Symmetries of the superconducting order parameters in the doped spin-liquid state

X. G. Wen*
Institute for Theoretical Physics, University of California—Santa Barbara, Santa Barbara, California 93106
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We show that the Fermi surfaces of the doped holes in the spin-liquid state consist of pairs of pockets separated by crystal momentum Q, in the low-doping limit. Due to the pocketlike structure of the Fermi surface and the A-B sublattice pairing, the spin-singlet superconducting order parameters have unusual symmetry properties. The order parameters are found to have nontrivial quantum numbers of 90° rotation and, sometimes, nonzero crystal momenta. The nonzero crystal momenta carried by the order parameters can be detected by measuring the unusual values (e.g., $hc/4e$) of the flux confined to various dislocations in superconducting thin films.

Many people believe that the spin fluctuations play an important role in high-T_c superconductors. Experiments have established that the Cu spins form a spin-liquid state in the superconducting phase. Thus it is very important to understand the properties of doped holes in the spin-liquid state in order to understand the basic properties of the high-T_c superconductors. The properties of the low-energy spin fluctuations in the undoped samples have been interpreted nicely in terms of the effective nonlinear σ model.[1]

It was first suggested in Ref. 2 that there are two kinds of holes in the [resonating-valence-bond (RVB-type)] spin-liquid state. The holes on A sublattice and B sublattice do not mix with each other, at least at low energies. Later it is shown that the interactions between the spin fluctuations and the doped holes in the spin-liquid state can be described by an effective U(1) gauge interaction in $1+2$ dimensions.[3-8] A nonlinear σ-model approach to the quantum spin-liquid state (with holes) was first suggested in Ref. 4. It was independently developed in Ref. 5 in greater details. The nonlinear σ model approach only applies to the spin-liquid state with a spin-spin correlation length much larger than the lattice spacing a. To describe the spin-liquid state with short spin-spin correlation length, a quantum dimer model[9] is developed, based on the RVB theory.[10] Surprisingly, the quantum dimer model approach[7,8] and the nonlinear σ model approach[4,5] give a very similar physical picture for the spin-liquid state, despite the fact that the mathematical appearance of the two approaches are very different. In both approaches, the A holes and B holes are found to be independent and carry opposite charges of the effective U(1) gauge field. Thus they are attractive to each other.[4,5,7,8] This attraction may be used to explain the high-T_c superconductivity. In Refs. 11 and 12, the problems associated to finite hole concentration are addressed and some interesting physical properties are derived. A discussion about quantum numbers of quasiparticles in the spin-liquid state can be found in Ref. 13.

These works are for the time-reversal and parity-symmetric spin-liquid state. The properties of time-reversal and parity-broken (chiral) spin-liquid state are discussed in Ref. 14. In this paper we are going to discuss the symmetry properties of the superconducting state in time-reversal and parity-symmetric spin-liquid state in the *low-doping* limit.

Assume at temperatures just above T_c the doped holes are described by the Fermi-liquid theory. As indicated in Ref. 5 and 15, the Fermi surfaces are small pockets in the low-doping limit. The Luttinger theorem is broken down. Due to the pocket structure of the Fermi surfaces and the A-B sublattice pairing, the superconducting state may have nontrivial symmetries.[16]

Let us first review and extend some results obtained in Refs. 5 and 13. The dynamics of holes and spin fluctuations in the continuum limit is described by the effective Lagrangian (choosing the spin-wave velocity to be 1) (Ref. 13),

$$L=\frac{1}{2g^2}(\partial_\mu \mathbf{n})^2+\eta_A^\dagger i\partial_t\eta_A+\frac{1}{2m}\eta_A^\dagger\partial_i^2\eta_A +\eta_B^\dagger i\partial_t\eta_B+\frac{1}{2m}\eta_B^\dagger\partial_i^2\eta_B \; . \tag{1}$$

The unit vector $\mathbf{n}$ in (1) is the local antiferromagnetic (AFM) order parameter that is chosen to be parallel to the spins on the A sublattice. The local AFM order parameter can be defined if the spin-spin correlation length is much larger than the lattice spacing.

If we fix $\mathbf{n}$ to be a constant, (1) can be regarded as the effective theory of the spin-density-wave (SDW) state. η_A and η_B are fields that describe A sublattice and B sublattice holes near the *top* of the valence band. η_A and η_B are related to A sublattice and B sublattice electron operators c_A and c_B according to

$$\begin{aligned}\begin{bmatrix}\eta_{A\uparrow}\\ \eta_{A\downarrow}\end{bmatrix}&=e^{-ik_0x_A}\begin{bmatrix}c_{A\uparrow}\\ c_{A\downarrow}\end{bmatrix},\\ \begin{bmatrix}\eta_{B\uparrow}\\ \eta_{B\downarrow}\end{bmatrix}&=e^{-ik_0x_B}\begin{bmatrix}c_{B\uparrow}\\ c_{B\downarrow}\end{bmatrix},\end{aligned} \tag{2}$$

