Major normal and superconducting parameters of high-T_c oxides

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The major normal and superconducting parameters of the high-T_c oxides have been evaluated. The analysis based on a Fermi-liquid approach is carried out in reciprocal space: the anisotropy of the system is reflected in the shape of the Fermi surface. The analysis is valid for any form of the Fermi curve. The small value of the Fermi energy and the Fermi velocity we obtained along with the presence of a layered structure are the key features of the cuprates. The coherence length ξ_0 is evaluated and appears to be short. The value of H_{c2} is calculated in a self-consistent way. The method of evaluation of the strength of the electron-phonon coupling is developed and leads to a strong coupling which, however, is not sufficient to give the observed value of T_c. The effects of anisotropy and multigap structure are discussed.

I. INTRODUCTION

Over the past two years, there have been a large number of experimental and theoretical efforts devoted to understanding the properties of the new class of superconducting cuprates.[1-3] In this paper we present a theoretical framework, based for the most part on a Fermi-liquid approach, which has allowed us to evaluate many of the normal and superconducting parameters of these materials (especially the $La_{1.8}SrCuO_4$ compound).

We have developed this framework over the past two years and some of the results have been presented in several short communications.[4] The present paper contains a detailed description of the method and its consequences and, in addition, several new results. In particular, we have estimated the effective mass, the Fermi energy, momentum and velocity, the electron-phonon coupling constant λ, the coherence length, and the upper critical field H_{c2}. We have found that the Fermi energy and velocity are much smaller than in typical metals[4,5] and also that the superconducting coherence length is very small compared to conventional superconductors. The effect of these unusual parameters on the properties of these cuprates is discussed in detail. We also find that the electron-phonon coupling parameter is quite large (but not large enough to explain the measured transition temperature) and consequently there is a need for additional coupling.

The theory presented below is for the doped compounds which are metallic. One can distinguish two interrelated, but nevertheless, different directions in the physics of high T_c, one involving the problem of carrier doping and the transition to the metallic state and the second being the description of the metallic state. It is important that this metallic phase undergoes the transition into superconducting state; as a result, our analysis is directly related to the origin of high T_c. We are using a quasi-two-dimensional (2D) Fermi-liquid model to estimate the fundamental parameters of these very interesting materials.

The paper is divided up into several parts. Section II deals with a detailed description of our Fermi-liquid approach and the estimation of the normal-state properties. Section III discuss the superconducting parameters; the electron-phonon coupling parameter, the coherence length, and the upper critical field H_{c2}. Finally we discuss the interesting possibility of observing energy-gap anisotropies and multiple-gap structures due to the unusual Fermi surface and the very short coherence length that we have found.

II. NORMAL PARAMETERS: A FERMI-LIQUID APPROACH

The new high-T_c cuprates are characterized by a large anisotropy which is caused by the presence of low-dimensional substructures such as layers or chains (in 1:2:3 compound).

This anisotropy manifests itself in the normal (see, e.g., Ref. 6), as well as in the superconducting state. One should note that many conventional superconductors display strong anisotropy of their normal properties, whereas the effect in the superconducting state is not well pronounced.

The fact that one can observe anisotropy of the superconducting state is a peculiar property of the new superconductors. We will discuss it later (see Sec. IV): at present, we focus on the normal properties of cuprates.

We think that the most efficient way to analyze the anisotropy and to evaluate the normal parameters is to describe the system in momentum (reciprocal) space; our method is based on the use of a Fermi-liquid approach. The anisotropy of the system is reflected in the topology of the Fermi surface (FS): $\varepsilon(\mathbf{p})=\varepsilon_F$.

As is known, there are different approaches in reconstructing the FS (Fermiology). One is based on band-structure calculations; the parameters of the FS are calculated. Another approach is based on an analysis of certain experimental data, which are sensitive to the topology of the FS; special theoretical analysis allows us to reconstruct its shape. The attenuation of ultrasound in a magnetic field represents an example of such an experiment: the attenuation oscillates as a function of the field, and the period of oscillation is directly related to the geometry of the FS and its parameters. The parameters of the FS of many metals and compounds have been evaluated (see, e.g., Ref. 7) and detailed results have been obtained with the use of such an approach.

In this paper we are going to evaluate the major parameters using the former method. As a concept, the approach is similar to the development of tunneling spectroscopy, where theoretical analysis allows us to use the tunneling experimental data in order to evaluate the major parameters describing the electron-phonon interaction.

Consider La-Sr-Cu-O; in this paper we are going to focus mainly on this compound. We think that this material is exceptionally important for the understanding of the physics of high T_c. This importance is connected with the relative simplicity of its structure. At the same time, it contains the essence of the behavior of the whole class of the cuprates. In addition, there are highly reliable experimental data (see below) for this material.

