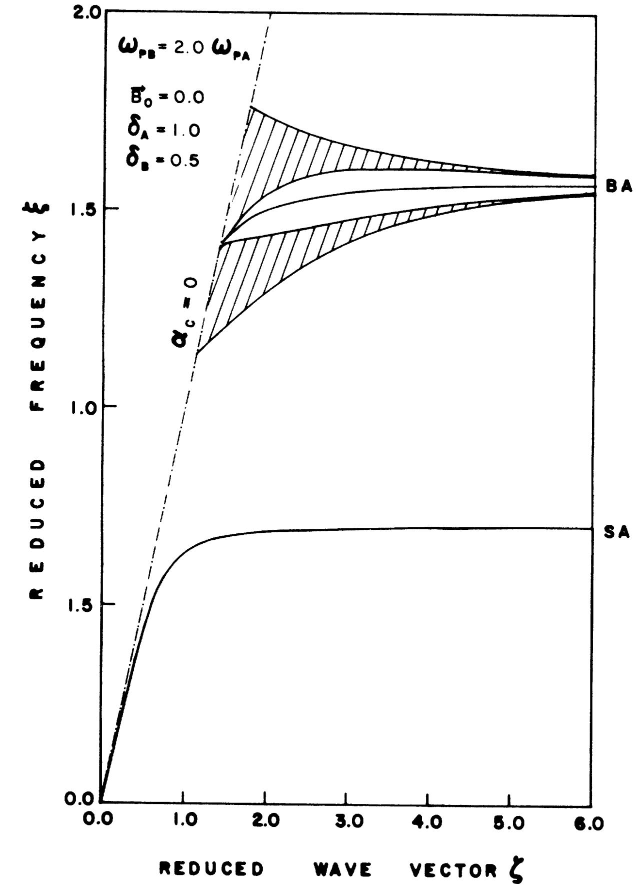

FIG. 2. Dispersion curves for the surface polaritons of the truncated superlattice and the allowed bulk bands (cross-hatched regions) of the infinite superlattice in the absence of a magnetic field, with $\delta_B=0.5\delta_A$. The parameters used are listed in the figure; $\alpha_C=0$ is the light line. The letters BA and SA refer to the bulk and surface asymptotic limits.

wave vectors. There are two bulk bands above ω_{pA}, separated by a gap. In the gap lies an another SP mode which remains inside the gap but does not merge into any of the two bulk bands. This SP mode (in the gap) and the two bulk bands become asymptotic at higher wave vectors. Note that there is no bulk band existing below ω_{pA}, and the inner branches of the two bulk bands seem to be touching each other at a wave vector a little left of the light line. The existence of touching points of the bulk bands has been noted previously by some workers.[17,28]

In the case of the nonzero magnetic field, $\omega_c=0.5\omega_{pA}$ (Fig. 3), there is nothing interesting in the pseudosurface wave region below the cyclotron frequency. The lowest bona fide SP mode starts almost at ω_c, rises toward the right of the light line and reaches an asymptotic limit at $\xi=\omega_{HA}^{*}$ $(=\omega_{HA}/\sqrt{2}$, ω_{HA} being the hybrid plasmon-cyclotron frequency with respect to layer A). Attention is drawn to the parabolic nature of this SP mode at lower wave vectors. This reveals that our general dispersion relation (II.34), at low wave vectors, approximates to the expression for the helicon-like mode in the metallic su-

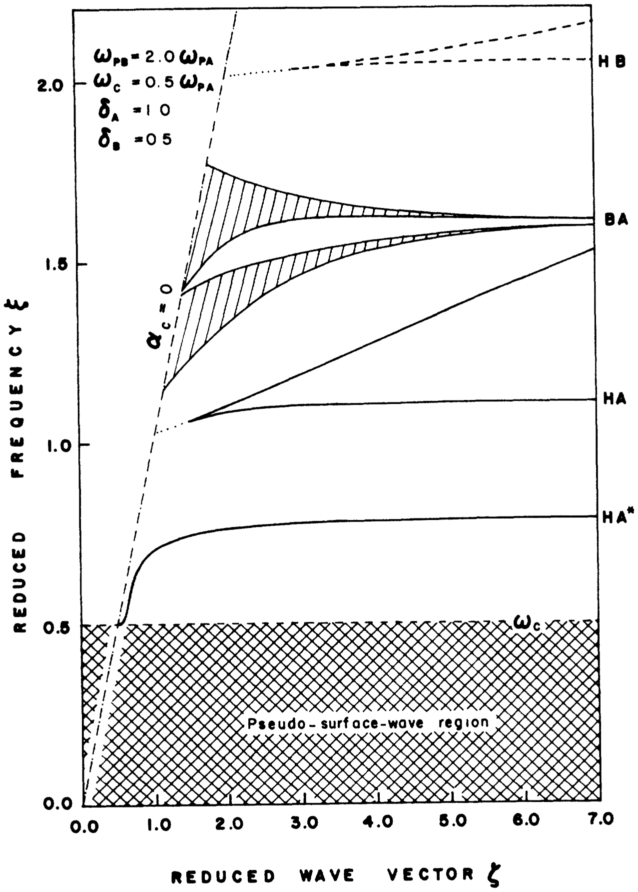

FIG. 3. Dispersion curves for the surface polaritons of the truncated superlattice and the allowed bulk bands (cross-hatched regions) of the infinite superlattice in the presence of a magnetic field, with $\omega_c=0.5\omega_{pA}$ and $\delta_B=0.5\delta_A$. The letters HA^*, HA, and HB refer, respectively, to the asymptotic limits $\omega_{HA}/\sqrt{2}$, ω_{HA}, and ω_{HB}. The letters BA stand for the asymptotic limit approached by the bulk bands.