where k_0 is the momentum at the top of the valence band (for a detail discussion of k_0, see Ref. 5 and 16) and $x_A(x_B)$ is the coordinate of $A(B)$ sublattice. For the holes near the *top* of the valence band, the A holes (B holes) have spin parallel (antiparallel) to $\mathbf{n}$.[16,5,13] Therefore, η_A and η_B in (1) further satisfy the following constraint:

$$\eta_A^\dagger(1-\mathbf{n}\cdot\boldsymbol{\sigma})\eta_A=0\ ,$$
$$\eta_B^\dagger(1+\mathbf{n}\cdot\boldsymbol{\sigma})\eta_B=0\ , \tag{3}$$

which enforce the $\eta_A(\eta_B)$ field to have spin parallel (antiparallel) to $\mathbf{n}$. In this paper we will allow $\mathbf{n}$ to fluctuate and study the spin-liquid state resulting from the orientation fluctuations of $\mathbf{n}$. Equation (3) describes the interaction between the spin fluctuations and the doped holes. Note that the effective Lagrangian (1) and the constraint (3) respect the following symmetry:

$$\mathbf{n}\to-\mathbf{n}\ ,$$
$$\eta_A\to\eta_B\ , \tag{4}$$
$$\eta_B\to\eta_A\ .$$

The transformation (4) (denoted as T) is associated with the transition by one lattice spacing.

When g^2 in (1) is large enough, the fluctuations of $\mathbf{n}$ destroy the long-range AFM order. In this case we may have a spin-liquid state with finite spin-spin correlation length. In the CP^1 formalism the effective Lagrangian describing the spin-liquid state is given by[5,13]

$$\begin{aligned}L_{\text{eff}}=&\frac{1}{2g^2}|(\partial_\mu-ia_\mu)z|^2-\lambda z^\dagger z\\&+\frac{1}{2m}\chi_A^\dagger(\partial_i-ia_i)\chi_A+i\chi_A^\dagger(\partial_t-ia_0)\chi_A\\&+\frac{1}{2m}\chi_B^\dagger(\partial_i+ia_i)\chi_B+i\chi_B^\dagger(\partial_t+ia_0)\chi_B\\&+\frac{1}{2\gamma M}(f_{\mu\nu})^2\end{aligned} \tag{5}$$

where

$$\mathbf{n}\sim z^\dagger\boldsymbol{\sigma}z\ ,$$
$$\begin{bmatrix}\eta_{A\uparrow}\\ \eta_{A\downarrow}\end{bmatrix}\sim\begin{bmatrix}z_1\\ z_2\end{bmatrix}\chi_A\ , \tag{6}$$
$$\begin{bmatrix}\eta_{B\uparrow}\\ \eta_{B\downarrow}\end{bmatrix}\sim\sigma^2\begin{bmatrix}z_1^*\\ z_2^*\end{bmatrix}\chi_B\ ,$$

and a_μ is the effective U(1) gauge field. χ_A and χ_B is the operator for the holes in A and B sublattice, respectively. They carry the same electric charge e and opposite z charge—the charge for the effective gauge field a_μ. χ_A and χ_B are spin singlets. z describes the spin fluctuations, which carries spin $\frac{1}{2}$ and zero electric charge.

Since the U(1) gauge field in $1+2$ dimensions is confining, the physical quasiparticles (with finite energies) are z-charge neutral bound states: χ_A-z, χ_B-z^*, χ_A-χ_B, and $z^\dagger$-z. χ_A-z and χ_B-z^* bound states correspond to dressed electrons (holes), $z^\dagger$-z correspond to the spin wave (with spin 1), and χ_A-χ_B corresponds to the electron pair. A single z quantum (or χ quantum) carries a nonzero z charge and creates a long-range a_μ field, which results in an infinity energy. Thus z and χ fields alone do not correspond to the physical quasiparticles.

The transformation T when acting on the new fields takes the form:

$$z\to\sigma^2z^*,\ z^*\to-\sigma^2z\ ,$$
$$\chi_A\to\chi_B,\ \chi_B\to-\chi_A\ , \tag{7}$$
$$a_\mu\to-a_\mu\ .$$

Using (6), one can check that (7) reproduces (4). Note that according to (7) we have $T^2=(-1)^q$ where q is the z-charge operator. This seems inconsistent with (4). However, z and χ are not physical operators, i.e., they do not generate physical states. When restricted to the physical states (which have $q=0$) we do have $T^2=1$. Therefore all the physical states have $T=\pm1$. We would like to remark that (7) is not the most general realization of T on the new fields. Since z and χ are not physical fields, we may include some phase factors on the right-hand side of (7). Those phase factors do not change T when restricted in the physical subspace (with zero z charge). Notice that (5) is invariant under (7), which implies that the spin-liquid state respects the translation symmetry (by one lattice spacing).