The La-Sr-Cu-O system has a layered structure, so that the interlayer distance $d_c \gg d_a, d_b$. The dispersion relation $\varepsilon(\mathbf{p})$ is highly anisotropic, where $\mathbf{p}=\{\mathbf{k},p_z\}$ is a quasimomentum ($\mathbf{k}$ is a 2D momentum); the Z axis has been chosen to be perpendicular to the layers. As a first approximation, one can assume the Fermi surface to be cylindrically shaped which corresponds to neglecting the interlayer transitions. Of course, the interlayer transitions lead to small deviations from the cylindrical shape. It is important that we are not assuming the Fermi curve (the Fermi curve is defined as the cross section of the FS by the plane $p_z=$const) to be a circle. It appears that one can estimate the values of the Fermi energy ε_F and the effective mass m^* [for its definition see below, Eq. (2)] without specifying the shape of the Fermi curve. In the case of cylindrically shaped FS the dispersion relation is $\varepsilon(\mathbf{k})$ and does not depend on p_z. A large anisotropy of the normal conductivity justifies such an approximation. One should note that this approach is applicable to hole as well as to electron-carrier types of materials. For example, the hole surfaces at the corners of the first zone can be viewed in the quasi-2D case a cylinder as can be seen from a simple translation in momentum space.

Let us estimate, by using this approach, the values of the normal and superconducting parameters. The total energy can be written in the form

$$E=2\int \frac{d\mathbf{k}\,dp_z \varepsilon f}{(2\pi\hbar)^3} . \tag{1}$$

Here f is the Fermi function: integration over p_z is restricted by $|p_{z\max}|=\pi/d_c$, where d_c is the interlayer distance. With the use of Eq. (1) and making the transformation into integrals over constant energy curves and energy, we obtain the following expression for the Sommerfeld constant $\gamma=C_e/T$: $\gamma=(\pi/3\hbar^2)m^*k_B^2 d_c^{-1}$ where the average effective mass is defined as

$$m^*=(2\pi)^{-1}\int dl\, v_\perp^{-1} \tag{2}$$

and the integration is taken over the Fermi curve: $\varepsilon(k)=\varepsilon_F$, $p_z=$const, $v_\perp=(\delta\varepsilon/\delta k)_F$. In this analysis we have assumed that the effective mass as defined by Eq. (2) does not depend strongly on energy. Note that the mass defined by (2) corresponds to the cyclotron mass (see, for example, Ref. 8). In the case of a simple parabolic band this cyclotron mass is equal to the usual effective mass. Our method can be used for an estimation of this last quantity. Equation (2) can be used in order to determine the value of m^*

$$m^*=3(\hbar^2/\pi)k_B^{-2}d_c\gamma . \tag{3}$$

Note that for an isotropic system (spherical FS) the density of states N_F and, hence, the Sommerfeld constant are proportional to m^*p_F (where p_F is the Fermi momentum which depends on the carrier concentration), whereas for a layered structure (cylindrical FS), $N_F \sim m^* d_c^{-1}$. This enables us to obtain a one-to-one correspondence between the Sommerfeld constant and the effective mass [see Eq. (3)].

As was said above, we are not using the effective-mass approximation, that is, the dependence $\varepsilon(\mathbf{k})$ has not been assumed to be quadratic. In this paper the effective mass is defined by Eq. (2), and, therefore, Eq. (3) is valid for any shape of the Fermi curve $\varepsilon(\mathbf{k})$.

The carrier concentration n is equal to $2\int_F dk\, dp_z(2\pi\hbar)^{-3}$ (integration is taken over the filled states at $T=0$). Again transforming this into integrals over curves F of constant energy and energy, we obtain $\varepsilon_F=(\pi n d_c\hbar^2/m^*)$, where m^* is defined by Eq. (3) and therefore

$$\varepsilon_F=(\pi^2 k_B^2/3)n/\gamma . \tag{4}$$

Equations (3) and (4) express m^* and ε_F in terms of the experimentally measured quantities d_c, n, and γ. These values of m^* and ε_F does not depend on any assumption about the shape of the Fermi curve. On the contrary, these values should be taken into account when reconstructing the Fermi surface.

The values of γ and n can be determined from heat-capacity data (see, e.g., Refs. 9 and 10 and the review[11]) and Hall-effect measurements (see, e.g., Refs. 12 and 13). It is important to stress that because the FS corresponds to the ground state of the system, Eqs. (3) and (4) contain a value of γ in the low-temperature region near $T=0$ K [that is γ in Eqs. (3) and (4) is equal to $\gamma(0)$]. This remark is essential because γ depends strongly on T (see Sec. III A). Note also, that the Hall-effect measurements of La-Sr-Cu-O do not display a strong temperature dependence[13] (in Y-Ba-Cu-O the dependence is much stronger, see, e.g., Ref. 13). This is connected with a relatively simple band structure of this system which allows us to use

the carrier concentration obtained by the Hall measurements in La-Sr-Cu-O. The situation in Y-Ba-Cu-O is different because of a more complicated band structure (see below).

The determination of $\gamma(0)$ is not a simple task because the system is in a superconducting state and the value of the critical field is large. The problem has been solved in Ref. 9 by analyzing the dependence of γ on H, so that $\gamma(0)\cong(\delta\gamma/\delta H)H_{c2}$. (For a more detailed description see Refs. 9 and 11). Note also that the value $H_{c2}\cong 80-100$ T for La-Sr-Cu-O can be evaluated in a self-consistent way (see Sec. III B).