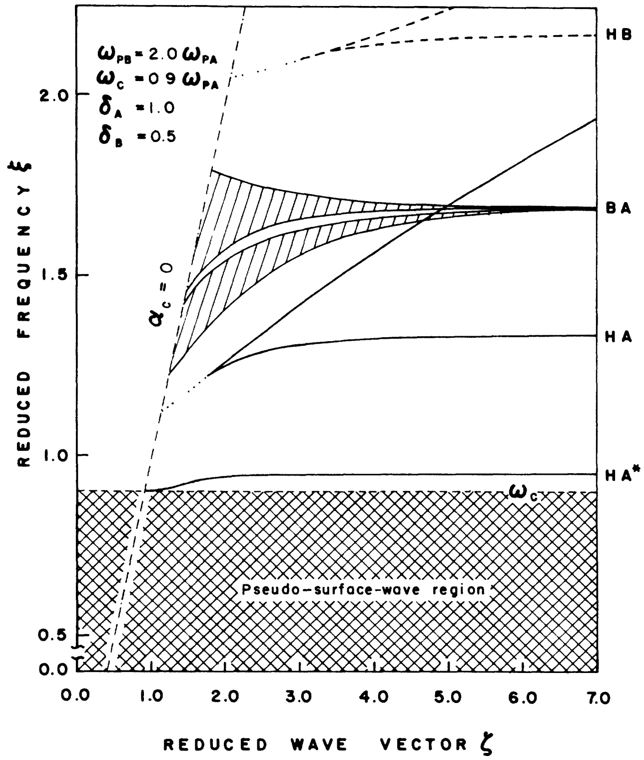

FIG. 4. Dispersion curves for the surface polaritons of the truncated superlattice and the allowed bulk bands (cross-hatched regions) of the infinite superlattice in the presence of a magnetic field, with $\omega_c=0.9\omega_{pA}$ and $\delta_B=0.5\delta_A$.

perlattice. The second surface mode which starts at $\xi\simeq1.033$ is not a bona fide one until it reaches $\zeta\simeq1.52$ where it attains a pure SP character and splits into two branches, shown by a fork in the figure. The lower branch of this fork becomes asymptotic to ω_{HA}. The lower edge of the lower bulk band, the upper edge of the upper bulk band, and their inner branches happen to start at frequencies little higher than their $\mathbf{B}_0=0$ counterparts. There does not occur any SP mode in the gap between the two bulk bands. A third surface mode starts above ω_{pB} at $\xi\simeq2.016$ but becomes a bonafide surface mode only after $\zeta\simeq2.93$ where it splits into two, as shown by a fork. The lower branch of this fork becomes asymptotic to ω_{HB} (the hybrid plasmon-cyclotron frequency with respect to layer B). This fork is characterized by real $\alpha_{B\pm}$ and complex conjugate $\alpha_{A\pm}$ (with $\mathrm{Re}\alpha_{A\pm}\gg\mathrm{Im}\alpha_{A\pm}$), and hence belongs to the HSPGC modes (dashed curves).

The dispersion relations at a higher magnetic field $\omega_c=0.9\omega_{pA}$ are plotted in Fig. 4. The lowest surface mode is a pure SP mode which starts at ω_c, rises upward and becomes asymptotic to ω^*_{HA}. It is observed that increament in the magnetic field results into flattening the parabolic part of the SP mode near ω_c. The second surface mode starts at $\xi\simeq1.12$ from the light line and attains a pure SP character at $\zeta=1.75$ and splits into two branches shown by a fork in the figure. The lower mode in this fork approaches the asymptotic limit defined by $\omega=\omega^*_{HA}$ and the upper branch increases almost linearly and intersects the two bulk bands. We find that, in general, the magnitude of the wave vector corresponding to the intersection points is inversely proportional to the magnetic-field strength (see, for example, Figs. 3 and 4). It is noted that the gap between the two bulk bands decreases with the increasing magnetic field. The asymptotic limits attained by the bulk bands at higher vectors, designated as BA, are in agreement with our analysis (see following section). The third surface mode starts above ω_{pB} at $\xi\simeq2.051$, becomes a bonafide HSPGC mode (shown by a dashed curve) as it reaches $\zeta\simeq2.99$, and splits into two, as shown by a fork. Again the lower mode of this fork approaches asymptotic limit $\omega=\omega_{HB}$. A direct inspection of the results in Figs. 3 and 4 reveals that, at a given wave vector, the energy of the magnetoplasma (bulk and surface) excitations is directly proportional to the intensity of the applied magnetic field.

In general, the effect of an applied magnetic field results in the Zeeman-like splitting of the two surface modes at large wave vectors. The presence of a magnetic field does not permit the propagation of a surface mode in the gap between the two bulk bands. Unlike the semiconductor superlattices, the bulk bands here do not experience any Zeeman-like splitting. (See Sec. IV.)