Now let us discuss the T quantum numbers for various physical quasiparticles. The spin-singlet z-z^* bound state is described by $z^\dagger z$, which transforms as $z^\dagger z\xrightarrow{T}z^\dagger z$ under T. Thus $z^\dagger z$ is even under T. The spin triplet z-z^* bound state is given by $z^\dagger\boldsymbol{\sigma}z$ which transforms as $z^\dagger\boldsymbol{\sigma}z\xrightarrow{T}-z^\dagger\boldsymbol{\sigma}z$. Thus $z^\dagger\boldsymbol{\sigma}z$ is odd under T. Similarly we find that $z\chi_A\pm\sigma^2z^*\chi_B$ have $T=\pm1$, respectively, and $\chi_A\chi_B$ have $T=+1$ (note χ_A and χ_B anticommute). Because T is related to the translation by one lattice spacing, the preceding results can be translated into the crystal momenta carried by the quasiparticles:

$$\begin{aligned}&z^\dagger z:\ k=0\ ,\\&z^\dagger\boldsymbol{\sigma}z:\ k=Q\ ,\\&\chi_A\chi_B:\ k=2k_0\ ,\\&z\chi_A^\dagger+\sigma^2z^*\chi_B:\ k=k_0\ ,\\&z\chi_A-\sigma^2z^*\chi_B:\ k=k_0+Q\ ,\end{aligned} \tag{8}$$

where $Q=(\pi/a,\pi/a)$. The shift of k_0 comes from (2).

At the low-doping limit the fermionic quasiparticles in the spin-liquid state are χ-z bound states. According to (8) they carry crystal momenta near k_0 and k_0+Q at low energies. If the holes are described by the Fermi-liquid theory, they form two small Fermi surfaces[5] around k_0 and k_0+Q.

In realistic models[16] the top of the valence band, some times, may appear at $k_0=(\pi/2a,0)$, in which case the Fermi surfaces (in the spin-liquid state) are two pockets near $(\pi/2a,0)$ and $(0,\pi/2a)$ [Fig. 1(a)]. For different coupling constants, the top of the valence band may appear at $k_0=(\pi/2a,\pi/2a)$ and $k_0=(-\pi/2a,\pi/2a)$ as well.[16] In this case the Fermi surfaces are four pockets near $(\pi/2a,\pi/2a)$, $(\pi/2a,-\pi/2a)$, $(-\pi/2a,\pi/2a)$, and $(-\pi/2a,-\pi/2a)$ [Fig. 1(b)]. In general, the Fermi surfaces always appear in pairs separated by momentum Q. This is another way to say that the A holes and the B holes are independent (i.e., they do not mix with each other). We would like to emphasize that the holes here carry spin $\frac{1}{2}$. They are not the holons χ_A and χ_B which carry spin 0. Our results are not about the nonmixing of the unphysical particles χ_A and χ_B. What we have shown is the nonmixing of the physical quasiparticles $z\chi_A$ and $\sigma^2 z^* \chi_B$. This is different from the nonmixing of the A holons and the B holons suggested in Ref. 2.

Now let us assume that the superconductivity is due to the pairing induced by the spin (orientation) fluctuations in the spin-liquid state, i.e., by the attractive U(1) gauge interaction between χ_A and χ_B. In this case the superconductivity can be regarded as the boson condensation of the $\chi_A\chi_B$ pairs, which at least gives the correct symmetry properties of the superconducting state. The preceding assumption also implies that the superconducting state must be spin singlet ($\chi_A\chi_B$ carriers zero spin). The spin triplet superconducting states are very unlikely because they must arise from the pairing between $z\chi_A$ and $\sigma^2 z^* \chi_B$. The spin (orientation) fluctuations induce little attractions between those z-charge neutral bound states.

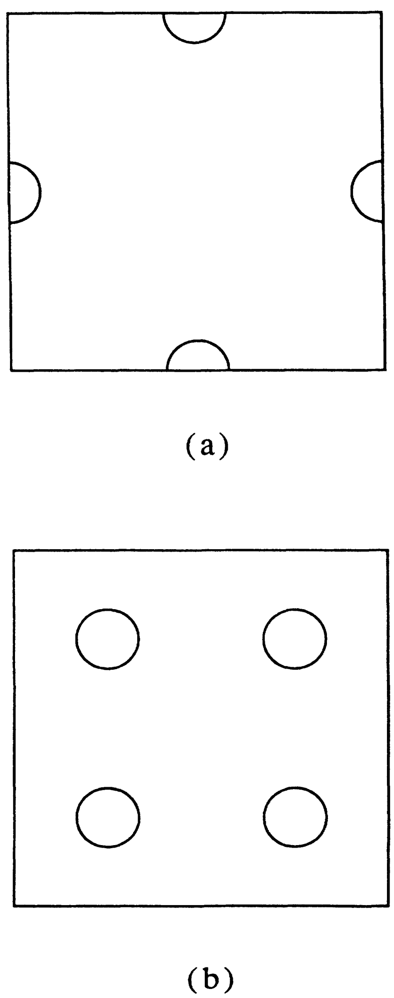


FIG. 1. The small circles represent the Fermi surfaces when the top of the valence band is (a) at $k_0=(\pi/a,0)$ (b) at $k_0=(\pi/2a,\pi/2a)$ and $k_0=(-\pi/2a,\pi/2a)$.

