Enhanced backscattering of electrons in a magnetic field

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We calculate the exact shape of the enhanced coherent backscattering peak for electrons in the presence of an external magnetic field. The interference phenomena that cause the backscattered enhancement are reduced due to the breaking of time-reversal symmetry. It is shown that the form of the peak in the presence of a magnetics field $I(q,H)$ can be obtained (to a good approximation) from $I(q,H=0)$ by replacing q with $\tilde{q}=[q^2+(3L_H^2)^{-1}]^{1/2}$, where $L_H=(2\hbar c/eH)^{1/2}$. We have also calculated $I(q,H)$ at finite temperatures and proposed it as the most sensitive tool for extracting inelastic processes.

Recently, a renewed interest in the backscattering of optical waves from random systems led to the discovery[1–5] of a narrow coherent backscattering peak of width $\sim\lambda/2\pi l$, where λ is the wavelength and l the transport elastic mean free path. This phenomenon is caused by interference between each optical trajectory with its time-reversal trajectory. The narrow shape of the peak is broadened when the long trajectories are cut off either by the finiteness of the sample[6–8] or by injecting an absorbing dye[9,10] that reduces the existing probability of long trajectories. The remaining shorter trajectories still obey time-reversal symmetry, and the ratio between the enhanced backscattered peak and the background remains 2. In order to reduce this ratio one needs to break time-reversal symmetry. It was recently proposed[11] that a magnetic field may cause such a mechanism in certain magneto-optical systems due to the Faraday effect. In general, a magnetic field couples very weakly with the electromagnetic waves and the Faraday effect is quite small.

In this paper, we study the effect of a magnetic field on backscattering of *electrons* from random media. It was recently proposed that the enhanced coherent backscattered peak is a general phenomenon and may be observed for backscattering of neutrons[12] or electrons[13] at low temperatures, where inelastic scattering is frozen out. Electrons, of course, couple strongly to an applied magnetic field, which breaks the time-reversal symmetry. This breaking symmetry reduces all the interference phenomena, and in particular the enhancement ratio of the backscattering peak will be reduced below 2. The effect of a magnetic field on the coherent backscattered peak resembles to some extent the effect of a magnetic field on the electrical conductivity, which results[14–18] in a *negative* magnetoresistence. It should, however, be noted that the coherent backscattering peak is a *first*-order effect in the sense that its width is proportional to λ/l, whereas the negative magnetoresistance effect is much smaller being proportional to $(\lambda/l)^2$.

The magnetoresistance measurements were widely used[17–19] to study interference phenomena in general and to extract inelastic scattering times in particular.[17–19] (For a review of precise measurements of inelastic lengths see Arnov and Sharvin.[20]) We calculate here the exact shape of the coherent backscattering of electrons $I(q,H)$ in the presence of an external magnetic field and the presence of inelastic scattering processes. We suggest that measuring $I(q,H)$ at different temperatures may serve as an important tool in extracting inelastic scattering times. We use two independent methods. The first is an approximate real-space approach that reveals the physical insight of the problem. The second approach is a rigorous diagrammatic calculation. We show that the real-space approach is in excellent agreement with the rigorous approach.

We first use the real-space approach to calculate the angular dependence of the electron backscattered intensity by using the formulation of Kaveh *et al.*[5] for the backscattered intensity.

$$I=\sum_{i,j,l,m} A_{ij}A_{lm}\exp\{i[\mathbf{k}_i\cdot(\mathbf{r}_i-\mathbf{r}_l)+\mathbf{k}_f\cdot(\mathbf{r}_m-\mathbf{r}_j)]\}\exp[i(\phi_{ij}-\phi_{lm})]\ , \tag{1}$$

where $\mathbf{k}_i$ and $\mathbf{k}_f$ are the initial and final electron wave vectors correspondingly, A_{ij} is the amplitude of the electron wave, which performs a trajectory that started at position $\mathbf{r}_i$ and is emitted from the material at the final position $\mathbf{r}_j$, and similarly for A_{lm}. θ_{ij} is the phase acquired by a trajectory that started at r_i and ended at r_j, and similarly for θ_{lm}.

