Nonclassical scattering dynamics in the quantum Hall regime

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We consider a large class of two-dimensional systems of electrons in a static disorder potential and subject to an in-plane electric field and to a strong perpendicular magnetic field. The time evolution of the single-particle states is investigated and found to be crucial for quantum Hall behavior. The macroscopic Hall current is carried by the nonadiabatic states. Quantum Hall behavior results when the Fermi energy lies in a range of adiabatic levels. Linear-response theory is inadequate to describe the quantum-mechanical single-particle scattering process in the quantum Hall regime. The general results are illustrated by an explicit weak-disorder model, where the scattering process and the nature of dc-insulating and -conducting states can be understood in detail. We stress analogies with one-dimensional conductance in the presence of disorder.

I. INTRODUCTION

The quantum Hall effect[1] describes the transport of electrons in two dimensions in the presence of a Hall electric field and of a strong perpendicular magnetic field. It is generally accepted that the integer quantum Hall effect (IQHE) results from a localization-delocalization process caused by a static disorder potential. The details of this process are not sufficiently known, since the corresponding quantum-mechanical scattering mechanism is not in general understood.[2,3] Most theories of the IQHE apply to a situation where there are no conducting states at the Fermi energy, and are based on arguments which do not require a detailed knowledge of the scattering process. Such a knowledge is, however, necessary for a complete understanding of the physics in the quantized plateau region, and it is indispensable for understanding the properties of the system outside this region and for other effects like breakdown, frequency, and temperature behavior.

Potential scattering in crossed electric and magnetic fields has recently been investigated in the presence of a square barrier and of a horizontal wall, and it was found that an extremely complex classical dynamics results.[3] Preliminary results indicate complex quantum dynamics as well.[3] A quantum-mechanical investigation[4] of scattering across a smooth barrier in the presence of disorder has shown that rather unusual nonclassical particle dynamics may result, which reflects previously[5] derived general properties of Schrödinger functions of particles subject to an electric field in the presence of disorder. The importance of this nonclassical behavior for the IQHE has been investigated in an explicit model system,[6] and it was found that the nature of the time evolution of the scattered wave functions is crucial for the IQHE in the system.

In the present paper we investigate the time evolution of the scattering process from a more general point of view. We will consider a large class of two-dimensional systems, and we will clarify the physical conditions and the properties of the time-dependent scattering process that are essential for the occurrence of the IQHE in these systems. Simultaneously we further investigate our previously[6] introduced explicit model system, where the nature of insulating and conducting states and the time dependence of the scattering mechanism can be understood in detail. This explicit model will serve as an illustration of our general developments. The time evolution of the electron states will show some remarkable nonclassical features which do not seem to have been sufficiently noticed before. We will point out that there exists an analogy with ordinary conduction at sufficiently low electric fields.

II. DEFINITION OF THE MODEL

We consider independent electrons (charge $q<0$, mass m), on a long strip (of width L_y) in the x direction, subject to a potential $V(x,y)$, a magnetic field $\mathbf{B}=(0,0,B)$, and an electric field $\mathbf{E}=(0,E_y,0)$. The single-particle Hamiltonian can be written in the form

$$H=\frac{1}{2m}\left\{\left[\frac{\hbar}{i}\frac{\partial}{\partial x}\right]^2+\left[\frac{\hbar}{i}\frac{\partial}{\partial y}-\frac{q}{c}\left[Bx+\frac{\phi(t)}{L_y}\right]\right]^2\right\}+V(x,y)\ , \tag{1}$$

with

$$\phi(t)=-cE_yL_yt \tag{2}$$

and the boundary conditions

$$\psi(x,y,t)=\psi(x,y+L_y,t)\ . \tag{3}$$

Such a model is justified by the results of microwave experiments,[7] which show that effects due to contacts and edge states are not essential for the occurrence of the IQHE. This is analogous to conduction based on Bloch theory, where also only bulk properties are investigated.

We consider potentials $V(x,y)$ which do not describe a macroscopic electric field, i.e., the integral of $V(x,y)$ over

x and over y are zero. If $V(x,y)$ has no symmetry, e.g., in the presence of static disorder, it follows[4,5] from the structure of the Hamiltonian that adiabatic states [for which E_y is sufficiently small such that $\phi(t)$ in (1) can be considered as a parameter] have currents which are periodic in time with zero average and with period

$$\tau=|h/(qE_yL_y)| \ . \tag{4}$$

For realistic values of E_yL_y, the period τ is extremely small (e.g., $\tau=4\times10^{-12}$ s for $E_yL_y=1$ mV). Hence adiabatic states are dc insulating. As a consequence any dc current must result from nonadiabatic states (see Ref. 6 for a more detailed discussion of this point).