B. $\delta_A = \delta_B = 1.0$

Here we study the case of equal layer thicknesses. The numerical results are plotted in Figs. 5–7, respectively, for $\mathbf{B}_0 = 0$, $\omega_c = 0.5\omega_{pA}$, and $\omega_c = 0.9\omega_{pA}$. In the case of the zero magnetic field, we find that the gap between the two bulk bands decreases with increasing wave vector. The surface mode propagating in the gap loses its significance after $\zeta \simeq 4.0$ where it merges with the upper edge of the lower bulk band. It is worth mentioning that occurrence of this mode is a consequence of the inclusion of the retardation effect. In the nonretardation limit this SP mode will not exist at all, because then there will be no gap between the bulk bands. The lower SP mode existing below the lower bulk band remains almost intact, but the frequency of the upper SP mode (in the gap) decreases with increasing thickness of the denser medium. It is noted that the asymptotic limits (of bulk bands as well as of SP modes) are independent of the superlattice period.

In the presence of an applied magnetic field (Figs. 6 and 7), the basic difference caused by changing the layer thickness of the denser medium happens to occur in the propagation characteristics of the bulk bands. The touching points regarding the inner edges of the bulk bands near the light line disappear altogether. The variation of the gap width resembles more with the zero field case than with the nonzero one. We point out that apart from the small differences in the frequencies of the surface modes, the rest of the discussion related to Figs. 3 and 4 regarding their behavior in the ω-q space is still valid.

C. $\delta_A = 1.0$ and $\delta_B = 2.0$

In this case we examine the situation where the thickness of the denser medium is greater than that of the rarer one. The results are illustrated in Figs. 8–10, for $\mathbf{B}_0 = 0$, $\omega_c = 0.5\omega_{pA}$, and $\omega_c = 0.9\omega_{pA}$, respectively. In the case of the zero magnetic field, we find that the widths of the bulk bands decrease as compared to Fig. 5 and the gap between them increases. The SP mode existing in the gap starts at a relatively smaller frequency and retains its identity up to very large wave vectors as com-

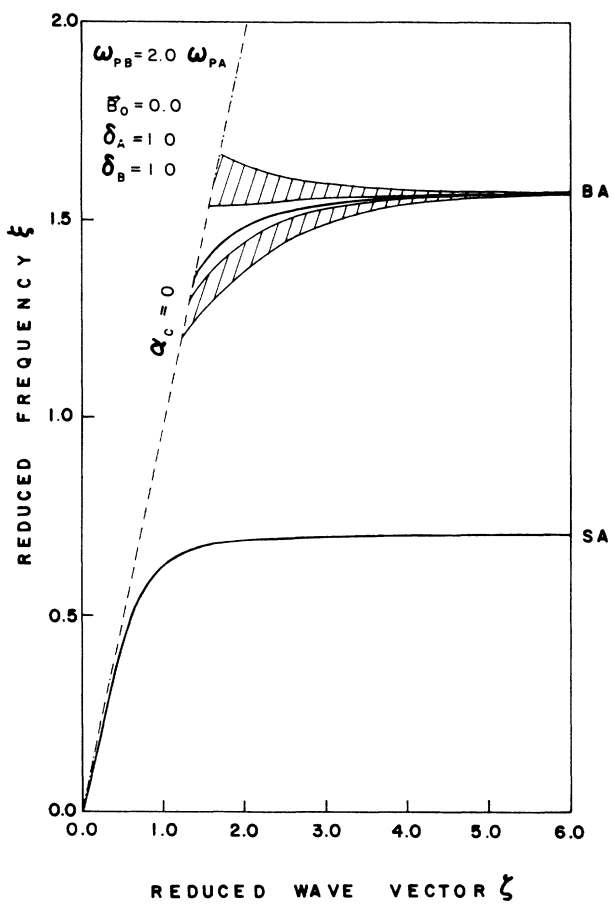

FIG. 5. Dispersion curves for the surface polaritons of the truncated superlattice and the allowed bulk bands (cross-hatched regions) of the infinite superlattice in the absence of a magnetic field, with $\delta_B = \delta_A$.

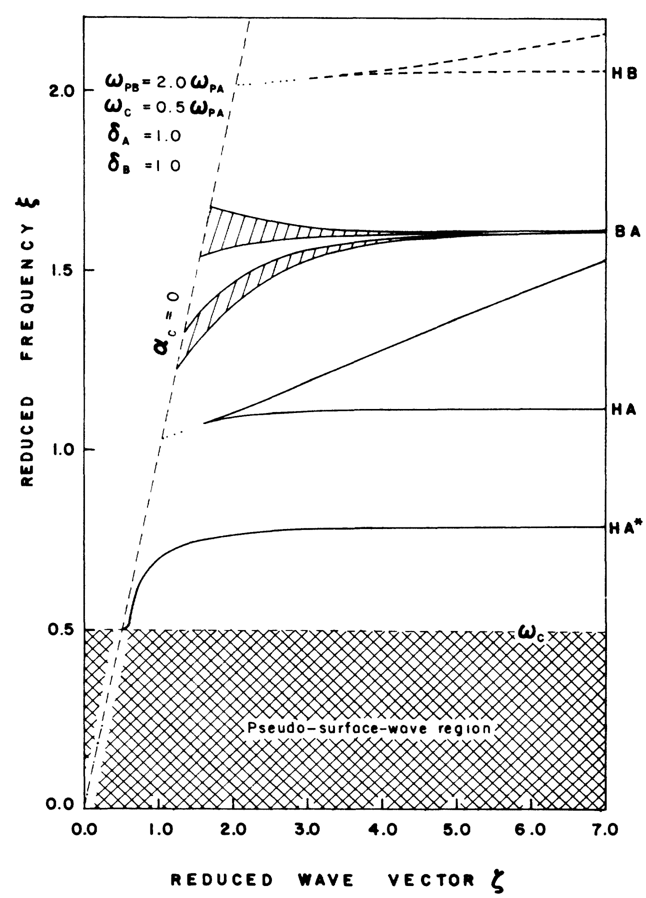

FIG. 6. Dispersion curves for the surface polaritons of the truncated superlattice and the allowed bulk bands (cross-hatched regions) of the infinite superlattice in the presence of a magnetic field, with $\omega_c = 0.5\omega_{pA}$ and $\delta_B = \delta_A$.

