Critical indices for high-T_c superconductors

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We study the critical indices of a high-T_c superconductor near its transition temperature T_c by assuming, phenomenologically, that superconductivity arises from a Bose-Einstein condensation of charged bosons (holons) in a resonating-valence-bond vacuum in $2+\epsilon$ dimensions. A self-consistent version of the static random-phase approximation leads to a quasiparticle spectrum $\epsilon(\mathbf{k})=Ak^{\sigma}$, with $\sigma=1.766$.

I. INTRODUCTION

The discovery of high-temperature superconductivity at 40 K in La-Sr-Cu-O systems by Bednorz and Müller,[1] and subsequently at 90 K in Y-Ba-Cu-O systems by Wu *et al.*,[2] has stimulated enormous experimental and theoretical activity recently. There are several mechanisms proposed for this revolutionary behavior of high-T_c materials.[3-5] Even though there exists, to our knowledge, no definitive and foolproof theory for high-T_c superconductivity, the resonating-valence-bond (RVB) model of Anderson and his co-workers appears to be promising.[6,7]

As is well known, the structure of high-T_c materials is a very peculiar one, in which the planes of the Cu-O_2 system (Cu being situated at the center of the square lattice formed by O) are either separated by oxygen or Cu-O chains and the coupling between the Cu-O_2 planes is weak in the third direction (the z direction). In RVB it is proposed that[8] it is the $d_{x^2-y^2}$ orbitals of oxygen that overlap with the p orbitals of copper via superexchange so that a RVB state is formed between two copper holes and it is these holes that are the charge carriers. The excitations in the RVB vacuum are spinons, holons, and charged electrons.[8] Since the interlayer coupling is weak, the motion of holes takes place more or less in two dimensions and the phenomenon of superconductivity may be thought of as a Bose-Einstein condensation of charged holons, in $2+\epsilon$ dimensions, an idea explored recently by Wen and Kan[9] to obtain some of the properties characterizing the superconducting phase transition in high-T_c materials. Wen and Kan model a high-T_c material with an energy spectrum in which the effective mass of the hole in the z direction exceeds that in the x-y plane. They obtain the value for La as well as Y systems by equating the grand potential in $2+\epsilon$ dimensions to the grand potential in three-dimensions but with different masses in different directions. Subsequently, Wen and Kan introduce a hard-sphere interaction between two holes. This, we feel, is rather drastic because the screening effects for charged bosons near T_c are very peculiar and long range in nature.[10] We hope to correct for this aberration and obtain the critical indices as demonstrated in the next section. Admittedly, such an approach is phenomenological with some artifacts of it; however, towards an affirmative end, we hope to bring out some aspects of the phase transition in the high-T_c material in the vicinity of T_c.

II. THEORY

In order to determine the value of ϵ, we write the thermodynamic grand potential Ω_0 for an ideal Bose gas in $2+\epsilon$ dimensions as

$$\Omega_0=\frac{k_BTL^{2+\epsilon}}{(2\pi)^{1+\epsilon}}\int_0^{\infty}dk\,k^{1+\epsilon}\ln(1-e^{[\mu-\varepsilon(\mathbf{k})]/k_BT}) . \tag{1}$$

Here, $L^3=V$ is the actual volume of the system. After some simplification, one is led to

$$\Omega_0=\frac{-L^{2+\epsilon}}{4(2\pi)^{1+\epsilon}(2+\epsilon)}\left[\frac{2mk_BT}{\hbar^2}\right]^{1+\epsilon/2}\times\Gamma(1+\epsilon/2)g_{1+\epsilon/2}(e^{\mu/k_BT}) , \tag{2}$$

with $g_t(s)=\sum_{l=1}^{\infty}s^l l^{-t}$. This, in conjunction with

$$-\frac{\partial\Omega_0}{\partial\mu}=n_{2+\epsilon}-n_{2+\epsilon}^s \tag{3}$$