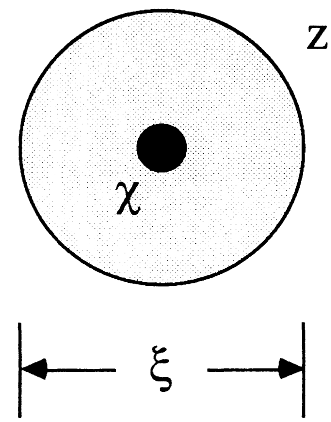


FIG. 2. A quasiparticle in the spin-liquid state can be regarded as a χ quantum dressed by a cloud of a z quantum.

For the Fermi surfaces described in [Fig. 1(a)] $k_0=(\pi/a,0)$ and $\chi_A\chi_B$ carry zero crystal momentum. Under 90° rotation (around a site in A sublattice) $k_0 \to (0,\pi/a)=k_0+Q$. From (2) we find that

$$\begin{aligned} &\eta_A \to \eta_A,\ \chi_A \to \chi_A \\ &\eta_B \to -\eta_B,\ \chi_B \to -\chi_B \end{aligned} \tag{9}$$

under 90° rotation. Therefore $\chi_A\chi_B$ is odd under 90° rotation. The condensation of $\chi_A\chi_B$ leads to a *nodeless d*-wave spin singlet superconducting state.

We can also derive the preceding symmetry properties from the instability of the Fermi surfaces[16] in Fig. 1(a). Let us first discuss the interactions between the doped holes based on the effective theory (5). We may view the doped hole as a χ particle dressed by a cloud of z particles.[5] We may view the doped hole as a χ particle dressed by a cloud of z particles.[5] The quasiparticle (the χ-z bound state) has a size of order spin-spin correlation length ξ (Fig. 2). The potential between two z charges q_1

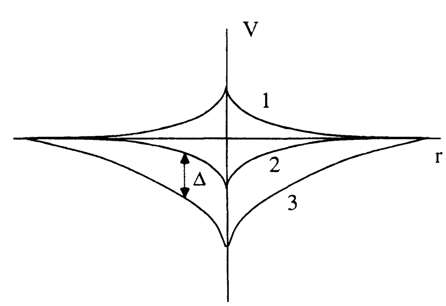


FIG. 3. The pairing potentials between two doped holes, (1) when the two holes on the same sublattice, (2) when the two holes on the different sublattices and in the spin-triplet state, (3) on the different sublattices and in the spin-singlet state.

and q_2 induced by the U(1) gauge field a_μ is given by

$$-q_1 q_2 \frac{\gamma M}{4\pi} \ln|r_1 - r_2| + \text{const} .$$

The interaction between two doped holes can be readily calculated and the results are sketched in Fig. 3. The interaction between an A hole and a B hole in spin-singlet channel has to be treated separately. In this case the two z quanta bounded to χ_A and χ_B may annihilate. This will reduce the energy of the hole pair by an amount of order spin gap Δ. Therefore, there is a strong attraction between an A hole and a B hole in the spin-singlet channel as plotted in Fig. 3. For simplicity, we will ignore all other interactions except the interaction between A holes and B holes in the spin-singlet channel. Mathematically we assume that the potential between the holes in the spin-singlet channel take the form

$$V(i-j) = [(-1)^{i-j} - 1] f(i-j) , \tag{10}$$

where $f(i)$ is a positive smooth function, and the potential to be zero in the spin-triplet channel. In the momentum space the interaction Hamiltonian is

$$\begin{aligned} H_I &= \frac{1}{2} \sum_{q,k_1,k_2} [C^\dagger_{k_1} C_{q+k_1} V(q) C^\dagger_{k_2} C_{-q+k_2} - C^\dagger_{k_1} \sigma C_{q+k_1} \cdot V(q) C^\dagger_{k_2} \sigma C_{-q+k_2}] \\ &= - \sum_{q,k_1,k_2} (C^\dagger_{k_1} i\sigma^2 C^\dagger_{k_2} V(q) (C_{q+k_1} i\sigma^2 C_{-q+k_2}) , \end{aligned} \tag{11}$$

where C_i is the quasiparticle operator

$$C_i = \begin{cases} z\chi_A , & \text{if } i \text{ is in } A \text{ sublattice} , \\ \sigma^2 z^* \chi_B , & \text{if } i \text{ is in } B \text{ sublattice} \end{cases}$$

From (10) we see that $V(q)$ has a negative peak near $q=0$, a positive peak near $q=Q$ and is zero elsewhere (see Fig. 4). From the symmetry of the Fermi surface [Fig. 1(a)], we can see that if k lies near a Fermi surface, $-k$ also lies near a Fermi surface. Therefore we may consider a pairing between C_k and C_{-k}. Such a pairing in the spin-singlet channel is described by the superconducting order parameter

$$\langle C_{k\uparrow} C_{-k\downarrow} - C_{k\downarrow} C_{-k\uparrow} \rangle = \gamma(k) = \gamma(-k) . \tag{12}$$