We now include the effect of a magnetic field. For weak magnetic fields we may assume that the electron motion still remains diffusive. This means that A_{ij} and

A_{lm} in Eq. (1) remain independent of the magnetic field. The magnetic field, however, changes the phases θ_{ij} and θ_{lm},

$$\phi_{ij}=\phi_{ij}^0+\phi_{ij}^1(H) , \tag{2}$$

where θ_{ij}^0 is the phase acquired in the absence of a magnetic field and $\theta_{ij}^1(H)$ is the phase caused by the magnetic field

$$\phi_{ij}^1(H)=(ec/\hbar)\int_{\mathbf{r}_i}^{\mathbf{r}_j}\mathbf{A}\cdot d\mathbf{l} , \tag{3}$$

where $\mathbf{A}$ is the vector potential. The magnetic field breaks the time-reversal symmetry and for a given trajectory

$$\phi_{ij}^1(H)=-\phi_{ji}^1(H) . \tag{4}$$

We now calculate the ensemble averaged intensity from Eq. (1) and get,

$$I(\mathbf{q},H)=2\sum_{i>j}|A_{ij}|^2(1+\exp\{-2\langle[\phi_{ij}^1(H)]^2\rangle\}\cos[\mathbf{q}\cdot(\mathbf{r}_i-\mathbf{r}_j)]) , \tag{5}$$

where $\mathbf{q}=\mathbf{k}_i+\mathbf{k}_f$. Only diagonal terms survive the ensemble average. Note that θ_{ij}^0 cancels out in (5) because of the time-reversal property $\theta_{ij}^0=\theta_{ji}^0$. The phase θ_{ij}^1, which does not preserve time-reversal symmetry due to (4), does *not* cancel out. This introduces a new factor in (5) that depends explicitly on the magnetic field. Using Eq. (4) we get for the reduction factor in (5)

$$\langle\exp[i(\theta_{ij}^1-\theta_{ji}^1)]\rangle=\exp[-2\langle(\theta_{ij}^1)^2\rangle] .$$

When $H\rightarrow 0$, Eq. (5) coincides with the expression for $I(q,H=0)$ for the coherent backscattering peak in the absence of a magnetic field. The new procedure here is to calculate the reduction factor $\exp[-2\langle(\theta_{ij}^1)^2\rangle]$. We now calculate $\langle(\theta_{ij}^1)^2\rangle$ by using the diffusive nature of the electron motion in which the averaged square of the distance between $\mathbf{r}_i$ and $\mathbf{r}_j$ is Dt, where $D=\frac{1}{3}vl$ is the electron diffusion constant (v is its velocity) and t is the average time needed to travel between the two points. By using Eq. (3) it can be shown[20] that

$$\langle(\phi_{ij}^1)^2\rangle=2(Dt)(e/\hbar c)^2\langle A^2\rangle . \tag{6}$$

For an applied magnetic field in the x direction $\mathbf{A}=Hy\hat{\mathbf{z}}$ and $\langle A^2\rangle=H^2\langle y^2\rangle$. We have to calculate the expectation value of y^2 by using the electron wave functions in the presence of a magnetics field. We approximate $\langle y^2\rangle$ by taking the ground-state wave functions for the electron that decays exponentially with the Landau magnetic length $L_H=(2\hbar c/eH)^{1/2}$. Thus, we use $\langle y^2\rangle=(\frac{1}{3})L_H^2$. Inserting this result in Eq. (6) we get

$$\langle(\phi_{ij}^1)^2\rangle=\tfrac{1}{3}Dt/L_H^2 . \tag{7}$$

Defining a magnetics cutoff time $\tau_H=3L_H^2/D$, we get $\langle\exp(i\theta_{ij})\rangle=\exp(-t/\tau_H)$. Inserting this result in (5) and using the random walk probability with the appropriate boundary conditions[21] for $|A_{ij}|^2$ we get