Using the value $\gamma(0)\cong 8.8$ mJ/mole K^2 (Refs. 9 and 11) (a similar value was obtained in Ref. 10 with the use of a different method), we obtain

$$m^*\approx 5m_e \ . \tag{5}$$

Therefore, the material is characterized by a set of "heavy" carriers. It is worth noting that the set of the parameters is unique even relative to other layered systems (e.g., for graphite $m^*\cong 0.7m_e$). Such a value of the effective mass is due to strong renormalization, $m^*=m^b(1+\lambda)$, λ is the strength of the coupling (see Sec. II).

Let us turn to the evaluation of the Fermi energy with the use of Eq. (4). The carrier concentration has been obtained by Hall-effect measurements[10–12] and is equal to $n\cong 3\times 10^{21}$ cm^{-3}. Using this value and Eqs. (4) and (5) we obtain

$$\varepsilon_F\approx 0.1 \text{ eV} \ . \tag{6}$$

This value of ε_F is much smaller than in conventional metals where the value is between 5 and 10 eV. We think that a small value of ε_F is a key feature of the high-T_c oxides.

The consequences of a small value of ε_F were discussed by us in Ref. 4 and in the paper by Deutscher and coworkers.[5] Many properties, including, e.g., transport properties of the cuprates, are due to this small ε_F. For example, the electronic thermal conductivity in the normal state K_e^n is proportional to ε_F (see, e.g., Ref. 14). A small value of ε_F should lead to a significant decrease in the electronic contribution to the total thermal flow. This result corresponds to the experimental data (see, e.g., Ref. 15); contrary to the conventional picture the thermal conductivity in the cuprates is dominated by the lattice so that $K_{\rm ph}\gg K_{\rm el}$.

We have already evaluated m^* and ε_F. When we consider the Fermi momentum p_F and the Fermi velocity v_F these quantities depend strongly on the shape of the Fermi curve and the direction. In this paper we can estimate the values of p_F and v_F by making the isotropic approximation for the Fermi curve $\varepsilon(k)=\varepsilon_F$, that is $p_F=(2m^*\varepsilon_F)^{1/2}$ and $v_F=p_F/m^*$. One can estimate, for La-Sr-Cu-O,

$$k_F\equiv p_F/\hbar=3.5\times 10^{-7} \text{ cm}^{-1}$$

and $v_F=8\times 10^6$ cm sec^{-1}. Note that the value of the Fermi momentum is smaller but does not differ drastically from its value in conventional metals. This is important because it allows a large phase space for pairing.

The small value of the Fermi velocity, v_F, is another unique feature of the cuprates [in conventional superconductors v_F is usually $(1.5-2)\times 10^8$ cm sec^{-1}].

The comparison of the evaluated major normal parameters of the high-T_c material and the usual normal metal can be seen in Table I.

In this paper we have mainly focused on the one-particle excitations. However, our results have many implications about the behavior of the collective excitations. For example, the quasi-2D low-lying acoustic plasmon branch has a slope of the same order of magnitude as the Fermi velocity (see, e.g., Refs 16 and 17). This small slope of the "electronic sound" makes the branch similar to the usual phonon acoustic branch.

We have evaluated the major normal parameters for the La-Sr-Cu-O compound, but we think that the basic conclusions are valid for other high-T_c cuprates as well, it is connected with the presence of the Cu-O conducting planes as a universal feature. Indeed, the Y-Ba-Cu-O (1:2:3) compound has been analyzed in the author's paper[4] and in the paper by Deutscher and co-workers.[5] It has been shown with another method based on the analysis of fluctuations (see Ref. 5) that the Fermi energy is also small ($\cong 0.25$ eV). As similar conclusion follows from Ref. 18; the authors[18] have obtained narrow bands for Y-Ba-Cu-O and it has been confirmed by the analysis of the transport data. The more complicated band structure of Y-Ba-Cu-O also makes the FS more complicated, but the FS must have a cylindrical part similar to La-Sr-Cu-O. Upon the availability of appropriate experimental data aimed toward a Fermi-liquid approach, we intend to carry out a detailed reconstruction of the FS of 1:2:3 material.

Our above approach is based on the Fermi-liquid theory. It means that concepts such as dispersion relation, Fermi surface, and Fermi velocity have a direct meaning. We certainly followed this approach in our previous papers.[4,5] Recent experimental photoemission spectra on 1:2:3 (Ref. 19) shows the presence of a sharp Fermi edge. These data present direct experimental support for an approach based on these concepts. There are also positron annihilation data,[20] which have been interpreted as support of a Fermi-liquid picture. Note that their analysis[20] reveals a cylindrical FS in accordance with our theoretical analysis (see above). Note also that a small value of ε_F leads to a large value of the parameter $\tilde{\Omega}/\varepsilon_F$ ($\tilde{\Omega}$ is an average phonon frequency). This parameter describes a nonadiabaticity (see, e.g., the review in Ref. 21).