Substituting (12) into (11) we find that the mean-field Hamiltonian of superconducting state takes a form

$$H_I = - \sum_{q,k} [\gamma(k) V(q) (C_{q+k\uparrow} C_{-q-k\downarrow} - C_{q+k\downarrow} C_{-q-k\uparrow}) + \text{H.c.}] . \tag{13}$$

Such a mean-field Hamiltonian leads to a gap equation

$$\Delta_k = \tfrac{1}{2} \sum_q \frac{-V(q)\Delta_{k+q}}{(E^2_{k+q} + \Delta^2_{k+q})^{1/2}} \tag{14}$$

where

$$\Delta_k = 2\gamma(k)(E^2_k + \Delta^2_k)^{1/2}$$

The gap equation can be easily solved in the low-doping limit. We find a solution satisfying $\gamma(k) = -\gamma(k+Q)$.[16] $\gamma(k)$ is nonzero only near $k=(\pi/a,0)$ and $k=(0,\pi/a)$ and is plotted in Fig. 5. The superconducting state has a d-wave symmetry.

Notice the interaction (11) does not couple to the spin-triplet superconducting order parameter $\langle C_{k\uparrow} C_{-k\downarrow} + C_{k\downarrow} C_{-k\uparrow} \rangle$. Therefore H_I can never induce a spin-triplet superconducting state.

For the Fermi surfaces in Fig. 1(b) we have two families of the hole operators, χ_A and χ_B arising from $k_0 = (\pi/2a, \pi/2a)$ and χ'_A and χ'_B arising from $k'_0 = (-\pi/2a, \pi/2a)$. Both the bound states $\chi_A\chi_B$ and $\chi'_A\chi'_B$ have a crystal momentum Q. The condensation of $\chi_A\chi_B$ and $\chi'_A\chi'_B$ lead to an unusual *nodeless* spin-singlet

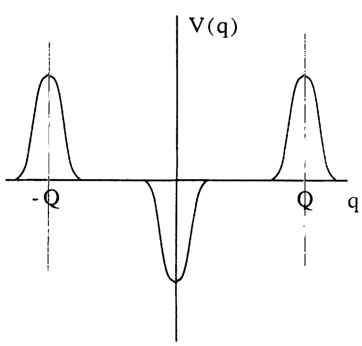


FIG. 4. The pairing potential $V(q)$ is plotted in the momentum space.

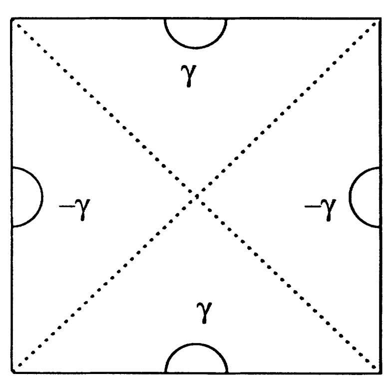


FIG. 5. The solution of the gap equation, $\gamma(k)$, is plotted. $\gamma(k)$ is peaked near the small circles. $\gamma(k)=0$ on the dotted lines.

superconducting state. Although the superconducting order parameter carries nonzero crystal momentum, I do not want to say the translation by one lattice spacing is broken in the superconducting state. This is because under such a translation the superconducting order parameter just changes sign. Therefore the order parameter is invariant under the combined transformation of the translation and the gauge transformation. The superconducting state does not carry supercurrent despite the nonzero crystal momentum carried by the order parameter. The simplest way to see this is to notice that the state respects 180° rotation symmetry. The real characteristic physical effect of such an order parameter is that a dislocation in $\hat{\mathbf{x}}$ or $\hat{\mathbf{y}}$ direction (Fig. 6) has to bound with $hc/4e$ flux. This result can be understood by noticing that the superconducting order parameter takes opposite sign on the A sublattice and the B sublattice [Fig. 7(a)]. The dislocation described in Fig. 6 creates frustrations in such a superconducting state [Fig. 7(b)], which costs infinite energy. However, the frustration can be removed by binding $hc/4e$ flux to the dislocation (notice that the superconducting order parameter carries charge $2e$). Therefore if the superconducting order parameter carries a crystal momentum Q, the dislocation in the superconducting state is confined with $hc/4e$ flux. We would like to emphasize that the flux quantum in the superconducting state is still $hc/2e$. The $hc/4e$ flux can only appear (and must appear) with the dislocations.

The two order parameters $\chi_A\chi_B$ and $\chi'_A\chi'_B$ are related by 90° rotation. Under 90° rotation

$$\begin{aligned} &k_0 \to k'_0, \quad k'_0 \to k_0 + Q , \\ &\chi_A \to \chi'_A, \quad \chi_B \to \chi'_B , \\ &\chi'_A \to \chi_A, \quad \chi'_B \to -\chi_B , \end{aligned} \tag{15}$$

$$\begin{bmatrix} \chi_A\chi_B \\ \chi'_A\chi'_B \end{bmatrix} \to \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} \chi_A\chi_B \\ \chi'_A\chi'_B \end{bmatrix} . \tag{16}$$

The order parameters are odd under 180° rotation and correspond to p-wave pairing. Notice that the order parameters are spin singlet despite their p-wave symmetry. This is possible because the order parameter carry nonzero crystal momentum.