In view of our explicit model system we will now consider a potential of the type

$$V(x,y)=V(x)+V^1(x,y) \ , \tag{5}$$

where $V^1(x,y)$ represents homogeneous disorder, and $V(x)$ is a sequence of smooth barriers and wells, whose first and second spatial derivatives (piecewise) vary slowly over a magnetic length $\lambda=(\hbar c/qB)^{1/2}$ [see, e.g., $V(x)$ of Fig. 1].

In the absence of $V^1(x,y)$, the adiabatic solutions of the time-dependent Schrödinger equation in the presence of such a potential $V(x)$ have the approximate form[4,6] (n is the Landau band index)

$$\psi_{p,n}(x,y,t)=(L_y)^{-1/2}\exp(i2\pi py/L_y)u_{p,n}(x,t) \ , \quad p \text{ integer} \tag{6}$$

where $u_{p,n}(x,t)$ is the product of a Hermite polynomial and a Gaussian g_p,

$$g_p(x,t)=\exp\{-[x-x_p(t)]^2/(2\lambda^2)\} \tag{7}$$

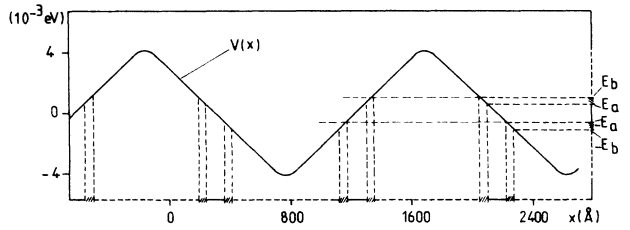

FIG. 1. Smooth substrate potential $V(x)$ and the nature of the orbitals in the case where, in addition to $V(x)$, an electric field E_y and a magnetic field B are present. The orbitals are characterized by the position of the localization centers x_p of the corresponding unperturbed ($V^1=0$) orbitals $\psi_p(x-x_p)$ (see text). Solid-line regions, fully nonadiabatic (classically conducting) orbitals; shaded regions, intermediate nonadiabatic orbitals (they are composed of a classically and a nonclassically conducting part); dashed-line region, adiabatic (nonconducting) orbitals. The parameter values are $d_{pp'}=0.001$ eV, $L_y=0.1$ cm, $B=6$ T, and $E_y=2.37\times10^{-7}$ V cm^{-1}. $V(x)$ also represents the energy E_p of the unperturbed orbitals ψ_p; see (9). On the scale of the figure, unperturbed and perturbed energies coincide for orbitals situated outside the dashed region.

with

$$x_p(t)=chp/(qBL_y)-\phi(t)/(BL_y)-(mc^2/q^2B^2)V'(x_p) \tag{8}$$

and energies

$$E_{p,n}=\hbar\omega(n+\tfrac{1}{2})+V(x_p)+(m/2)\left[\frac{cV'(x_p)}{qB}\right]^2 \tag{9}$$

[the prime denotes d/dx and $\omega=qB/(mc)$]. In the following we consider a single band and the index n will be dropped. The unperturbed ($V^1=0$) adiabatic functions $\psi_p(x,y,t)$ are localized in the x direction at sites $x_p(t)$, which move with the constant, classical velocity $v=cE_y/B$. The adiabatic variation of $E_p[\phi(t)]$ is essentially given by the term $V[x_p(t)]$. Therefore the energy curves $E_p[\phi(t)]$, $E_{p'}[\phi(t)]$ *intersect* (Fig. 2) if $x_p(t)$ and $x_{p'}(t)$ are situated on different sides of a barrier or of a well (Fig. 1). Note that the *set* of eigenvalues of $H[\phi(t)]$ is periodic in t with period τ (corresponding to the ϕ period hc/q), but the individual energies $E_p[\phi(t)]$ are not periodic.

In the presence of the disorder potential $V^1(x,y)$ the previously intersecting energy levels anticross and become individually periodic with period τ. We now consider those adiabatic states $w_s(x,y,t)$, which can be described by a weak-disorder approximation. This is the case if (see Ref. 4)

$$|V^1_{pp'}|\equiv\langle\psi_p|V^1|\psi_{p'}\rangle = f_{pp'}|E_p(t^*+\tau/2)-E_{p'}(t^*+\tau/2)|$$