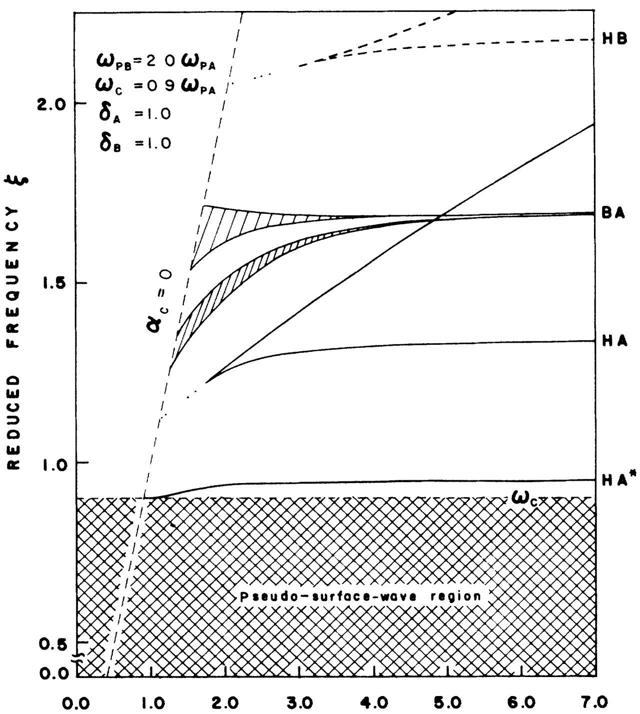

FIG. 7. Dispersion curves for the surface polaritons of the truncated superlattice and the allowed bulk bands (cross-hatched regions) of the infinite superlattice in the presence of a magnetic field, with $\omega_c=0.9\omega_{pA}$ and $\delta_B=\delta_A$.

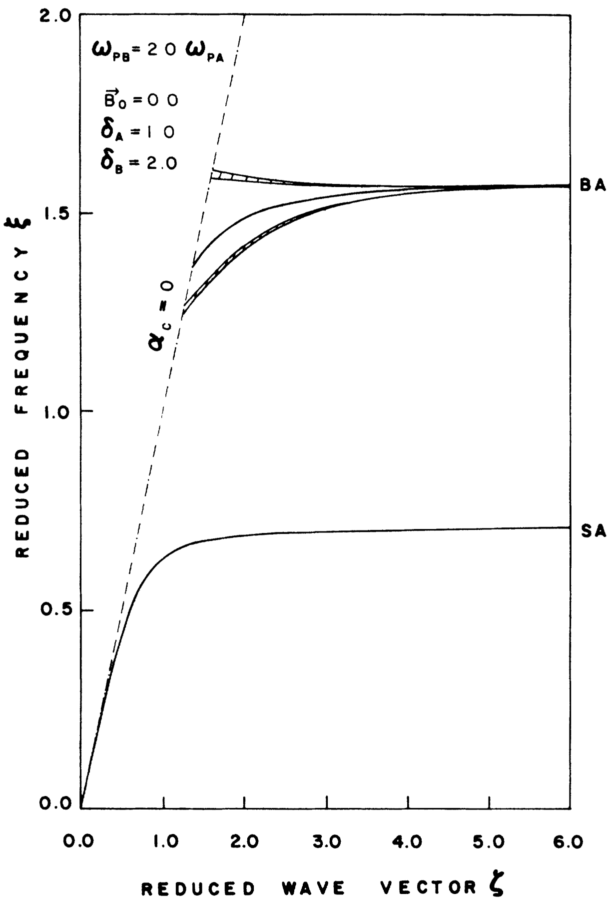

FIG. 8. Dispersion curves for the surface polaritons of the truncated superlattice and the allowed bulk bands (cross-hatched regions) of the infinite superlattice in the absence of a magnetic field, $\delta_B=2.0\delta_A$.

pared to the previous case, depicted in Fig. 5. The lower SP mode does not reveal any difference in its behavior from the corresponding modes in Figs. 2 and 5.

We now turn to examine the situation for nonzero magnetic field. It is observed that at $\omega_c=0.5\omega_{pA}$ (Fig. 9), the bulk bands start and reach the asymptotic limit in almost the same fashion, as in the absence of a magnetic field. A special feature of the upper bulk band in Figs. 2–9, irrespective of the presence or absence of $\mathbf{B}_0$, is the negative group velocity of its upper edge at the lower wave vectors. This trend of the upper edge of the upper bulk band, however, seems to be changed at the higher magnetic field (Fig. 10) where it shows, like other bulk-band edges, a positive group velocity right from the starting point. It is evident that while bulk bands are considerably sensitive to the change in the relative thickness of the layers, the surface modes do not exhibit any appreciable dependence on it. The latter (former), however, experience a strong (weak) effect of the magnetic-field strength. This remark is valid in all the cases we have studied in the present work.