$$I=I_1\int\int\int e^{-z/\mu_i l}e^{-z'/\mu_f l}[1+e^{-t/\tau_H}\cos(\mathbf{q}\cdot\mathbf{R})]\rho(\mathbf{r},\mathbf{r}',t)dt\,d^2R\,dz\,dz' , \tag{8}$$

where $\mu_i=\cos\theta_i$ and $\mu_f=\cos\theta_f$ and

$$\rho(\mathbf{r},\mathbf{r}',t)=\frac{1}{(4\pi Dt)^{3/2}}\exp(-R^2/4Dt)\{\exp[-(z-z')^2/4Dt]\exp[-(z+z'+2z_0)^2/4Dt]\} \tag{9}$$

with $Z_0=0.7104l$.

We thus see that trajectories longer than L_H contribute only to the background intensity but *not* to the second interference term. This reduces the enhancement factor below 2. As the magnetic field increases, L_H decreases and the enhanced peak is reduced. From the reduction factor $\exp(-t/\tau_H)$ we find that the peak becomes rounded and reduced for $qL_H<1$. For $qL_H>1$ the form of the peak is hardly affected. Thus, L_H acts as a cutoff length only for the interference between time-reversal trajectories.

Inserting (9) in (8) and taking $\mu_i=\mu_f\approx 1$, we get,

$$I(q,H)=I_0(1+2z_0/l)^{-1}\left[1+\frac{2z_0}{l}+\{1+[q^2+(D\tau_H)^{-1}]^{1/2}\}^{-2}\left[\frac{1-\exp\{-2[q^2+(D\tau_H)^{-1}]^{1/2}z_0\}}{l[q^2+(D\tau_H)^{-1}]^{1/2}}\right]\right] . \tag{10}$$

This coincides with the result obtained by Akkermans *et al.*[21] if we set $H=0$. For a nonzero magnetic field we may define a generalized q wave vector

$$\tilde{q}=[q^2+(D\tau_H)^{-1}]^{1/2} .$$

Thus, $I(q,H)$ can be obtained from $I(q,H=0)$ by replacing q with $\tilde{q}$ in the last expression. From Eq. (10) we get

$$I(q,H)=I(\tilde{q},H=0) . \tag{11}$$

We have also calculated $I(q,H)$ for a sharp injection boundary condition in which the exponentials in (8) $\exp(-l/z\mu_i)$ and $\exp(-l/z'\mu_f)$ are replaced by $\delta(z-l/\mu_i)$ and $\delta(z'-l/\mu_f)$, respectively. This leads to

$$I(q,H)=I_0\left[1+\frac{1-\exp\{-2a[q^2+(D\tau_H)^{-1}]^{1/2}\}}{2a[q^2+(D\tau_H)^{-1}]^{1/2}}\right], \tag{12}$$

where $a=l+z_0$, which again coincides with the result first obtained by Akkermans *et al.*[21] for $H=0$. For $H\neq 0$, $I(q,H)$ can be extracted from $I(q,H=0)$ by using Eq. (11) which is independent of the boundary conditions.

We now give a rigorous derivation of $I(q,H)$ by using a diagrammatic approach. In the limit of small magnetic fields where $L_H \gg l$, we may neglect the influence of the magnetic field on the Ladder diagrams $L(\mathbf{r},\mathbf{r}')$. Only the maximally crossed diagrams $C(\mathbf{r},\mathbf{r}')$ are of course affected by the magnetic field, which suppress their contribution to the intensity. The contribution of the crossed diagrams to the coherent backscattered peak is given by[22]

$$I_c=A\int d^3r_1d^3r_2R(\hat{\mathbf{s}},\mathbf{r}_1,\mathbf{r}_2)C(\mathbf{r}_1,\mathbf{r}_2)\phi(r_2)\phi^*(r_1), \tag{13a}$$