with

$$f_{pp'}\lesssim0.2 \ . \tag{10}$$

Here t^* is the time, where two unperturbed energies $E_p(t)$ and $E_{p'}(t)$ intersect (see Fig. 2). In the weak-disorder approximation the corresponding perturbed adiabatic states $w_s(x,y,t)$ are linear combinations $c_p(t)\psi_p(x,y,t)+c_{p'}(t)\psi_{p'}(x,y,t)$ of only two unperturbed states at a given value of t, but the pair of indices (p,p') changes periodically after each time interval $\tau/2$. For example, in Fig. 2 the indices of $w_s(t)$ are (p,p') for $t\in[t^*-\tau/4, t^*+\tau/4]$ and $(p',p+1)$ for $t\in[t^*+\tau/4, t^*+3\tau/4]$, and so on. The properties of these weakly perturbed adiabatic states are discussed in Refs. 4 and 6 and can be summarized as follows: Each state $w_s(x,y,t)$ describes a wave packet moving with the velocity v, but which is alternately localized in a small neighborhood of one of the two fixed sites $x_p(t^*)$ and $x_{p'}(t^*)$, which are situated on different sides of a barrier (or of a well) and whose energies $E_p(t)$ and $E_{p'}(t)$ intersect at $t=t^*$. This corresponds to a discontinuous motion of a mass point between the two spatially distinct sites. The whole motion is periodic in time with period τ.

In Ref. 6 the explicit potential $V(x)$ shown in Fig. 1 was used. (The choice of equal shapes of subsequent barriers and wells is made only for simplifying the calculations.) Here the weakly perturbed states [i.e., for which $f_{pp'}\lesssim0.2$ in Eq. (10)] are situated in the center of each

Landau band. This follows from the fact that due to the Gaussian factors (7) the matrix elements $|V_{pp'}|$ exponentially decrease with increasing distances $|x_p - x_{p'}|$.

III. ADIABATIC VERSUS NONADIABATIC EVOLUTION

At sufficiently low fields E_y ($E_y \to 0$) all states of a perturbed system behave adiabatically, independent of the particular form of $V(x,y)$. The adiabatic states are denoted by $w_s(x,y,t)$. If E_y is increased, the true time-dependent states become progressively nonadiabatic, i.e., they develop into time-dependent linear combinations of the states $w_s(x,y,t)$.

For the weakly perturbed states considered above the transition from adiabatic to nonadiabatic behavior can be understood by means of the Zener theory,[8] which gives the probability P for nonadiabatic transition according to the dashed line in Figs. 2 and 3. If $P=1$ the energy expectation value follows the dashed line, and the perturbed wave function essentially behaves as the unperturbed function $\psi_{p'}(x,y,t)$ in this time interval, with the localization center $x_{p'}(t)$ moving with the constant velocity v. Using the developments of Ref. 4 one readily finds that

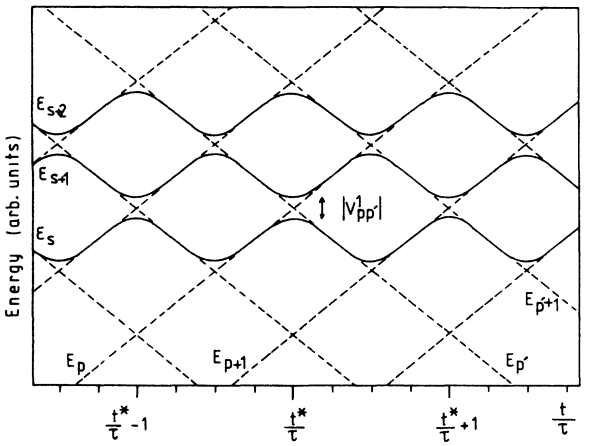

FIG. 2. Adiabatic evolution of the single-particle energies in the absence of the disorder potential $V^1(x,y)$ (dashed lines) and in the presence of $V^1(x,y)$ (solid lines). Shown is the weak-disorder case. $\tau = |h/(qE_yL_y)|$.

the probability $P_{s,s+1}=P_{pp'}$ for Zener tunneling from a weakly perturbed state $w_s(t)=c_p(t)\psi_p + c_{p'}(t)\psi_{p'}$ at $t=t^*-\tau/4$ to the state $w_{s+1}(t)$ at $t=t^*+\tau/4$ in Figs. 2 and 3 is given by

$$P_{pp'}=P_{|p-p'|} = \exp\{-|\pi^2 d^2_{pp'}\exp[-2(x_p-x_{p'})^2/(2\lambda)^2]B/[V'(x_p)chE_y]|\}\ , \tag{11}$$

where $d_{pp'}$ is a Fourier coefficient of V^1 as defined in Ref. 4. Due to a succession of such tunneling events in the presence of disorder V^1, any function becomes a time-dependent linear combination of the adiabatic basis functions $w_s(x,y,t)$, which themselves are linear combinations of the unperturbed functions $\psi_p(x,y,t)$.

From (11) we see that according to the distances $|x_p - x_{p'}|$ of the unperturbed functions ψ_p and $\psi_{p'}$, which are localized on opposite sides of a barrier or well, the corresponding perturbed function describes a fully *nonadiabatic* state (if $P_{pp'} \gg \delta$), an *intermediate* state (if $\delta \le P_{pp'} \le 1-\delta$), or an *adiabatic* state (if $P_{pp'} < \delta$). Here δ is a small number chosen such that in the first case the probability $1-P_{pp'} < \delta$ for adiabatic behavior is negligible and in the last case the probability $P_{pp'} < \delta$ for nonadiabatic behavior (i.e., for Zener tunneling).