Now it seems worthwhile to recall that we started with the general formalism for the collective (bulk and surface) excitations in truncated superlattices whose constituents were assumed to have a background dielectric constant ϵ_L (see II). In II we have presented exact calculations for a semiconductor superlattice and here for a metallic superlattice. The only difference we took into account in the theory is the value of constant ϵ_L. But we find two important differences in the numerical results in the two cases: (i) the typically small effect of magnetic field on the collective (bulk) excitations, and (ii) an extra SP mode starting at ω_c and approaching an asymptotic limit ω^*_{HA}, in the metallic superlattices. We intend to explain these points in the following sections.

IV. ASYMPTOTIC LIMITS

In this section we analyze the exact dispersion relations, both in the absence and in the presence of an applied magnetic field, in the nonretardation (NR) limit ($c\rightarrow\infty$) in order to understand the asymptotic limits attained by the bulk bands and by certain surface excitations.

A. Zero magnetic field

First we recall (I.23) for the collective (bulk) excitations in the absence of an applied magnetic field. Subjecting (I.23) to the NR limit, we obtain, with $q_y\rightarrow\infty$,

$$\epsilon_A+\epsilon_B=0 , \tag{1}$$

where ϵ_i $[=\epsilon_L(1-\omega_{pi}^2/\omega^2)]$ is the frequency-dependent dielectric function in the local theory for the ith layer. Equation (1) can be rewritten in the form

$$\xi=\frac{1}{\sqrt{2}}\left[1+\frac{\omega_{pB}^2}{\omega_{pA}^2}\right]^{1/2} . \tag{2}$$

Substituting the parameters used in the present calculations leads us to obtain $\xi=1.581\,14$. This is the asymptotic limit attained by the bulk bands in Figs. 2, 5, and 8.

Similarly, Eq. (I.31), which is the dispersion relation for collective (surface) excitations in the absence of an applied magnetic field, in the NR limit, assumes the form

$$(\epsilon_A+\epsilon_B)(\epsilon_A+\epsilon_C)(\epsilon_B-\epsilon_C)=0 , \tag{3}$$

where $\epsilon_C=1.0$ is the dielectric constant of the insulating medium truncating the superlattice system at $z\leq 0$. Equating the first factor to zero results in Eq. (2). This implies that one of the surface modes attains the same asymptotic limit as approached by the bulk bands. This is clearly the SP mode propagating inside the gap (see Figs. 2, 5, and 8).

Equating the second factor to zero gives

$$\xi=\frac{1}{\left[1+\dfrac{1}{\epsilon_L}\right]^{1/2}} . \tag{4}$$

Substituting $\epsilon_L=1$, just as in the present calculations for metallic superlattices, yields $\xi\simeq 0.707\,11$. Note that this is the asymptotic limit attained by the lower SP mode starting from the origin in the ω-q space (in Figs. 2, 5, and 8).

Finally, equating the third factor to zero yields

$$\xi=\frac{1}{\left[1-\dfrac{1}{\epsilon_L}\right]^{1/2}}\frac{\omega_{pB}}{\omega_{pA}} . \tag{5}$$

It is obvious from this equation that this is not an appropriate asymptotic limit to be realized, particularly in the metallic superlattices, since $\epsilon_L=1$ implies that $\xi=\infty$.

B. Nonzero magnetic field

In the presence of an applied magnetic field we start with (II.21), for the collective (bulk) magnetoplasma excitations. Imposing the NR limit, with $q_y\rightarrow\infty$, reduces this to

$$(\epsilon_{xx}^A\epsilon_{zz}^A)^{1/2}+(\epsilon_{xx}^B\epsilon_{zz}^B)^{1/2}=0 . \tag{6}$$

Using the expressions for dielectric tensor elements (see

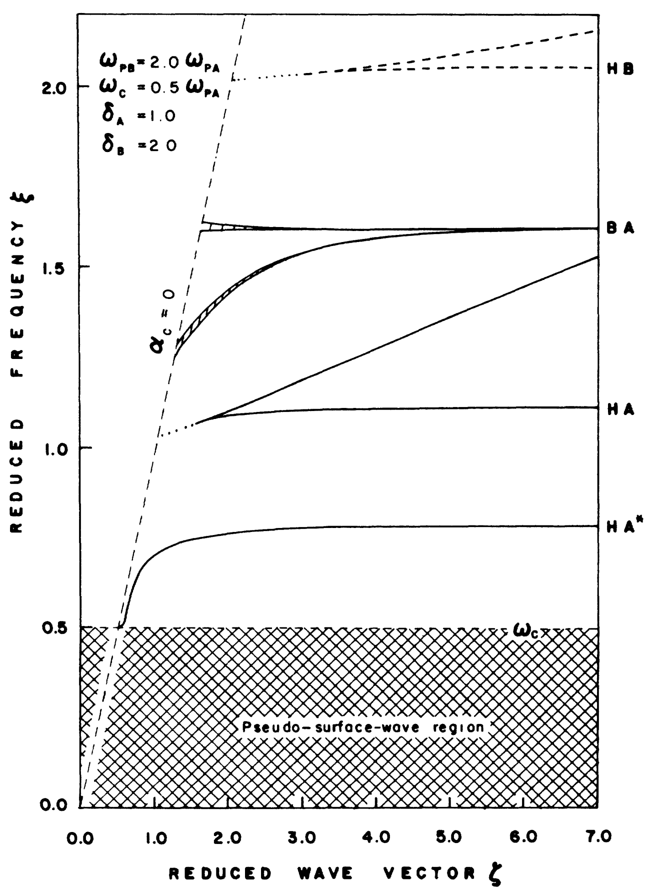

FIG. 9. Dispersion curves for the surface polaritons of the truncated superlattice and the allowed bulk bands (cross-hatched regions) of the infinite superlattice in the presence of a magnetic field, with $\omega_c=0.5\omega_{pA}$ and $\delta_B=2.0\delta_A$.