Again the preceding superconducting state can be obtained through the instability of the Fermi surfaces in Fig. 1(b). The interaction between the holes is still given by (11). Let us first consider the spin-singlet pairing between C_k and C_{-k} [see (12)]. The gap equation for such a superconducting state is given by (14), however, now E_k give rise to the Fermi surfaces in Fig. 1(b). In the weak coupling limit, Δ_k is nonzero only near the Fermi surfaces, in our case, near $k=(\pm\pi/2a,\pm\pi/2a)$. Because $\Delta(\pi/2a,\pi/2a)=\Delta(\pi/2a,-\pi/2a)$ as implied by (12), one can easily check that the gap equation (14) only supports a zero solution $\Delta_k=0$, at least in low-doping limit. This is because the pairing potential V_q (Fig. 4) requires

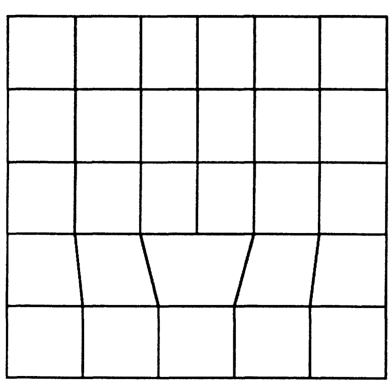
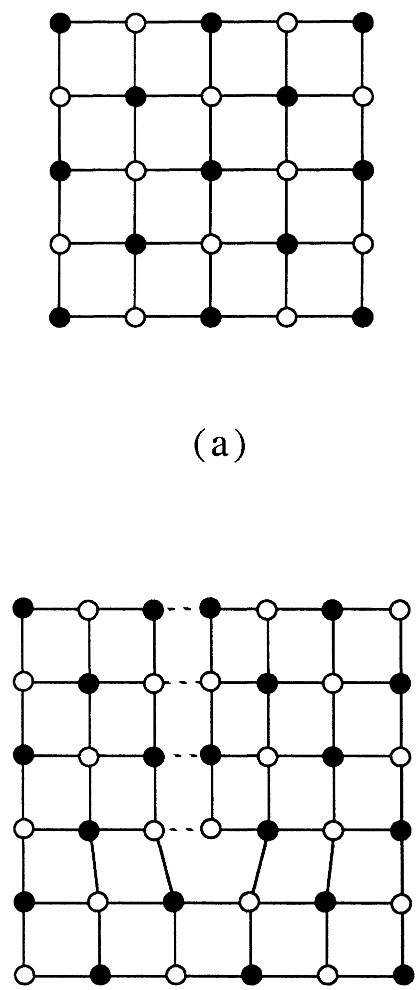


FIG. 6. A dislocation in $\hat{\mathbf{x}}$ direction.



(a)

(b)

FIG. 7. (a) The superconducting order parameter carrying a crystal momentum Q has opposite signs on the two sublattices. ○ represents a positive value of the order parameter and ● represents a negative value of the order parameter. (b) A dislocation introduces frustrations (represented by the dotted lines) in the superconducting order parameter. The frustrations (mismatches of a minus sign) can be removed by adding $hc/4e$ flux to the dislocation.

the superconducting gap to satisfy

$$\Delta_{(\pi/2a,\pi/2a)}=-\Delta_{(\pi/2a,\pi/2a)+Q}=-\Delta_{(-\pi/2a,-\pi/2a)} .$$

However, notice that if $(\pi/2a,\pi/2a)+k$ is near a Fermi surface in Fig. 1(b), $(\pi/2a,\pi/2a)-k$ is also near a Fermi surface, at least in the low-doping limit. Let us denote $q_1=(\pi/2a,\pi/2a)$. We may consider the superconducting state arised from the pairing between C_{q_1+k} and C_{q_1-k}. The spin-singlet order parameter is given by

$$\langle C_{q_1+k\uparrow}C_{q_1-k\downarrow}-C_{q_1+k\downarrow}C_{q_1-k\uparrow}\rangle=\gamma_1(k)=\gamma_1(-k) . \tag{17}$$

The Bardeen-Cooper Schrieffer (BCS) mean-field Hamiltonian for the order parameter in (17) is given by

$$H_I=-\sum_{q,k}[\gamma(k)V(q)(C_{q+k+q_1\uparrow}C_{q-k+q_1\downarrow}-C_{q+k+q_1\downarrow}C_{q-k+q_1\uparrow})+\text{H.c.}] . \tag{18}$$