where $\theta(r)\alpha e^{i\mathbf{K}_i\cdot\mathbf{r}-r/2l}$ and $\mathbf{K}_i$ is the incidence direction:

$$R(\hat{\mathbf{s}},\mathbf{r}_1,\mathbf{r}_2)=F(\hat{\mathbf{s}},\mathbf{r}_1+\mathbf{r}_2/2)e^{iK_0\hat{\mathbf{s}}\cdot(\mathbf{r}_2-\mathbf{r}_1)}, \tag{13b}$$

and for $\hat{\mathbf{s}}\sim -\hat{\mathbf{z}}$ (around the backscattering direction)

$$F(\hat{\mathbf{s}},\mathbf{r})=\pi e^{-z/l}\delta(x)\delta(y). \tag{13c}$$

The main task is to calculate $C(\mathbf{r}_1,\mathbf{r}_2)$ in the presence of a magnetic field. Following Altshuler *et al.*,[20] $C(\mathbf{r}_1,\mathbf{r}_2)$ is a solution of the following equation:

$$D\left[-i\nabla-\frac{2e}{c}\mathbf{A}\right]^2C(\mathbf{r},\mathbf{r}')=\frac{\delta(r,r')}{\tau}. \tag{14}$$

The solution of Eq. (14) for $H=H\hat{\mathbf{z}}$ is given by Kawabata,[16]

$$C(\mathbf{r},\mathbf{r}')=\frac{1}{\tau}\sum_{n,K_y,K_z}\frac{\psi_{n,K_y,K_z}(r)\psi^*_{n,K_y,K_z}(r')}{DK_z+(4DeH/C)(n+\frac{1}{2})}, \tag{15}$$

where ψ_{n,K_y,K_z} are solutions of the homogeneous part of Eq. (14), which is equivalent to the Schrödinger equation for a charged particle with charge $2e$ and mass $\frac{1}{2}D$, which moves in a magnetic field with a vector potential $\mathbf{A}$. This was solved by Landau[23] and is given by

$$\psi_{n,K_y,K_z}=\phi_n(x-chK_y/2eH)\exp[i(K_yy+K_zz)], \tag{16}$$

where θ_n are the eigenfunctions of the harmonic oscillator. Inserting (16) in (15) and performing the sum on K_y and K_z we get,

$$G(\mathbf{r},\mathbf{r}')=\frac{1}{h\tau DL_H}\sum_n\frac{1}{(2n+1)^{1/2}}\exp[i(x-x')(y+y')/2L_H^2]\exp[-(x-x')^2-(y-y')^2/4L_H^2]$$
$$\times L_n[(x-x')^2+(y-y')^2]/2L_H^2\exp(|z-z'|/L_H)(2n+1)^{1/2}, \tag{17}$$

where $L_n(x)$ are the Legendre polynomials, $L_n(x)=(1/n!)(d^nx^ne^{-x}/dx^n)$. We need the solution $C(\mathbf{r},\mathbf{r}')$ for a half space with an absorbing boundary at $z=0$ and therefore use the image method to get our final result,

$$C(r,r')=\frac{1}{4\tau DL_H}\sum_n\frac{1}{(2n+1)^{1/2}}\exp[i(x+x')(y-y')/2L_H]\exp[-(x-x')-(y-y')/4L_H]$$
$$\times L_n\left[\frac{(x-x')^2+(y-y')^2}{2L_H^2}\right]\exp[-|z-z'|(2n+1)^{1/2}L_H]-\exp[|z+z'+2z_0|(2n+1)^{1/2}/L_H], \tag{18}$$

where $z_0=0.7104$. Inserting (18) in (13a) we get our final answer for $I_c(\mathbf{q})$, which is correct for sharp boundary conditions and weak magnetic fields, $L_H \gg l$,

$$\frac{I_c(q,H)}{I_c(q=\infty,H=0)}=\frac{L_H}{a}\sum_n\frac{(-1)^n}{(2n+1)^{1/2}}\exp(-L_H^2q^2)\{1-\exp[-2a(2n+1)^{1/2}/L_H]\}L_n(2L_H^2q^2). \tag{19}$$