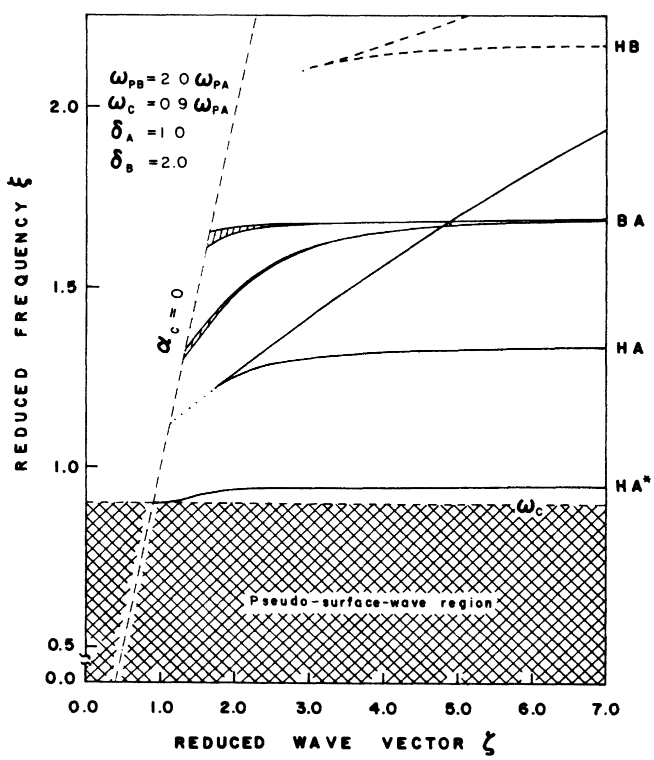

FIG. 10. Dispersion curves for the surface polaritons of the truncated superlattice and the allowed bulk bands (cross-hatched regions) of the infinite superlattice in the presence of a magnetic field, with $\omega_c=0.9\omega_{pA}$ and $\delta_B=2.0\delta_A$.

Sec. II.A in Ref. 19) in Eq. (6) yields, after a few algebraic steps,

$$\xi=\frac{1}{\sqrt{2}}\left[1+\frac{\omega_{pB}^2}{\omega_{pA}^2}+\frac{\omega_c^2}{\omega_{pA}^2}\right]^{1/2} . \tag{7}$$

Note that in the special case $\mathbf{B}_0=0$, Eq. (7) reduces to Eq. (2). Substituting the material parameters as used in the present computation results in

$$\xi\simeq\begin{cases}1.620\,18 \text{ for } \omega_c/\omega_{pA}=0.5 ,\\ 1.704\,41 \text{ for } \omega_c/\omega_{pA}=0.9 .\end{cases} \tag{8}$$

We see, from the figures depicting the dispersion curves for the nonzero magnetic field, that these are the asymptotic limits approached by the bulk bands at large wave vectors for the respective values of the magnetic field.

The most difficult task is the analytical diagnosis of the dispersion relation (II.34) for the collective (surface) magnetoplasma excitations. In order to understand the asymptotic limits attained by certain surface excitations in the presence of an applied magnetic, we prefer to start with (II.26)–(II.31) instead of with (II.34). The reason is just not to ignore the simplicity in handling the situation. Equations (II.26)–(II.31) in the NR limit can be written as

$$y_1E_{1A}+y_2E_{2A}+y_4E_{4A}=0 , \tag{9a}$$

$$z_1E_{1A}+z_2E_{2A}=0 , \tag{9b}$$

$$u_1E_{1A}+u_2E_{2A}=0 , \tag{10a}$$

$$v_1E_{1A}+v_2E_{2A}=0 , \tag{10b}$$

$$w_1E_{1A}+w_2E_{2A}+w_3E_{3A}=0 , \tag{10c}$$

$$x_1E_{1A}+x_2E_{2A}+x_4E_{4A}=0 , \tag{10d}$$

where $u_1,\ldots,u_4$, $v_1,\ldots,v_4$, $w_1,\ldots,w_4$, and $x_1,\ldots,x_4$ refer to the coefficients of the respective electric field components in Eqs. (II.28)–(II.31). In writing Eqs. (9) and (10), we have used the fact that

$$y_3=z_3=z_4=u_3=u_4=v_3=v_4=w_4=x_3=0$$

in the NR limit. Moreover, it is not difficult to see that $z_1=-z_2$ and $Q_2=0$ in the NR limit (see II). Clearly then, we obtain from Eq. (9b) that

$$z_1(E_{1A}-E_{2A})=0 . \tag{11}$$

This is equivalent to

$$(\epsilon_{xx}^A-\epsilon_{zz}^A)(\epsilon_{xx}^A)^{1/2}(E_{1A}-E_{2A})=0 . \tag{12}$$