In the numerical example corresponding to Fig. 1, δ was set equal to 0.01, and the physical parameters have been chosen so that all the basis functions $w_s(x,y,t)$, which belong to the nonadiabatic (including intermediate) energy region, can be expressed in the weak-disorder formalism [i.e., (10) holds for the corresponding unperturbed states]. On the other hand, most of the states in the adiabatic energy region (dashed line in Fig. 1) have to be described in a "strong"-disorder formalism since (10)

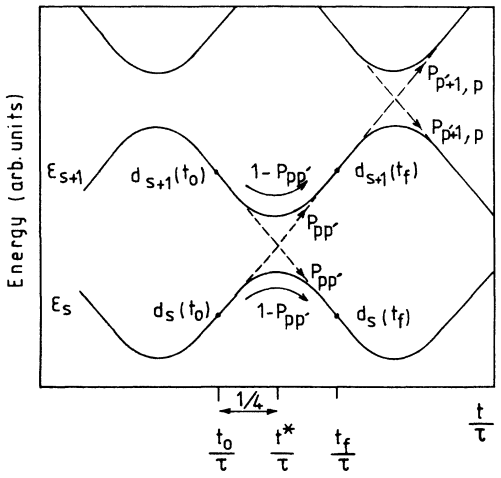

FIG. 3. Anticrossing situation around the value $t=t^*$ (mod τ). $P_{pp'}$ denotes the Zener tunneling probability for the nonadiabatic process from t_0 to t_f. $1-P_{pp'}$ is the probability for the two corresponding adiabatic processes. $d_s(t)$ and $d_{s+1}(t)$ denote the time-dependent expansion coefficients of a weakly perturbed orbital expressed in the basis of the adiabatic orbitals $w_s(t)$ and $w_{s+1}(t)$ in the interval $t^*-\tau/4 \le t \le t^*+\tau/4$.

no longer holds. According to (11) these adiabatic states are linear combinations of (more than two) functions ψ_p with localization centers x_p situated in the vicinity of the center of a barrier or of a well. As we have seen, adiabatic states do not contribute to the dc current. Their exact mathematical form is therefore irrelevant for the following discussion.

Figure 4 schematically shows the structure of the single-particle energies of a Landau band (corresponding to Fig. 1) as a function of time. In the center of the band (between E_a and $-E_a$) are situated the nonadiabatic levels, which are represented by the energies $E_p(\phi)$ of the unperturbed levels $\psi_p(\phi)$ of which they are composed, as we have discussed (see below). In Fig. 4 only unperturbed levels belonging to functions ψ_p with centers x_p situated on opposite sides of the *same* barrier or well are shown, since levels $E_p(\phi)$ belonging to functions $\psi_p(\phi)$ with centers situated on slopes further away do not interact appreciably (their anticrossing probability $1-P$ is negligible).

In our model system intermediate states split into two components at each level anticrossing of Fig. 2. One component develops according to the anticrossing (i.e., adiabatic) branch with probability $1-P$, the other component according to the unperturbed (i.e., nonadiabatic) branch with probability P (see Fig. 3). In our system the adiabatic basis functions $w_s(x,y,t)$ of this intermediate region are expressed in the weak-disorder approximation, i.e., they are linear combinations $\sum_p c_p(t)\psi_p(x,y,t)$ of only two unperturbed functions $\psi_p(x,y,t)$ at a given time. In this approximation, the basis functions $w_s(x,y,t)$ are identical to a single unperturbed function $\psi_p(x,y,t)$ for times exactly halfway between two successive anticrossings; see Refs. 4 and 6. The coefficients $c_p(t)$ can be calculated at times $t=t_0+m\times\tau/2\equiv T$ (m integer) situated halfway between two consecutive splittings. For example, in Fig. 3 the two coefficients $d_{s+1}(t_f)$ $[=c_p(t_f)]$ and $d_s(t_f)$ $[=c_{p'}(t_f)]$ are obtained from the corresponding coefficients at $t=t_0$ by a complex unitary 2×2 matrix. Since the basis states $w_s(x,y,t)$ of the adiabatic region do not mix with those of the nonadiabatic region in the course of time (since the corresponding nonadiabatic transitions between the two regions are negligible per definition), the set of coefficients $c_p(T)$ associated with all the nonadiabatic states can be obtained from the corresponding set of the coefficients $c_p(t_0)$ by a large unitary matrix, which is a product of matrices, each of which is composed of 2×2 matrices.