In Fig. 1, we plot $I(q,H)$ as a function of $q=(2\pi/\lambda)\theta$ for different values of the magnetic field. The solid curves represent our diagrammatic calculation and the dashed curves represent out approximate real-space calculation. We see that the real-space approach accounts quite accurately for the shape of the coherent backscattered peak. We therefore conclude that our statement that $I(q,H)$ can be obtained from $I(q,H=0)$, by replacing q with

$$\tilde{q}=[q^2+(D\tau_H)^{-1}]^{1/2},$$

is an excellent approximation. Moreover, the onset of the reduction in the peak should hold for $q<(D\tau_H)^{-1}$, since then $\tilde{q}$ depends weakly on H. This leads to $q<L_H^{-1}$ or $\theta<\lambda/2\pi L_H$.

From Fig. 1, we see that only for $q<q_c=L_H^{-1}$, $I(q,H)$ is reduced below $I(q,H=0)$. For $q>q_c$, we get a crossover effect and $I(q,H)\approx I(q,H=0)$. This means that

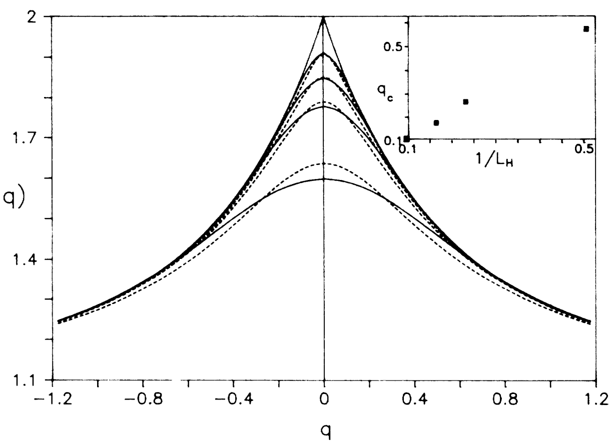

FIG. 1. Backscattering intensity for different values of the magnetic length L_H. We plot $I(q)/I(q=\infty)$ for the following values of L_H/l: 2 (the lowest curve), 4, 6, 10, ∞. The solid curves correspond to the diagrammatic calculation and the dashed curves to the real-space approach. The insert shows the value of q_c as a function of L_H.

short trajectories are not affected by the magnetics field and continue to contribute constructively to the backscattered peak. In the insert of Fig. 1, we plot q_c as a function of $1/L_H$; we see that $q_c \approx L_H^{-1}$. For small magnetic fields where $L_H/l > 6$ the two approaches yield the same curves for $I(q,H)$. For larger magnetic fields for which $L_H/l < 4$, $I(q \to 0,H)$ in the real space approach is somewhat higher than the diagrammatic approach. The two approaches however, should in principle give the same $I(q,H)$. The small deviations for large magnetic fields in Fig. 1, between the two curves result mainly from our approximation for $\langle y^2 \rangle$, which we used to get Eq. (7). This approximation underestimates the effect of the magnetic field causing $I(q \to 0,H)$ to be somewhat larger. In the diagrammatic approach, the magnetic field brings in *all* the Landau levels (not just the ground state as in the real-space approach) yielding a somewhat more quenched coherent peak.