Here either the first, second, or third factor is zero. The possibility of the vanishing of the first factor is ruled out, because $\mathbf{B}_0\neq 0$. The second factor equated to zero yields

$$\omega=\omega_{HA} . \tag{13}$$

Now, equating the third factor in Eq. (12) to zero leaves us with

$$E_{1A}=E_{2A} . \tag{14}$$

Consequently, one obtains from Eqs. (10a) and (10b) that

$$(u_1+u_2)=0 \tag{15}$$

and

$$(v_1+v_2)=0 . \tag{16}$$

Equation (15), after some algebraic steps, reduces to

$$P_1+Q_1\tanh(A_+)-P_1e^{-\lambda}e^{-B_+}\operatorname{sech}(A_+)=0 , \tag{17}$$

where A_+ (in the NR limit) $=(\epsilon_{xx}^A/\epsilon_{zz}^A)^{1/2}q_yd_A$. The rest of the quantities in Eq. (17) are as defined in II. Similarly, Eq. (16) reduces to the form

$$P_1-Q_1\tanh(A_+)-P_1e^{-\lambda}e^{B_+}\operatorname{sech}(A_+)=0 . \tag{18}$$

In the case that $q_y\to\infty$, Eqs. (17) and (18), respectively, assume the form

$$P_1+Q_1=0 \tag{19}$$

and

$$P_1-Q_1=0 . \tag{20}$$

These two results are compatible only if (see II)

$$\begin{aligned}P_1&=A_1B_3\alpha_{B_+}\\&=(\epsilon_{xx}^A-\epsilon_{zz}^A)(\epsilon_{xx}^B-\epsilon_{zz}^B)(\epsilon_{xx}^B)^{1/2}=0\end{aligned} \tag{21}$$

and

$$\begin{aligned}Q_1&=A_3B_1\alpha_{A_+}\\&=(\epsilon_{xx}^B-\epsilon_{zz}^B)(\epsilon_{xx}^A-\epsilon_{zz}^A)(\epsilon_{xx}^A)^{1/2}=0 .\end{aligned} \tag{22}$$

We know that the vanishing of the first two factors in these equations is ruled out for well-known reasons. Therefore, the result is

$$\omega=\omega_{HB}$$

from Eq. (21) and

$$\omega=\omega_{HA}$$

from Eq. (22). Thus, we have established that ω_{HA} and ω_{HB} are the asymptotic limits attained by the lower modes of the two "forks" appearing in the dispersion curves of the collective magnetoplasma excitations in the metallic superlattices.

The lowest SP mode in the present situation ($\omega_{pB}>\omega_{pA}$) happens to approach an asymptotic limit at[29]

$$\xi=\frac{1}{\left[1+\dfrac{1}{\epsilon_L}\right]^{1/2}}\left[1+\frac{\omega_c^2}{\omega_{pA}^2}\right]^{1/2} . \tag{23}$$

In the $\mathbf{B}_0=0$ case, this reduces to Eq. (4). In the metallic superlattices ($\epsilon_L=1$), Eq. (23) becomes

$$\omega=\omega_{HA}^*=\frac{1}{\sqrt{2}}\omega_{HA} . \tag{24}$$

In the semiconductor superlattices where ϵ_L can be approximated to much greater than unity (as, e.g., in II), Eq. (23) is simply

$$\omega \simeq \omega_{HA} . \tag{25}$$

That is to say that the lowest SP mode in the same situation (i.e., $\omega_{pB} > \omega_{pA}$) in the semiconductor superlattices is virtually a doubly degenerate surface mode. This is an explanation to the point (ii) raised in Sec. III C.

C. General comments

It should be pointed out that when $\omega_{pB} > \omega_{pA}$, the lowest SP mode propagates in the window $\omega_c < \omega < \omega_{pA}$ and approaches an asymptotic limit ω_{HA}^{*}. In the case that $\omega_{pB} < \omega_{pA}$, the situation is rather different. The aforesaid lowest mode (when $\omega_{pB} > \omega_{pA}$) will then (when $\omega_{pB} < \omega_{pA}$) appear above the upper bulk band and never approach an asymptotic limit. This leads us to believe that the location of the bonafide surface mode is not independent of the relative carrier concentration of the constituent layers.

It also seems worthwhile to discuss the location of the surface excitations in the semiconductor and metallic superlattices in the *absence* of an applied magnetic field. This is now a well-established fact that if $\epsilon_L/\epsilon_C > 1$ (<1), the second SP mode, apart from the one propagating in the gap between two bulk bands, will appear above (below) the upper (lower) bulk band. In this connection it is noteworthy that the above-mentioned statement is, by all means, justified if and only if $\omega_{pB} < \omega_{pA}$. The situation takes a different turn if $\omega_{pB} > \omega_{pA}$. In the latter case the second SP mode appears below the lower bulk band, even though $\epsilon_L/\epsilon_C > 1$. This is the criterion for the existence of the SP mode outside the bulk bands in the semiconductor superlattices. This mode in the metallic superlattices always appears below the lower bulk band—irrespective of whether $\omega_{pB} < \omega_{pA}$ or $\omega_{pB} > \omega_{pA}$. This again emboldens our confidence in the statement we made in the last paragraph.

As mentioned in the previous section, the numerical results in the metallic superlattices exhibit a considerable difference from their counterparts in the semiconductor superlattices, even though we use exactly the same theory; apart from the value assigned to the background dielectric constant. This is not surprising for the following basic reasons. Suppose in a semiconductor, in the presence of an applied magnetic field, we come across the situation when

$$\epsilon_{xx} = 1 \tag{26}$$

or, more explicitly,

$$\omega^2 = \omega_c^2 + \frac{1}{1-\dfrac{1}{\epsilon_L}}\omega_p^2 . \tag{27}$$

The result is quite reasonable because $\epsilon_L > 1$. In the metals, with $\epsilon_L = 1$, one obtains an absurd answer, as $\omega \rightarrow \infty$.