gives

$$T_c=\left[\frac{\hbar^2}{2mk_B}\right](2\pi)^{(1+\epsilon)/(1+\epsilon/2)}\times\left[\frac{2n_{2+\epsilon}}{\int_0^{\infty}dx\,x^{\epsilon/2}\Big/(e^x-1)}\right]^{2/(2+\epsilon)} , \tag{4}$$

where $n_{2+\epsilon}$ is the density of holes in $2+\epsilon$ dimensions and the superscript s indicates superfluid. From the available experimental data,[11] we take $m=1.6m_e$; $n=2.273\times10^{27}$ m^{-3} in Eq. (4) which then yields a value of $T_c\simeq90$ K for Y-Ba-Cu-O systems for $\epsilon=0.03$, a value remarkably close to that obtained by Wen and Kan.[9] In order to determine the critical exponents, we obtain the quasiparticle energy spectrum $\varepsilon(\mathbf{k})$ around $T=T_c$ in the long-wavelength ($|\mathbf{k}|\rightarrow0$) limit, following Panat *et al.*[10] We first note that the polarization propagator $\Lambda(\mathbf{q})$ in d dimensions as

$$\Lambda(\mathbf{q})=\frac{k_BT}{(2\pi)^d}\int d^dk\frac{1}{\varepsilon(\mathbf{k})}\frac{1}{\varepsilon(\mathbf{k}+\mathbf{q})} . \tag{5}$$

This, in turn, determines the screened interaction in the limit $|\mathbf{q}|\rightarrow0$

$$V(\mathbf{q})\equiv\frac{u(\mathbf{q})}{1+u(\mathbf{q})\Lambda(\mathbf{q})}\rightarrow\frac{1}{\Lambda(\mathbf{q})} , \tag{6}$$

$u(q) \equiv 2\pi/q$ being the unscreened Coulomb interaction between two holes. In the long-wavelength limit, the quasiparticle energy spectrum at $T \simeq T_c$ is thus given by[10]

$$\varepsilon(\mathbf{p}) = \frac{k_B T}{(2\pi)^d} \int \frac{d^d k}{\varepsilon(\mathbf{k})} \left[\frac{1}{\Lambda(\mathbf{k}+\mathbf{p})} - \frac{1}{\Lambda(\mathbf{k})} \right]. \quad (7)$$

Equations (5)–(7) may be solved self-consistently with an *ansatz* for the quasiparticle energy spectrum:

$$\varepsilon(\mathbf{p}) = A p^{\sigma} \quad (8)$$

$(1 < \sigma < 2)$. We observe that

$$\Lambda(q) = \frac{k_B T L^{2+\epsilon}}{A^2} \frac{q^{2+\epsilon-2\sigma}}{(2\pi)^{2+\epsilon}} C(\sigma), \quad (9)$$

with

$$C(\sigma) \equiv \int_0^\infty dx\, x^{1+\epsilon-\sigma} \int_0^{2\pi} d\theta (1+2x\cos\theta + x^2)^{-\sigma/2}.$$

Moreover, from Eq. (7) above,

$$\varepsilon(\mathbf{p}) = A p^{\sigma} D(\sigma)/C(\sigma),$$

where

$$D(\sigma) \equiv \int_0^\infty dx\, x^{1+\epsilon-\sigma} \int_0^{2\pi} d\theta [1/(1+2x\cos\theta+x^2)^{1+\epsilon/2-\sigma} - 1/x^{2+\epsilon-2\sigma}].$$

Simultaneous validity of Eqs. (8) and (9) demands that $D(\sigma) = C(\sigma)$, leading to $\sigma = 1.766$ and $C(\sigma) = D(\sigma) = 29.987$.