TABLE I. Comparison of normal-state properties of conventional metals with $La_{1.8}Sr_{0.2}CuO_4$.

Quantity	Conventional metals	$La_{1.8}Sr_{0.2}CuO_4$
m^*	$(1-15)m_e$	$5m_e$
k_F (cm^{-1})	10^8	3.5×10^7
v_F (cm sec^{-1})	$(1-2)\times 10^8$	8×10^6
ε_F (eV)	5–10	0.1

We think that small values of ε_F and v_F along with the anisotropy are key physical properties of the new high-T_c oxides.

III. SUPERCONDUCTING PARAMETERS

A. Electron-phonon coupling

Evaluation of the strength of the coupling which is responsible for the pairing in high-T_c oxides is a problem of great interest. Needless to say that this problem is directly related to the origin of high T_c. As is known, tunneling spectroscopy represents a very reliable and powerful method of determining the value of the electron-phonon coupling constant λ (see, e.g., Ref. 22). However, a number of factors and, in particular, a small value of the coherence length, make tunneling spectroscopy very difficult. That is why other methods of determining λ are important.

In this paper we use a different method for evaluating λ. This method is based on an analysis of heat-capacity data. The electron-phonon interaction leads to a deviation of the electronic heat capacity $C_{\rm el}=\gamma T$ from a linear law; in other words, Sommerfeld's constant γ depends on temperature.[23-25] This temperature dependence is described by the equation:[23,25]

$$\gamma(T)=\gamma(0)\left[1+\rho\left[\frac{\kappa(T)}{\kappa(0)}-1\right]\right] . \tag{7}$$

Here $\gamma(0)=\gamma^0(1+\lambda)$ is the value of γ as T approaches 0 K, γ^0 is the band value of the Sommerfeld constant, $\rho=\lambda(1+\lambda)^{-1}$, and

$$\kappa(T)=2\int d\Omega\, g(\Omega)\Omega^{-1}Z(T/\Omega) ,$$

where $Z(x)$ is the universal function (see Refs. 23 and 25) and Ω is a phonon frequency (see below). In other words, in the low-temperature region the carriers are "dressed" by phonons. Thermal motion eventually leads to the nonrenormalized band value $\gamma\cong\gamma^0$. The coupling constant λ is equal to

$$\lambda=\frac{\gamma(0)}{\gamma^0}-1 . \tag{8}$$

Hence, if the values $\gamma(0)$ and γ^0 are known, the coupling constant λ can be calculated. The value of $\gamma(0)$ for La-Sr-Cu-O has been obtained in Ref. 9 with the use of a unique method utilizing the low-temperature measurements of heat capacity in a magnetic field (see Sec. II). As for γ^0, its value can be determined from the electronic heat capacity in the high-temperature region $T\geq\Theta_D$ [see Eq. (7)]. A method based on measurements of heat capacity at high temperatures ($T>\Theta_D$), has been proposed by Deutscher and co-workers.[5] A small value of ε_F (Ref. 4) and the dependence of the chemical potential $\mu(T)$ are taken into account in Ref. 5. It would be interesting to carry out these high-temperature data experiments to obtain a reliable value of λ. However, at present these high-temperature data do not exist.

Because of the large value of T_c for the new oxides, we can use another method of determining γ^0. Our approach in this paper is similar to Ref. 26 which also was based on the analysis carried out by Zaitsev and one of the authors.[23] The compound Y-Ba-Cu-O was studied in Ref. 26.

In this paper we evaluate λ for La-Sr-Cu-O. We think that the data [including the value of $\gamma(0)$] are more reliable for this material. One should also emphasize that the authors[26] used the value of the jump in heat capacity for weak-coupled superconductors. In this paper we use a more general expression[27] valid for a strong electron-phonon interaction (EPI); as a result, the approach appears to be self-consistent. One can see from Eq. (7) and Refs. 23 and 25 that for $T\geq\cong 0.25\Omega_c$, $\gamma\simeq\gamma^0$ [Ω_c is the phonon cutoff frequency such that $G(\Omega)\equiv g(\Omega)\Omega^{-1}$ is very small in the region $\Omega>\Omega_c$]. Therefore, if the values of γ can be measured in the regions $T\cong 0$ K and $T>0.25\Omega_c$, then λ can be determined with the use of the simple relation $\lambda=[\gamma(0)/\gamma^0]-1$ as mentioned above.