Similarly, in the case that the relation

$$\epsilon_{zz} = 1 \Longrightarrow \omega = \frac{1}{\left[1-\dfrac{1}{\epsilon_L}\right]^{1/2}}\omega_p \tag{28}$$

is satisfied in the ω-q plane in the semiconducting material, one is always safe to find an appropriate frequency. But, if the relation is applied to the metals, one will face the same situation as predicted from Eq. (5).

Finally, the most fundamental aspect regarding the response of the charge carriers in the metallic and semiconducting superlattices in the magnetic fields is their effective masses. The effective mass of the conduction electrons in metals is relatively large, i.e., almost on the order of the free-electron mass m_0. In semiconductors, on the other hand, the effective mass of the conduction electrons is quite small, on the order of $0.02m_0$ (in InSb). This is the reason that the effect of the moderate magnetic fields in semiconductors is larger than that in metals.

V. CONCLUDING REMARKS

In conclusion, we have provided a detailed investigation of the collective (bulk and surface) excitations in the metallic superlattices subjected to a transverse magnetic field. We have used a general theory based on Maxwell's equations with proper electromagnetic boundary conditions in the local approximation. Allowing sufficiently large thicknesses of the layers justifies the use of macroscopic dielectric functions and the neglect of the quantum-size effects. The analysis (in Sec. IV) clarifies various aspects pertaining to the difference in the numerical results obtained in the semiconducting and metallic superlattices. The approximate diagnosis in the NR limit has proved to be very useful in understanding the asymptotic limits attained by the bulk bands and certain surface excitations, both in the presence and in the absence of an applied magnetic field.

The collisional damping and the spatial dispersion have been neglected, but can be incorporated by allowing the imaginary parts in the dielectric tensor elements and by using the hydrodynamical model, respectively. It is worthwhile to redo the numerical computation by treating the frequency or wave vector as the complex variables. This will help to investigate the lifetime and the propagation length of the collective excitations. One can also calculate the inverse penetration depth (λ) from Eqs. (II.28)–(II.31), in order to be familiar with the limitation of some experimental techniques, e.g., low-energy electron spectroscopy which may not be useful if λ is very small. Inelastic light (or Raman) scattering[30] can be used to observe the effect of an applied magnetic field on the collective surface excitations.

To date, no metallic multilayer or superlattice system has been found which has the same degree of structural and chemical perfection as their semiconductor counterpart. The reason is the following. In semiconductors the directional nature of the covalent bond aids in the formation of high-quality crystals. In metals, on the other hand, the bonding is much less directional, making the incorporation of defects easier. At the same time, while highly perfect structures are required for semiconductors to exhibit useful electronic properties, metals can tolerate greater levels of structural and chemical imperfections and still exhibit phenomena of interest. The study of electronic and optical properties of metallic superlattices is not yet as developed as the study of the physical properties of semiconductor superlattices. Much progress has been made in just the past few years, but metallic systems do not yet exhibit the same degree of long-range structural coherence across the layers.

In view of the aforesaid, it is worthwhile to shed some light on the physical significance of the present results and their implications. We emphasize on the magnetoplasma polaritons of semi-infinite superlattices, i.e., the superlattices which exhibit a free surface. In these systems, the accumulation of interfaces gives rise to peculiar electromagnetic (EM) eigenmodes, distributed as continuous frequency bands. The truncation of the superlattice at the surface modifies the density of these modes, as compared to the mode density of an infinite, truly periodic, superlattice. In particular, one notices the appearance of isolated branches analogous to the surface polaritons of semi-infinite homogeneous materials. The present results will be helpful in the investigation of the local (or, total) density of these polaritons which provides the complete information on allowed EM excitations, as a function of frequency and/or wave vector, at any depth in the superlattice.[31] Consequently, we can discuss the infrared optical experiments performed on thick layered superlattices. One of the exotic properties of the metallic superlattices is the correlation of the structural and electronic transport properties.[32] Since the elementary collective excitations are fundamental to the knowledge of electronic properties, the present results can help us give a deeper insight of the effect of an applied magnetic field on such correlation. Finally, the collective surface excitations are currently of greater interest because these are potentially useful modes for the surface wave devices.

The superlattice activities have been extended recently from semiconductors to pure metals and various other combinations with different materials, e.g., semiconductor-metal, semiconductor-semimetal, semiconductor-insulator, and metal-insulator superlattice structures. The motivation behind the growth of such structures is the expectation of getting detailed information on the various interface properties which influence the band offsets in these systems. Out of the aforesaid combinations of different materials, the realization of semiconductor-metal structures is the most tedious task, if not impossible. This is mainly because the metal has to be deposited at or near room temperature in order to avoid reaction with the underlying material, whereas the semiconductor has to be deposited at elevated temperatures for well-ordered epitaxial growth. A realistic compromise between the two widely differing temperature requirements is, at present, not possible. This might, nevertheless, also be a promising area for future research. However, realizing the importance of an application of magnetostatic fields,[33] the semiconductor-insulator and metal-insulator superlattice structures, wherein no scientific efforts are restricted due to, for example, interface reactivity, are worth investigating in the framework of the model theory developed in II.[34]

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