We now calculate the reduction in the peak due to the applied magnetic field. For $H=0$, $I(q,0)/I(q=\infty)=2$. In the presence of a magnetic field $I(q=0,H)/I(q=\infty,H)$ will be reduced below 2. We define the height by

$$\varepsilon(H)=I(q=0,H)/I(q=\infty,H) \ . \tag{20}$$

Using Eq. (20) we plot in Fig. 2, $\varepsilon(H)$ as a function of $1/L_H$. We see that $\varepsilon(H)$ depends linearly on L_H^{-1}, and follows

$$\varepsilon(H)=2-0.53a/L_H \ . \tag{21}$$

This is very close to the result we get from our real-space approach where

$$\varepsilon(H)=2-3^{-1/2}a/L_H \ . \tag{22}$$

We now include inelastic scattering which introduces an additional length scale, the Thouless length $L_i=\sqrt{D\tau_i}$ (where τ_i is the inelastic scattering time). Without a magnetic field it was already shown[10] that

$$I(q,L_i)=I(q_{\text{eff}},L_i=\infty) \ , \tag{23}$$

where $q_{\text{eff}}=(q^2+L_1^{-2})^{1/2}$. In the presence of a magnetic field, by using the real-space method, we get

$$I(q,H,L_i)=I(\tilde{q},H=0,L_i=\infty) \ , \tag{24}$$

where

$$\tilde{q}=[q^2+(3L_H^2)^{-1}+L_i^{-2}]^{1/2} \ .$$

We have also calculated $I(q,H,L_i)$ diagrammatically by adding a term $-i/\tau_i$ to the left-hand side of Eq. (14). This leads to

$$\frac{I(q,L_i,H)}{I(\infty,L_i,H=0)}=\frac{L_i}{1-\exp(-2a/L_i)}\sum_{n=0}\frac{(-1)^n}{[(2n+1)/L_H^2+(1/L_i)]^{1/2}} \times\exp(-L_H^2q^2)(1-\exp\{-2a[(2n+1)/L_H^2+(1/L_i^2)]^{1/2}\})L_n(2q^2L_H^2) \ . \tag{25}$$

From (25), we find that $I(q,H,L_i)\approx I(q,H=0,L_i)$ in two limits for $L_H > L_i$ and for $q > L_H^{-1}$, even for $L_H < L_i$. Thus, the approximate equation (24) is justified. This is demonstrated in Fig. 3, where we plot $I(q,H,L_i)$ as given by Eq. (25) for (a) $I(q,L_H=\infty,L_i=5l)$, (b) $I(q,L_H=10l, L_i=5l)$, (c) $I(q,L_H=\infty,L_i=20l)$, and (d) $I(q,L_H=10l,L_i=20l)$. We see that curves (a) and (b) nearly coincide. This is because when L_i is the smallest length the magnetic field hardly affects the shape of the backscattering peak. Curves (c) and (d) coincide only for $q > L_H^{-1}$. Here L_H is the smallest length, and the rounding of the peak for $q < L_H^{-1}$ is not affected by the inelastic scattering.

We now discuss the observable possibilities of the effects discussed in this paper. There is a great advantage in measuring the interference effects in disordered systems directly by measuring intensities. The reason is because the incident and emerged directions are well defined. This indeed led to the successful observations of weak localization[1-5] for electromagnetic waves. The challenging question is how to measure the intensity of backscattered electrons. The main difficulty is to avoid strong inelastic scattering. We propose here two possibilities and hope that they will be followed up experimentally. The first is to inject high energetic electrons of order ~KeV into a disordered semiconductor like germanium. Here the estimated elastic mean free path is $l \simeq 500$ Å. Thus, multiple elastic scattering is possible. For such en-