The value of Ω_c can be estimated with the use of neutron spectroscopy data.[28,29] Based on this data one can conclude that $Z(T/\Omega)$ is small for La-Sr-Cu-O at $\Omega>\Omega_c/4$, where $\Omega_c\cong 14$ meV. Indeed, according to Ref. 28 the function $F(\Omega)$ has two peaks at $\Omega_1\cong 10$ meV and $\Omega_2\cong 20$ meV. These two peaks correspond to the low optical modes at frequencies Ω_1 and Ω_2 observed in Ref. 29 where the phonon dispersion curves were reconstructed by neutron scattering. The function $\alpha^2(\Omega)$ is a smooth function, which is usually larger for a lower peak. The contribution of the lower peak to $G(\Omega)$ is enhanced by the factor Ω^{-1}. As a result, with a high degree of confidence, one can put $\Omega_c\cong 14$ meV. Note that a value for Ω_c of less than 14 meV would lead to even larger values of λ (see below). Therefore, $T_c\cong 40$ K $>\Omega\cong 0.25(\Omega_c)$ an $\gamma(T_c)\cong\gamma^0$. The jump is specific heat ΔC is equal to

$$\Delta C=\beta\gamma T_c . \tag{9}$$

Here $\gamma\sim\gamma^0$ [we put $\kappa=0$ in Eq. (7)]. In the Bardeen-Cooper-Schrieffer (BCS) theory developed in a weak-coupling approximation ($\lambda\ll 1$) $\beta\equiv\beta_{\rm BCS}=1.43$. The effects of strong coupling on β have been studied by Parchomenko and one of the authors.[27] If $T>\Omega_c/4$, one can neglect the renormalization effect and we obtain[27] (see also Ref. 30)

$$\beta=1.43\{1+18(T_c/\tilde{\Omega})^2[\ln(\tilde{\Omega}/T_c)+\tfrac{1}{2}]\} . \tag{10}$$

Here $\tilde{\Omega}$ is a characteristic frequency (see discussion in Refs. 27 and 31). For example, if $g(\Omega)$ is characterized by two distinct peaks then $\tilde{\Omega}\sim\Omega_1$, where Ω_1 is the frequency of the lower peak. The value $\tilde{\Omega}=\langle\Omega^2\rangle^{1/2}$ can be also be used as an estimate. For our estimation we put $\tilde{\Omega}\cong 15$ meV for La-Sr-Cu-O and it gives us an approximate value of β. One can expect that the actual value is somewhat higher.

Based on Eq. (9) and the relations $\gamma^0=\gamma(0)(1+\lambda)^{-1}$ and $\gamma(0)=(\partial\gamma/\partial H)\,H_{c2}$ (see Sec. II), we arrive at the following expression for λ:

$$\lambda=\frac{\beta(\partial\gamma/\partial H)H_{c2}T_c}{\Delta C}-1=g-1\ , \tag{11}$$

where β is described by Eq. (10). Equation (11) expresses λ in terms of experimentally measured quantities ΔC, T_c, H_{c2}, $\tilde{\Omega}$, and $\partial\gamma/\partial H$.

Let us evaluate λ with the use of Eq. (11). For La-Sr-Cu-O, $\Delta C/T_c=10$ mJ/mole K^2,[11] $\tilde{\Omega}\cong 15$ meV (see above), $\partial\gamma/\partial H\cong 0.1$,[9] $H_{c2}\cong 80-100$ T, and we obtain $g\cong 2.8-3$ and $\lambda\cong 2$. Hence, we come to the conclusion that the electron-phonon interaction in La-Sr-Cu-O is *not* small. We think that it is natural to assume that the strength of the interaction in the oxides with a higher T_c is of the same order (or larger). The value of $\lambda\cong 2$ that we obtained for La-Sr-Cu-O is large and it means that the EPI plays an important role. Nevertheless, it is not large enough to provide the measured high T_c.

Indeed, let us discuss the question of how large should $\lambda\cong\lambda_{ph}$ be in order to provide the measured T_c in the cuprates. This question can be answered with the use of the equation obtained by the one of the authors:[32]

$$T_c=\frac{0.25\tilde{\Omega}}{(e^{2/\lambda_{\rm eff}}-1)^{1/2}}\ . \tag{12}$$

Here $\tilde{\Omega}=\langle\Omega^2\rangle^{1/2}$,

$$\lambda_{\rm eff}=(\lambda-\mu^*)[1+2\mu^*+\lambda\mu^*\tau(\lambda)]^{-1}\ .$$

the function $\tau(\lambda)$ is defined in Ref. 32. If we put $\mu^*=0$, we obtain

$$T_c=\frac{0.25\tilde{\Omega}}{(e^{2/\lambda}-1)^{1/2}}\ . \tag{13}$$

The expression (12) is valid for any λ and was obtained by the solution of the Eliashberg equation in the matrix representation.[33] For large $\lambda\gg 1$ one can obtain directly from (13) the expression $T_c=0.18\sqrt{\lambda}\tilde{\Omega}$, obtained numerically in Ref. 34 and analytically by Gutfreund, Little and one of the authors;[35] if $\mu^*\neq 0$, we obtain[35] $T_c=0.18\sqrt{\lambda_{\rm eff}}\tilde{\Omega}$. The McMillan expression modified by Dynes[36] for T_c can be obtained from Eq. (12) for $\lambda\leq 1.5$ (see a detailed analysis in Ref. 32).

It is important that Eqs. (12) and (13) are valid for any λ. Let us estimate the value of $\lambda\cong\lambda_{ph}$ which can provide $T_c\cong 40$ K for La-Sr-Cu-O.