Equation (18) is equivalent to the usual BCS mean-field Hamiltonian except the momenta of the electron operators are shifted by q_1. Equation (18) results in the following gap equation:

$$\Delta_{1,k}=\tfrac{1}{2}\sum_q \frac{-\Delta_{1,k+q}V(q)}{(E^2_{k+q+q_1}+\Delta^2_{1,k+q})^{1/2}} \tag{19}$$

where

$$\Delta_{1,k}=2\gamma_1(k)(E^2_{k+q_1}+\Delta^2_{1,k})^{1/2}$$

The "Fermi surface" of E_{k+q_1} is plotted in Fig. 8. $\gamma_1(k)$ is nonzero only near $k=0$, Q, $(\pm\pi/a,0)$, and $(0,\pm\pi/a)$. The pairing potential $V(q)$ favors a superconducting order parameter that satisfies $\gamma_1(k)=-\gamma_1(k+Q)$. Such a solution is parametrized by two complex numbers γ and γ'. $\gamma_1(k)$ is represented in Fig. 8. Notice that the order parameter γ_1 carries a crystal momentum Q. γ and γ' correspond to the order parameters $\chi_A\chi_B$ and $\chi'_A\chi'_B$.

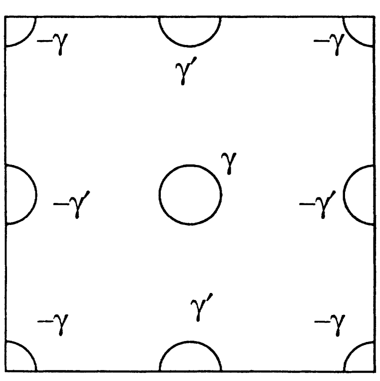


FIG. 8. The order parameter $\gamma_1(k)$ is plotted. $\gamma_1(k)$ is parametrized by two independent complex numbers, $\gamma_1(0)=\gamma$ and $\gamma_1(0,\pi/a)=\gamma'$.

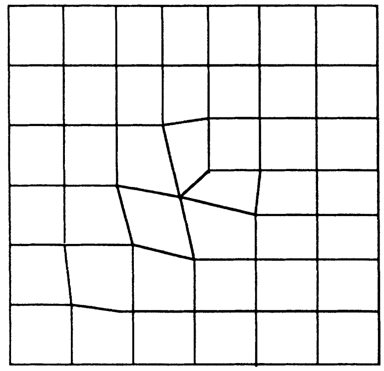


FIG. 9. A dislocation in $\hat{\mathbf{x}}+\hat{\mathbf{y}}$ direction.

Another possible superconducting state for the Fermi surfaces in Fig. 1(b) comes from the condensation of $\chi_A\chi'_B+\chi_B\chi'_A$ and $\chi_A\chi'_B-\chi_B\chi'_A$ which carry crystal momentum $(\pi/a,0)$ and $(0,\pi/a)$, respectively. Again the two order parameters are related by 90° rotation

$$\begin{bmatrix}\chi_A\chi'_B+\chi_B\chi'_A\\ \chi_A\chi_B-\chi_B\chi'_A\end{bmatrix}\to\begin{bmatrix}0 & -1\\ -1 & 0\end{bmatrix}\begin{bmatrix}\chi_A\chi'_B+\chi_B\chi'_A\\ \chi_A\chi'_B-\chi_B\chi'_A\end{bmatrix} . \tag{20}$$

The detailed properties of the superconducting state depend on how the two order parameters condense, which is determined by the potential energy term in the effective Ginzburg-Landau (GL) theory. However, in general, dislocations in certain directions are bounded with $hc/4e$ flux. For example, in the superconducting state characterized by $\chi_A\chi'_B=\chi_B\chi'_A\neq 0$ the dislocations in $\hat{\mathbf{y}}$ direction are bounded with $hc/4e$ flux. While in the superconducting state given by $\chi_A\chi'_B\neq 0$ and $\chi_B\chi'_A=0$, the dislocations in $\hat{\mathbf{x}}\pm\hat{\mathbf{y}}$ directions (Fig. 9) are bounded with $hc/4e$ flux.

In terms of C_i operator, the preceding superconducting state are described by the order parameters

$$\begin{aligned}&\langle C_{q_2+k\uparrow}C_{q_2-k\downarrow}-C_{q_2+k\downarrow}C_{q_2-k\uparrow}\rangle=\gamma_2(k) ,\\ &\langle C_{q_3+k\uparrow}C_{q_3-k\downarrow}-C_{q_3+k\downarrow}C_{q_3-k\uparrow}\rangle=\gamma_3(k) ,\end{aligned} \tag{21}$$

where $q_2=(\pi/2a,0)$ and $q_3=(0,\pi/2a)$. The solutions γ_2 and γ_3 of the gap equation are plotted in Fig. 10, they correspond to the two order parameters $\chi_A\chi'_B+\chi_B\chi'_A$ and $\chi_A\chi'_B-\chi_B\chi'_A$.