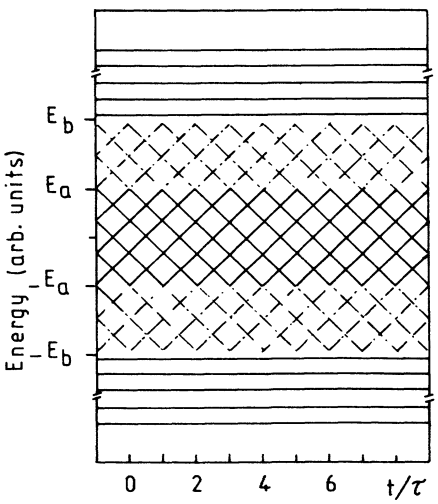

FIG. 4. Schematic time dependence of the single-particle energies corresponding to the model of Fig. 1. A single broadened Landau band is shown. Horizontal lines: adiabatic levels (with Zener probabilities $P\leq\delta$). Solid lines: fully nonadiabatic levels ($1\geq P\geq 1-\delta$). Dot-dashed lines: intermediate states ($\delta<P<1-\delta$).

Starting with the initial condition $c_p(t_0)=1$ for $p=q$ $[c_p(t_0)=0$ for $p\neq q]$, each expansion coefficient $c_p^q(T)$ can be expressed as a sum $\sum_j c_p^{j(q,p)}(T)$, where $j(q,p)$ labels the different splitting paths which lead from $c_q(t_0)$ to $c_p^q(T)$. The complex numbers $c_p^{j(q,p)}(T)$ are products of matrix elements of the 2×2 matrices, which describe the individual splittings encountered on the path $j(q,p)$. By an analysis of the 2×2 matrices it can be shown[9] that the $c_p^{j(q,p)}(T)$ and $c_p^q(T)$ have random phases. This is analogous to the time evolution of weakly perturbed Bloch electrons in a constant electric field.[10,11] The phase randomness implies that for sufficiently large T we have

$$|c_p(T)|^2=\sum_j |c_p^{j(q,p)}(T)|^2 \ . \tag{12}$$

This means that for the calculation of the final occupation numbers $|c_p(T)|^2$ only the splitting *probabilities* $P_{p,p'}$ and $1-P_{p,p'}$ (being absolute squares of splitting matrix elements) at the level anticrossings are needed. [It is highly probable that an analogous phase randomness also holds for the expansion coefficients in the general case, where each perturbed adiabatic function $w_s(x,y,t)$ is a linear combination of more than two unperturbed functions $\psi_p(x,y,t)$ at a given time.] In this way the time evolution in the x direction of the total particle density of a wave packet, which passes through a barrier or a well of $V(x)$ and which is scattered by the disorder potential $V^1(x,y)$, can be calculated explicitly in the weak-disorder case. Numerical calculations of such scattered particle densities are in progress.

The time-dependent scattering process leads to *delocalization* in the *v direction* of the shape of an initially localized Landau function ψ_p, since it develops into a superposition of different $\psi_{p'}$, which are localized at different centers $x_{p'}$. It also leads to a spreading of the energies of the nonadiabatic levels which will range over the whole zone from E_{-b} to E_b. Mathematically, this process is analogous to the scattering of Bloch waves in one-dimensional systems with static disorder in the presence of a constant electric field.[10-12] In both systems the time evolution leads to delocalization in k space, where k is parallel to the applied electric field. This implies localization in real space in the direction of the electric field. In the presence of a strong magnetic field it also implies delocalization in the direction perpendicular to the electric field (the x direction in our case) as a consequence of (6) and (8).

We remark that screening is not considered explicitly in this paper. The potential $V(x,y)$ in the Hamiltonian (1) is already supposed to be the total screened potential. In our explicit model system the slowly varying potential $V(x)$ of Eq. (5) could be obtained in a Hartree approximation along the lines developed in Ref. 13 (see, also, the remarks made in Ref. 6).

IV. ABSENCE OF DISSIPATION

We consider now the general case where we do not assume weak-disorder conditions and where $V(x,y)$ may be any sufficiently asymmetric potential [but the integrals over x and y are zero, i.e., $V(x,y)$ does not describe a macroscopic electric field]. We make the assumption, that the nonadiabatic states are sandwiched between adiabatic states. This is, e.g., the case in our model system (see Figs. 1 and 4). We let the system be in thermodynamic equilibrium through inelastic scattering with a surrounding heat bath. We consider the case where the Fermi level E_F is contained in the range of adiabatic levels and where the temperature is sufficiently low that the nonadiabatic states below E_F (and the empty nonadiabatic states above E_F) are not affected by inelastic scattering events. Therefore, these nonadiabatic states continue to evolve according to the time-evolution operator $\mathcal{U}(t)$ associated with the Hamiltonian (1). Since the nonadiabatic states do not mix with the adiabatic states, we have