Using the value $T_c\cong 40$ K and $\tilde{\Omega}\cong 15$ meV (see above) we obtain $\lambda\cong 5$ (for $\mu^*\cong 0.1$, larger values of μ^* lead to even larger λ). Note also that if we take $\tilde{\Omega}\cong 10$ meV, it also results in a larger value of the coupling constant. Hence, the value of λ should be greater than 5 in order to provide for such a high value of T_c. Therefore, there is a need for an additional mechanism.

This conclusion brings up the question of what the additional mechanism is. We think that the additional attraction is mediated by a peculiar acoustic plasmon branch[17] with weak additional coupling λ_{pl}. A small value of λ_{pl} does not mean that the contribution to T_c is also small. In fact, it depends not only on the strength of the coupling, but on its corresponding energy scale. A noticeable contribution of plasmons to T_c arises from the large scale of the plasmon energy relative to the phonon energy. This can be seen directly from the equation[17]

$$T_c=T_c^{\rm ph}\left[\frac{\tilde{\omega}_{\rm pl}}{T_c^{\rm ph}}\right]^{\nu}\ , \tag{14}$$

where $\nu=\lambda_{pl}(\lambda_{ph}+\lambda_{pl})^{-1}$. Here T_c^{ph} is described by Eq. (12) an λ_{pl} is the electron-plasmon coupling constant and $\tilde{\omega}_{pl}\approx\langle\omega\rangle_{pl}$. The presence of this additional mechanism does not affect the value of β [see Eqs. (9) and (10)] because of the small value of $(T_c/\tilde{\omega}_{pl})^2$. One should note, however, that λ_{pl} contributes to the renormalization of γ. In other words, our procedure should be slightly modified to include the plasmon contribution in a self-consistent way. Unlike phonons, because of the inequality $T_c\ll\tilde{\omega}_{pl}$ the renormalization by plasmons is not affected at T_c. As a result, the method of evaluating λ_{ph} employed above leads to a slightly larger value of λ_{ph}. Indeed, one can obtain the expression $\lambda_{ph}=(g-1)[1+\lambda_{pl}]$, where $g=\gamma(0)/\gamma^0$. Of course, the values of λ_{ph} and λ_{pl} are independent. Using experimental values of $g\approx 2.8-3.0$ [see Eq. (11)], $\tilde{\omega}_{pl}\approx 60_{\rm meV}$, and $T_c\approx 40$ K, we obtain $\lambda_{ph}\approx 2.25-2.5$ and $\lambda_{pl}\approx 0.25$. In the absence of a plasmon contribution, we evaluated λ_{ph} without knowing T_c [see Eq. (11)] and indeed obtained the lowest limit consistent with the experimentally obtained value of g. Even this lowest limit corresponds to strong electron-phonon coupling. We note that our result is different from that of Gurvitch and Fiory.[37] Their evaluation is based on an analysis of the normal conductivity at temperatures greater than the Debye temperature. This method is applicable to conventional isotropic metals where ε_F is much larger than the Debye temperature. The treatment of cuprates which are both highly anisotropic and characterized by an extremely small Fermi energy requires special treatment. Note also that a large value of λ means a strong renormalization of the effective mass: $m^*(0)=m^b(1+\lambda)$. That is why the value of m^* obtained above (see Sec. II and Table I) is large relative to its value m^b which follows from the band-structure calculations.

B. Other parameters

Let us turn to the problem of evaluating the other major superconducting parameters of the oxides such as coherence length and H_{c2}. This evaluation can be carried out with the use of values of the normal parameters (see Sec. II).

According to our analysis, the Fermi velocity v_F is small relative to the values in conventional metals. For La-Sr-Cu-O (see Table I) $v_F\cong 8\times 10^6$ cm sec^{-1}. One can see that a such small value along with large value of T_c leads to a short coherence length. Indeed, if we use the expression $\xi_c=0.18\hbar v_F/k_BT_c$ we obtain $\xi_0\cong 25$ A. One can use a more precise definition $\xi_c\cong\hbar v_F/\pi\Delta(0)$. As is known, the energy gap $\Delta(0)$ is directly related to T_c so that $2\Delta(0)=\alpha k_BT_c$; the value of α depends on the strength of the coupling (in a weak-coupling BCS approx-

imation $\alpha=3.52$). For La-Sr-Cu-O compound the coupling constant λ is about 2.25 (see above). Such strong coupling leads to a larger value of α, which appears to be equal to about 5. Such a value α has been observed before, e.g., for $Pb_{0.7}Bi_{0.3}$ ($\lambda\cong 2$, see Ref. 22). It can also be seen from the expression obtained by Geilkman and one of the authors[31] (see also the monograph[14])

$$\alpha\cong 2\Delta(0)/\kappa_B T_c$$
$$=3.52[1+5.3(k_B T_c/\tilde{\Omega})^2\ln(\tilde{\Omega}/k_B T_c)] \quad (15)$$

for La-Sr-Cu-O the characteristic phonon frequency $\tilde{\Omega}\cong 15$ meV (see above); we obtain $\Delta(0)\cong 5k_B T_c$. One should note that many tunneling experiments (see, e.g., Ref. 38) find that for La-Sr-Cu-O, $\alpha\cong 5$. The uncertainty of some tunneling data probably is connected with an energy-gap anisotropy (see discussion below, Sec. IV). With the use of this value of $\Delta(0)$, and the values of v_F (see above) and T_c one can obtain the coherence length and we obtain $\xi_0\cong 20$ A.