In general, a dislocation in two dimensions is characterized by the Burgers vector $\mathbf{b}=n_x\hat{\mathbf{x}}+n_y\hat{\mathbf{y}}$. If the superconducting order parameter carries a crystal momentum $\mathbf{K}$, the dislocation in the superconducting state must carry the magnetic flux given by

$$\Phi=(\mathbf{K}\cdot\mathbf{b}+2\pi n)\frac{\hbar c}{2e} , \tag{22}$$

where n is an integer. This result can be obtained by a similar argument presented in Fig. 7.

In the above we have considered spin-singlet superconducting states in the doped spin-liquid states. The dynamics of the effective theory (5) favors spin-singlet pairing. This is because the spin-singlet pairing corresponds

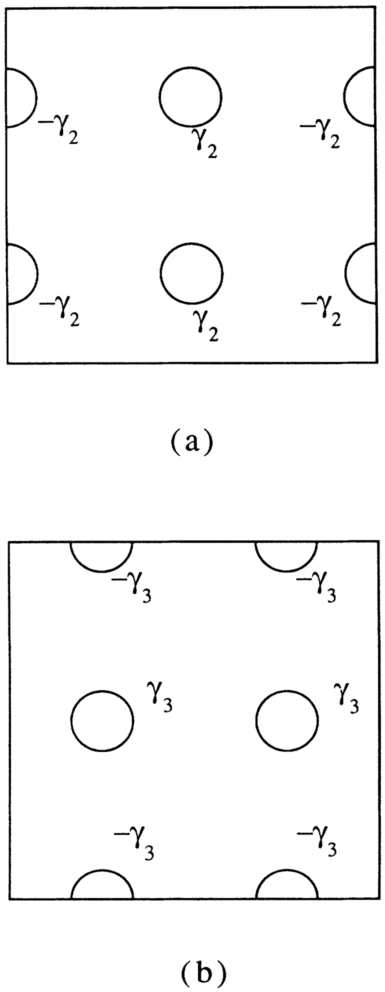


FIG. 10. The order parameters $\gamma_2(k)$ and $\gamma_3(k)$ are plotted in (a) and (b), respectively.

to the pairing between χ_A and χ_B, which we find to have a strong attractive interaction between them. The spin-triplet pairing is not favored by the spin-liquid state. For the Fermi surfaces in Fig. 1(a), the fact that the pairing is between holes in different sublattices and the pairing is spin singlet determines the superconducting state to be a *nodeless* d-wave superconducting state. For the Fermi surface in Fig. 1(b) the superconducting states remain to be *nodeless*. However, the A-B sublattice pairing and spin singlet conditions require the superconducting order parameters to carry nonzero crystal momenta. In this case, some dislocations must carry $hc/4e$ flux. Because the pocketlike structure of the Fermi surfaces and the A-B sublattice pairing, the superconducting order parameters in general form nontrivial representations of 90° rotation, at least in the low-doping limit.

The superconducting states in the spin-liquid state discussed in this paper are closely related to the superconducting states in SDW state discussed in Ref. 16. For example, the d-wave superconducting order parameter for the Fermi surfaces in Fig. 1(a) is just the projection of the d-wave superconducting order parameter in Ref. 16 to the spin-singlet and the zero crystal momentum tensor. The p-wave superconducting order parameters with crystal momentum Q for the Fermi surfaces in Fig. 1(b) is the projection of the p-wave superconducting order parameter in Ref. 16 to the spin singlet and the crystal momentum Q sector. (Note the superconducting order parameters in SDW state are a mixture of spin singlet and triplet, as well as 0 and Q crystal momenta.)

In this paper we discussed the shape of the Fermi surfaces of the doped holes in the spin-liquid state, based on the effective theory obtained in Refs. 5 and 13. The Fermi surfaces are found to consist of pairs of pockets separated by crystal momentum Q. We studied the symmetry properties of the spin-singlet superconducting state for the Fermi surfaces in Fig. 1(a) and Fig. 1(b). The superconducting order parameters are found to carry nontrivial 90° rotation quantum numbers, and in some cases nonzero crystal momenta. The nonzero crystal momenta carried by the order parameters can be measured by measuring the flux confined to dislocations in certain directions. It would be very interesting to determine experimentally whether an unusual flux quantum is bounded to a dislocation or not.

From the preceding discussions we see that the superconducting order parameters in the spin-liquid state of the high-T_c superconductors may have nontrivial symmetry properties, at least in the low-doping limit. It is very important to experimentally test those predictions in high-T_c superconductors.

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*Present address: School of Natural Science, Institute for Advanced Study, Princeton, NJ 08540.

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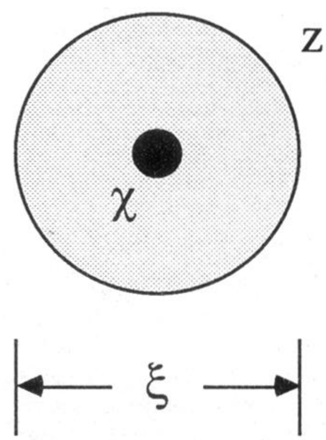

FIG. 2. A quasiparticle in the spin-liquid state can be regarded as a χ quantum dressed by a cloud of a z quantum.