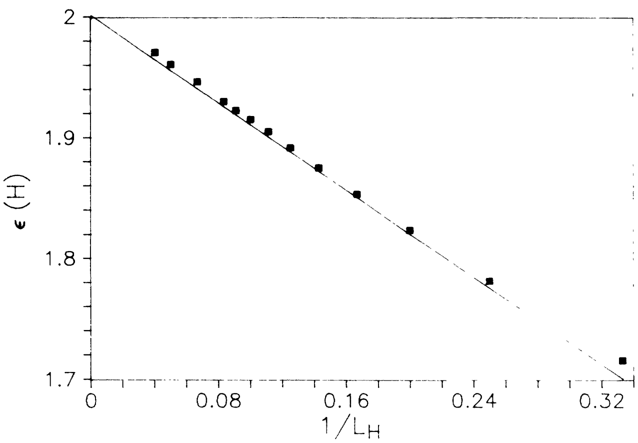

FIG. 2. The reduction of the peak due to the magnetic field [as was defined in Eq. (20)] as a function of $1/L_H$. The points correspond to the reduction as obtained from Eq. (19), while the solid line corresponds to the linear dependence that was suggested in Eq. (21).

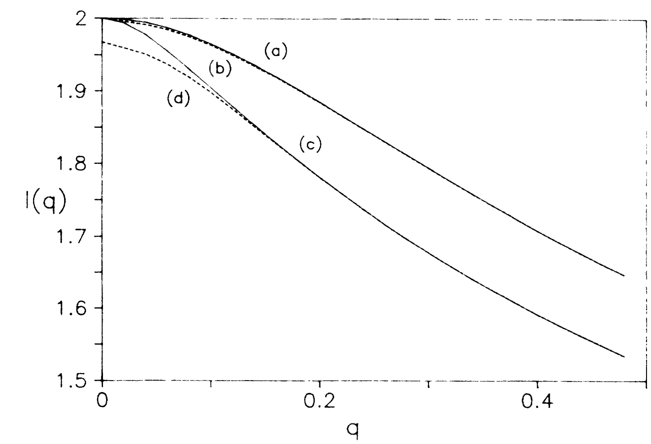

FIG. 3. Backscattering intensity dependence on the magnetic length L_H and the inelastic length L_i. Curves (a)–(d) correspond to (a) $L_H=\infty$, $L_i=5l$; (b) $L_H=10l$, $L_i=5l$; (c) $L_H=\infty$, $L_i=20l$; (d) $L_h=10l$, $L_i=20l$.

ergies, $\lambda_F/l \simeq 10^{-2}$ and the coherent backscattered peak may be possible to observe experimentally under these conditions. Of course, one should measure the dynamic structure factor $I(q,\omega)$ and look for $\lim_{\omega\to 0} I(q,\omega)$ by using an energy spectrometer.

Another interesting possibility is to use the mesoscopic transport regime in which the sample size is smaller than the inelastic length. Recently, it was demonstrated that a superlattice can be easily fabricated from a two-dimensional electron gas (GaAs) and electron states are ballistic (namely l is *larger* than the sample size). It is quite easy[24] to disrupt the order of this superlattice and form elastic scattering. The electron wavelength is quite small $\lambda_F \simeq 500$ Å and λ_F/l can be made quite large. The backscattered intensity when a magnetic field is applied should show the effects discussed in this paper.

In summary, we have calculated the form of the coherent backscattering peak of electrons in the presence of an external magnetic field. The effect of the magnetic field on the backscattering peak is as interesting as the negative magnetoresistence effect for transport properties of electrons. It is proposed that this effect at low temperatures is the strongest interference effect for electrons. The role of breaking time-reversal symmetry is revealed in reducing the enhanced peak. We find to an excellent approximation that the shape of the peak $I(q,H)$ can be obtained from the shape of the peak in the absence of a magnetic field $I(q,H=0)$ but with a renormalized q wave number $\tilde{q}=[q^2+(3L_H^2)^{-1}]^{1/2}$. In the presence of inelastic scattering $I(q,H,L_i)$ can be obtained from $I(\tilde{q},H=0,L_i=\infty)$ with

$$\tilde{q}=[q^2+(3L_H^2)^{-1}+L_i^{-2}]^{1/2} .$$

Thus, $I(q,H,L_i)$ should serve as a sensitive tool of extracting the inelastic scattering time of disordered systems.

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