This small value of the coherence length is due to a small value of the Fermi velocity and a large value of the energy gap. Such a short coherence length is a very important feature of the new materials of this feature is a unique opportunity to observe a multigap structure and energy-gap anisotropy.

Using the calculated value of the coherence length and the expression $H_{c2}=(\phi_0/2\pi\xi^2)$, one can evaluate the value of H_{c2} for La-Sr-Cu-O. Here ξ is the Ginzburg-Landau coherence length:

$$\xi_{GI}=a\xi_0[1-(T/T_c)]^{-1}\ .$$

In a weak-coupling approximation $a=0.74$ (Ref. 39) (see also the reviews in Ref. 40). Strong-coupling effects lead to an increase in a,[31] then $a\cong 0.95$ for $\lambda\cong 2$. Using this value of a and $\xi_0\cong 20$ A we obtain the value of $H_{c2}(0)\cong 90$ T. It is interesting to note that a value of $H_{c2}=88$ T was used above (see Ref. 9 and Sec. II). The fact that the calculated value of the coherence length leads to a nearly identical value of H_{c2} means a self-consistency of our approach.

A small value of the Fermi energy ε_F has also had a strong impact on the superconducting properties. The ratio $\Delta(0)/\varepsilon_F$ in conventional superconductors is small ($\sim 10^{-4}$), whereas in the high-T_c oxides, ε_F and Δ are comparable [$\Delta(0)/\varepsilon_F\leq 10^{-1}$]. A small value of the ratio $\Delta(0)/\varepsilon_F$ means that only a small number of states near the FS are involved in pairing. The picture is different in the oxides. A large value of the ratio corresponds to a situation when a significant fraction of the carriers are paired. This, of course, means a short coherence length.

The possibility of having a large value of Δ/ε_F and a short coherence length is directly related to the quasi-2D structure of the cuprates. Indeed, in conventional superconductors ($\Delta/\varepsilon_F\ll 1$) pairing can occur only near the Fermi surface (Cooper theorem). The states on the Fermi surface form a quasi-2D system in momentum space. This is an important factor because in the 2D case, any attraction leads to the formation of bound states (see, e.g., Ref. 41). The presence of the layered structure makes pairing possible even for states which are distant from the Fermi surface; this corresponds to the picture in the cuprates.

A large value of $\Delta(0)/\varepsilon_F$ leads to an unusual critical behavior near T_c. This has been predicted in Ref. 42 and observed experimentally in Ref. 43. A small value of the parameter $\Delta(0)/\varepsilon_F$ also effects the stability of the lattice; this problem will be discussed in detail elsewhere.

IV. ENERGY-GAP ANISOTROPY AND MULTIGAP STRUCTURE

Small values of the coherence length lead to the possibility of observing an energy-gap anisotropy and multigap structure. From the point of view of a Fermi-liquid approach, both these phenomena correspond to a deviation of the shape of the FS from a simple spherical form. Anisotropy of the energy gap has been observed in conventional superconductors; in particular, there are interesting data on the ultrasonic absorption (see the reviews in Ref. 6). However, the effect of anisotropy is relatively small, and the isotropic one-band model describes the properties of conventional materials with sufficient accuracy. According to Anderson's theorem[44] (see also, Ref. 45), this is connected with a large value of the coherence length. Indeed, in conventional superconductors ξ_0 is large and it leads to the inequality $\xi_0\gg l$, where l is the mean free path. The scattering by impurities and, as result, the transitions between different parts of the Fermi

The situation in the cuprates is entirely different. A small value of ξ_0 leads to the realization of the case $\xi_0<l$. As a result, one can observe the effects due to the energy-gap anisotropy. In addition, in the presence of overlapping energy bands, there are several energy gaps. The effect of anisotropy manifests itself in many phenomena. We would like to emphasize that the determination of the value of the energy gap with use of tunneling and/or infrared spectroscopy in the presence of the anisotropy and several gaps requires unconventional analysis. For example, the value obtained for 1:2:3 by an infrared bolometry technique $2\Delta(0)\cong 0.8k_B T_c$ (Ref. 46) corresponds to $\Delta_{\min}$.