$$\mathcal{U}(t)\mathcal{D}(0)\subseteq\mathcal{D}(t)\ , \tag{13}$$

where $\mathcal{D}(t)$ denotes the subspace of the highest occupied nonadiabatic states at the time t. $\mathcal{D}(t)$ is spanned by the corresponding subset of the adiabatic basis states $w_s(x,y,t)$. We now define an operator $\mathcal{U}'(t)$ by

$$\mathcal{U}(t)=\mathcal{U}'(t)\mathcal{U}_{\rm ad}(t)\ , \tag{14}$$

where $\mathcal{U}_{\rm ad}(t)$ is the adiabatic time evolution [which transforms each state $w_s(x,y,0)$ into $w_s(x,y,t)$]. Therefore, together with (13), and since $\mathcal{U}(t)$ and $\mathcal{U}_{\rm ad}(t)$ are unitary in our Hilbert space, it follows that $\mathcal{U}'(t)$ restricted to $\mathcal{D}(t)$ is unitary.

Suppose now the initial condition, where all the basis states $w_j(x,y,t=0)$ of $\mathcal{D}(0)$ are occupied with a probability of 1. According to (14) the time evolution $\mathcal{U}(t)$ of these states can be seen as a transformation by $\mathcal{U}_{\rm ad}(t)$ from $\mathcal{D}(0)$ to $\mathcal{D}(t)$ [which transforms each $w_j(x,y,0)$ into $w_j(x,y,t)$, and leaves the occupation numbers unchanged] followed by $\mathcal{U}'(t)$ restricted to the subspace $\mathcal{D}(t)$, which transforms each $w_j(x,y,t)$ into a linear combination $\sum_s d_s^j(t)w_s(x,y,t)$. Since $\mathcal{U}'(t)$ restricted to $\mathcal{D}(t)$ is unitary, we have

$$\sum_j |d_s^j(t)|^2=1 \tag{15}$$

for all s labeling the basis states $w_s(x,y,t)$ of $\mathcal{D}(t)$.

Since the left-hand side of (15) is the total occupation number of the state $w_s(x,y,t)$, Eq. (15) means that under the stated conditions the total occupation number of each basis state $w_s(w,y,t)$ in $\mathcal{D}(t)$ remains equal to 1 in the course of time. In our weak-disorder model, relation (15) can be checked immediately for $t=(\tau/2)$ times integer by using the fact that the final occupation numbers can be obtained from the individual splitting probabilities encountered in the course of the time evolution.

Now consider the total charge density originating from the nonadiabatic states of $\mathcal{D}(t)$:

$$\begin{aligned}\rho_{\mathcal{D}}(x,y,t)&=\sum_j \psi_j^*(x,y,t)\psi_j(x,y,t)\\&=\sum_j\sum_{s,s'} d_s^{j\,*}(t)d_{s'}^j w_s^*(x,y,t)w_{s'}(x,y,t)\\&=\sum_s\sum_j |d_s^j(t)|^2|w_s(x,y,t)|^2\\&\quad+\sum_{\substack{s,s'\\(s\neq s')}}\sum_j d_s^{j\,*}(t)d_{s'}^j(t)w_s^*(x,y,t)w_{s'}(x,y,t)\,.\end{aligned} \tag{16}$$

The last term in (16) is zero since the matrix of the coefficients $d_s^j(t)$ is unitary. We have seen that in the presence of disorder the modulus of the adiabatic basis states $w_s(x,y,t)$ is periodic in time with period τ (which is extremely small; it tends to zero for an infinitely large strip). Therefore, and as a consequence of (15), $\rho_{\mathcal{D}}(x,y,t)$ is periodic in time with period τ. Further, the occupation of the adiabatic states above $\mathcal{D}(t)$ (part of which are subject to inelastic scattering) is statistically constant in time.

As a consequence there is no self-consistent modification (due to charge redistribution) of the potential $V(x,y)$, provided at $t=0$ $V(x,y)$ represents the self-consistent potential of the ground state. This means that, under the considered assumptions, the presence of the field E_y does not modify the self-consistent potential $V(x,y)$ of the Hamiltonian (1) in the course of time (in particular, no macroscopic field E_x is created) and all adiabatic energies $\varepsilon_s(t)$ remain periodic with period τ. Since we have assumed a physical situation, where the nonadiabatic states (whose energies are not periodic with time and therefore could lead to energy change over macroscopic time intervals, i.e., over times greater than τ, in contrast to the adiabatic states) are not affected by inelastic scattering events, these states obey relation (15) also in the presence of the heat bath, and the total energy of these states remains equal to the ground-state energy. As a consequence the system cannot lose energy to the heat bath: there is no dissipation.

Since the density of dissipated power is given by $\mathbf{I}\cdot\mathbf{E}$, where $\mathbf{I}$ is the macroscopic current density, we have

$$\sigma_{xx}=0,\quad \sigma_{yy}=0\ , \tag{17}$$

whence $I_y=0$ (since $E_x=0$; see above).