With the quasiparticle energy spectrum (8) the critical exponents characterizing the superconducting transitions can be determined following Gunton and Buckingham.[12] Table I summarizes these results, in terms of a parameter $t \equiv |T - T_c|/T_c$. From the table, it is evident that the condensate density goes linearly to zero as $T \to T_c$. This gives rise to the divergence in London penetration depth as

$$\lambda_L = \left[\frac{mc^2 T_c}{4\pi e^2 n_0} \right]^{1/2} |T - T_c|^{-1/2}. \quad (10)$$

Since the Landau-Ginzburg equations are phenomenological in nature, we may assume that they are valid in the high-T_c realm as well. We define the critical field H_c, arising out of the difference between the free-energy densities of the normal (F_n) and superconducting (F_s) state as

$$F_n - F_s = (8\pi)^{-1} H_c^2. \quad (11)$$

TABLE I. Critical exponents for the superconducting phase transition.

Quantity	Critical exponent	Behavior	Value
Amplitude of order parameter, ψ	β	$\psi \sim t^{\beta}$	$\beta = \frac{1}{2}$
Condensate density $n_s = \lvert\psi\rvert^2$	2β	$n_s \sim n_0 t^{2\beta}$	$2\beta = 1$
Correlation function $(C(r))$	η	$C(r) \sim r^{2-d-\eta}$	$\eta = 0.234$
Correlation length, R_0	ν	$R_0 \sim t^{-\nu}$	$\nu = 3.788$
Specific heat, C_p	α	$C_p \sim t^{-\alpha}$	$\alpha = 0.850$
Superfluid density, ρ_s	ζ	$\rho_s \sim t^{-\zeta}$	$\zeta = 0.150$

Since the field H_c is related to the Landau-Ginzburg expansion coefficient $a(T)$ that goes to zero linearly with $T_c - T$, we see that $H = \mathrm{const}(T_c - T)$. H_{c1} and H_{c2}, the lower and the upper critical fields, are linearly related to H_c, leading to the same dependence on temperature. This remarkable behavior has been experimentally observed.[13] The present model is seen to conform to this observation.

III. RANGE OF VALIDITY

In the present model we have implicitly assumed that the vertex corrections are negligible. However, they become important near T_c. The zero-frequency part of the lowest-order correction to vertex in the long-wavelength limit can be evaluated and gives estimates of the temperature range around T_c in which the present model is valid. Thus, the vertex $\Gamma(\mathbf{p},q)$ differs from unity as

$$\Gamma(\mathbf{p},\mathbf{q}) - 1 = -\frac{k_B T}{(2\pi)^d} \int d^d k\, V(\mathbf{k}) G(\mathbf{p}-\mathbf{k}) G(\mathbf{p}-\mathbf{k}-\mathbf{q}). \quad (12)$$

This expression cannot be evaluated in closed form. To circumvent this difficulty, we introduce a lower cutoff $k = R_0^{-1}$ and retain the most divergent part of Γ, namely $\Gamma(0,0)$. Thus

$$\Gamma(0,0) - 1 \simeq -\frac{k_B T}{(2\pi)^{1+\epsilon}} \int_{R_0^{-1}}^\infty dk\, k^{1+\epsilon} G^2(\mathbf{k})/\Lambda(\mathbf{k}). \quad (13)$$

Putting the asymptotic forms of $\Lambda(\mathbf{k})$ and $G(\mathbf{k})$ in the limit $|\mathbf{k}| \to 0$, μ, the chemical potential going to zero, and the behavior of Γ with respect to t, we get

$$\Gamma(0,0) - 1 \sim \frac{2\pi}{C(\sigma)} \ln R_0^{-1} \simeq 0.797 \ln \left| \frac{T_c - T}{T_c} \right|. \quad (14)$$

This puts the limit that the theory is not valid when $t \ll 0.1$. Thus, too close to T_c, our power-law behavior of various physical quantities may not be valid. This should be contrasted with Bardeen-Cooper-Schrieffer superconductors where the region of criticality is $t \lesssim 10^{-14}$ (cf. Ref. 14).

It is thus felt that the description of high-T_c superconducting phase transition is facilitated in terms of a Bose-Einstein condensation of charged bosons in $2+\epsilon$ dimensions.

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