The two-gap model was introduced by Suhl, Mattiass, and Walker[47] and by Moskalenko.[48] A detailed analysis of the properties of superconductors with overlapping bands was carried out by Geilikman, Zaitsev, and one of the authors[49] (see also the monograph in Ref. 14)

In the presence of two overlapping bands (the generalization to the multigap case is straightforward), each of them is characterized by its own energy gap. The self-energy parts $\Delta_i(\mathbf{p},\omega_n)$ describing the pairing are satisfied by equations

$$\Delta_i(\mathbf{p},\omega_n)$$
$$=T\sum_{l=1,2}\sum_{\omega_n}\int d\mathbf{p}'\Gamma_{il}(\mathbf{p},\mathbf{p}';\omega_n-\omega_n')F_{ll}^{+}(\mathbf{p}'\omega_n')\ . \quad (16)$$

Here $F_{ii}^{+}=-i\langle T(\psi_t^{+}\Psi_t^{+})\rangle$ is the anomalous Green's function (see, e.g., Ref. 50), and Γ_{ik} are the total vertexes describing the effective interaction between the carriers. If the pairing is caused by the conventional mechanism, i.e., by the phonon exchange, then $\Gamma_{ik}=\lambda_{ik}D$, where D is the phonon Green's function, and λ_{ik} is the corresponding coupling constant.

One should stress a very important feature of the two-gap theory. Each band has its own condensate of pairs, but it does not mean that the pairs in each band form independently, and there is no effective interband interaction. The pairs in the "a" band (the same is valid for the "b" band) are formed first due to the attraction between carriers within the band (the coupling constant λ_{aa}), and secondly, due to the transition of pairs of particles from the other band (the coupling constant λ_{ab}). In other words, λ_{ab} describes the following transition: two carriers from the "b" band make a transition caused by the phonon exchange with a formation of the pair in the "a" band. Note that $\lambda_{ik}=\tilde{\lambda}_{ik}N_k$, where N_k is the density of states in the "k" band, $\tilde{\lambda}_{ik}=\tilde{\lambda}_{ki}^{*}$. One can show[49] that the presence of the overlapping bands favors pair formation.

It is important that, contrary to the one-gap case model, the two-gap model is characterized by several coupling constants λ_{aa}, λ_{bb}, λ_{ab}, and λ_{ba} (although $\lambda_{ab}/\lambda_{ba}=N_b/N_a$; for simplicity we consider the phonon contribution to the pairing). The expression (15) correlating the ratio $\Delta(0)/T_c$ with the strength of the coupling, has been derived for the isotropic one-band model. Only in this case one can obtain one-to-one correspondence between $\Delta(0)/T_c$ and λ [see Eqs. (13) and (15)]. One can see directly from Eqs. (13) and (15) that an increase in λ leads to an increase in the ratio $\Delta(0)/T_c$. The situation is different in the multigap case. Even if $\lambda_{ik}\ll 1$, the values of the ratios Δ_i/T_c may differ noticeably from the value $(\Delta/T_c)_{\mathrm{BCS}}$. In the presence of the two-gap structure, the analysis of the tunneling and infrared data should be carried out with considerable care.

Regarding the new high-T_c oxides, one can expect an appearance of the multigap structure an energy-gap anisotropy (because of the short coherence length). This is particularly true for the materials with a more complicated band structure than La-Sr-Cu-O.

The presence of the multiband structure is manifested in a strong temperature dependence of the Hall effect; such a dependence has been observed in Y-Ba-Cu-O. Recently a similar behavior has been reported for the Bi-Sr-Ca-Cu-O system[51] and interpreted as being a direct manifestation of two types of carriers (two bands) (see also the review in Ref. 52). It is worth noting that only a weak dependence of the Hall coefficient on temperature has been observed for La-Sr-Cu-O (see above); its connected with its simple band structure. The NMR data indicate the presence of two gaps in Y-Ba-Cu-O which differ noticeably from each other.[53]

The ideal method for analyzing the multigap structure and the anisotropy is ultrasonic attenuation. It would involve carrying out the corresponding experiments. Recent progress in materials makes such an experiment possible in the near future.

V. SUMMARY

The major normal and superconducting parameters have been evaluated. The anisotropy of the system is reflected in the shape of the FS. We focus mainly on the properties of the La-Sr-Cu-O compound, although we discuss properties of other cuprates as well. We think that the analysis of this oxide is extremely important for the understanding of physics of high T_c. The main results can be summarized as follows.

(1) The values of the effective mass, Fermi energy, Fermi velocity, and momentum p_F have been calculated (Sec. II). Small values of ε_F and v_F are very important features of high-T_c oxides.

(2) The method of evaluation of the strength of the electron-phonon coupling λ is developed (Sec. III). The materials are characterized by strong coupling. It leads to strong renormalization and a large value of the effective mass (Sec. II) which greatly exceeds the band value. However, the coupling is not strong enough to provide such a high T_c. There is a need for an additional attraction, such as a plasmon mechanism.

(3) The calculated value of the coherence length (Sec. III B) appears to be small. The calculation of H_{c2} demonstrates the self-consistency of the approach.

(4) A small value of ε_F leads to a peculiar picture of the pairing when a significant part of the carriers are paired. The quasi-2D nature of the system is favorable for the pairing (Sec. III B).

(5) The small value of ξ_0 leads to a unique opportunity to observe the strong effects of anisotropy and multigap structure (Sec. IV).

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