The prerequisites for deriving (15) are also fulfilled for a band of Bloch electrons in the presence of a constant electric field (which is not too high, so that no Zener transitions to the next-higher band occur), if E_F is situated in a band gap (semiconductor), or, in the presence of disorder, if one could create a situation where E_F is in the fully adiabatic levels near the upper edge of the band or near the lower edge of the next-higher band. Therefore

Eq. (15) holds and the same conclusions follow, i.e., no dissipation and absence of diagonal dc conductivity.

V. NONLINEAR PARTICLE CURRENTS AND QUANTIZED HALL CONDUCTANCE

The macroscopic dc current I_x results from the nonadiabatic states. [It is constant in time since the total energy (averaged over τ) associated with the nonadiabatic states is constant in time as long as E_F is situated in the range of adiabatic levels.] In the weak-disorder model which corresponds to Fig. 1 we find

$$I_x = nq/\tau = nq^2 E_y L_y/h \ , \tag{18}$$

and hence

$$\sigma_{xy} = nq^2/h \ . \tag{19}$$

Equation (18) follows from the fact that there is a spatial region which is occupied only by fully nonadiabatic states (see Fig. 1). These states are linear combinations of the basis states $w_s(x,y,t)$ such that their modulus is just equal to that of an unperturbed state $\psi_p(x,y,t)$, as we have discussed above. At each time $m\tau/2$ halfway between two splittings they represent a different adiabatic basis state $w_s(x, y, m\tau/2)$, since there is perfect Zener tunneling (see Figs. 2 and 3). Note that according to (15) the corresponding basis states $w_s(x,y,t)$, and hence the resulting fully nonadiabatic states ψ_p, are occupied with a total probability equal to 1 for all times. (In general, this probability is the sum of contributions from *different* time-dependent single-particle functions.) Since the localization center of $\psi_p(x,y,t)$ moves with the velocity $v = cE_y/B$, it describes the transport of exactly one charge q from $x_p(t_0)$ to $x_{p-1}(t_0)$ in the time interval $t_0 \le t \le t_0 + \tau$. In the next time interval of length τ, the orbital ψ_p is replaced by ψ_{p+1} and the scenario repeats itself in the same spatial interval, since $x_{p+1}(t+\tau) = x_p(t)$ according to (8) and because $q < 0$. Therefore in this region a current results which is exactly equal to q/τ per occupied band (including the highest occupied band with Fermi energy situated in the adiabatic levels above the nonadiabatic states).

On the other hand, the dc current of the fully adiabatic states is zero. These states are situated between the spatial regions occupied by nonadiabatic states (see Fig. 1). Since (18) holds in the spatially separated regions of fully nonadiabatic states and since the total charge of the system is conserved and since there is no charge redistribution in the course of time, one charge q (per band) must disappear per time interval τ in the intermediate region on the left of a barrier (well) and one charge q must reappear in the same time interval in the intermediate region on the right of the barrier (well). This means that, although the adiabatic orbitals situated in the dashed region of Fig. 1 carry no dc current, the dc current is not zero in this region. This is a nonclassical current, which corresponds to discontinuous propagation of single particles between the spatially distinct regions, where the nonadiabatic states are situated. This nonclassical particle propagation is the result of the time-dependent scattering process in the intermediate-energy zone; in particular, it is due to the adiabatic components of the intermediate states. For further discussion see Ref. 6.

The total current J_x [Eq. (18)] can also be expressed as the sum of all the single-particle currents of the system. Equating this sum with (18) gives a definition of the effective velocities associated with each nonclassical current of the intermediate states. The sum of these effective velocities just compensates the loss of current from the adiabatic states and from the adiabatic components of the intermediate states. The effective velocities are of the order of the distance between intermediate areas (shaded in Fig. 1) divided by τ times the number of intermediate states per shaded area. This means they are much higher than the classical velocity $v = cE_y/B$ of the fully nonadiabatic states ψ_p. A simple illustration would be the case, where there is only one state per shaded area. Then the effective velocity of this intermediate state is just v times the number of adiabatic states between the two shaded areas on opposite sides of a barrier or well (cf. Ref. 6).

Our analysis shows that these compensating currents are not created by increased *stationary* single-particle currents, as one might think on classical grounds. Instead, they result from a disappearance of charge in a small area and from a reappearance a definite distance further ahead after a short time interval. (As we have seen, this is the result of time-dependent scattering between Landau functions localized at different sites.) During this short time interval the effective particle velocity is higher than v. But at any time the *sum* of the *time-dependent* single-particle currents of all the nonadiabatic states of the n bands has a dc component equal to (18).

This macroscopic Hall current is linear in E_y and so are the single-particle currents associated with the fully nonadiabatic states and with the nonadiabatic components of the intermediate states. However, it is important to realize that the adiabatic single-particle currents, and hence the currents due to the intermediate states (which are partly composed of adiabatic basis states) are nonlinear, particularly the compensating currents. The nonclassical, nonlinear behavior of the adiabatic and intermediate states has been illustrated by our weak-disorder model in Fig. 1. This microscopic nonlinearity is a general property of all our disordered systems (independent of the weak-disorder assumption). It follows from the general form of the adiabatic time evolution[5] of systems which can be described by a Hamiltonian of type (1). In the presence of disorder this time evolution implies periodicity as a function of $\phi = -cE_y L_y t$ of all adiabatic currents and of the modulus of the adiabatic basis states $w_s(x,y,t)$. To describe such a behavior correctly by an expansion with respect to E_y, all orders have to be included. (A linear description would only be a good approximation for time intervals much smaller than half a period τ.) Now, we have seen that the *full* time-dependent behavior of each nonadiabatic state is necessary for a correct quantum-mechanical description of the system and, in particular, for the derivation of (15). As a consequence, linear-response theory cannot correctly describe the quantum-mechanical state which leads to quantum Hall behavior for all the systems which are de-

scribed by Eqs. (1)–(3). This result is quite remarkable, since most theoretical approaches to the IQHE presented so far have been placed into the framework of linear-response theory.

The IQHE starts to break down when E_y is sufficiently high such that the nonadiabatic component of the states at the Fermi level E_F are no longer negligible, i.e., when the states at E_F become intermediate. Here the conditions for the validity of the key equation (15) are no longer fulfilled. This leads to dissipation (hence $\sigma_{xx} \neq 0$), since the occupied nonadiabatic states, especially those at the Fermi level E_F, have energies or energy fluctuations which, in the course of time, would go beyond E_F if they were not continuously absorbed by inelastic interaction with the surrounding heat bath.

If E_y (hence I_x) increases beyond some threshold value E_y^{th} (which corresponds to a threshold value for I_x), the adiabatic components of all the states in the highest occupied band become negligible with respect to their nonadiabatic components. In this case, each state essentially describes again a linear classical current $qv = qcE_y/B$. Therefore, for sufficiently high Hall currents I_x the Fermi level is always in the range of energies of fully (or almost fully) nonadiabatic states, and the total Hall current I_x essentially shows the linear classical behavior without pleateaus.

In the context of the IQHE the nonlinear behavior of single-particle currents may seem surprising. However, we emphasized before that the general structure of the time evolution is analogous to that of one-dimensional Bloch electrons in a static disorder potential in the presence of a homogeneous electric field (but in the absence of a magnetic field). In this case of one-dimensional conduction, it was pointed out previously[12,14] that linear-response theory is inadequate to give a correct quantum-mechanical description of the currents whenever the electric field is below a threshold value.

VI. SUMMARY

We have investigated the time evolution of the single-particle states in a large class of two-dimensional quantum Hall systems in the presence of a static disorder potential. It was found that the nonadiabatic states carry the dc-Hall current, whereas adiabatic states are dc insulating. If the Fermi energy is in a range of adiabatic energy levels (at sufficiently low temperatures), the diagonal conductivities vanish and the dc-Hall current shows a plateau. This result follows directly from the unitarity of the time-evolution operator and from the special form of the adiabatic time evolution. No further assumptions are needed. Linear-response theory is inadequate for a correct quantum-mechanical description of all the single-particle currents in the quantum Hall regime, particularly of the so-called compensating currents, which constitute an important part of the quantized macroscopic Hall current. Our general results are illustrated by a weak-disorder model system, where the time-dependent scattering process of the current-carrying states can be understood explicitly. Here the compensating currents correspond to discontinuous particle motion (which has no classical analogy), and the macroscopic Hall conductance in the plateau region is found to be equal to nq^2/h (n integer).

A widely accepted conventional approach to the explanation of the plateaus in the Hall conductance is the localization model, which assumes the existence of localized and extended states in the tails and the centers of the disorder-broadened Landau bands, respectively. Our results are consistent with this general picture, since adiabatic states are nonconducting and nonadiabatic states are conducting, and therefore can be identified with the localized and extended states assumed in the localization model. The mobility edge corresponds to the separation between intermediate and fully adiabatic states. This separation is not completely sharp, since the intermediate states gradually go over into fully adiabatic states. As the electric field E_y increases, this mobility edge moves away from the center of the conducting states (as illustrated by the weak-disorder case[6]). This leads to the disappearance of the conductance plateaus for sufficiently high Hall fields (currents). For the systems considered in this paper we have shown that the understanding of conducting and nonconducting states is crucially based on their characteristic time dependence in the presence of the Hall field. We plan to give a detailed analysis (based on our present results) of previous theories of the IQHE in forthcoming papers.

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