Inelastic scattering in a doped polar semiconductor

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We calculate, within a many-body theory, the inelastic-scattering rate for hot electrons injected into doped GaAs as a function of electron energy and doping density by treating Coulomb and Fröhlich interactions on an equal footing. Our theory, which includes effects of quantum degeneracy, dynamical screening, plasmon-phonon mode coupling, and phonon self-energy correction, is in excellent quantitative agreement with recent experimental findings, and resolves a puzzle about why a recent observation of single-optical-phonon emission in highly doped GaAs can be quantitatively explained by unscreened theoretical results.

Ballistic transport of hot electrons in doped hot-electron transistor structures has been the subject of intense recent experimental activity.[1-5] These hot-electron structures are mostly made of thin-GaAs-well regions (where the electrons move ballistically) sandwiched between potential barrier layers made of $Al_xGa_{1-x}As$ alloy material. Even though a number of different structures differing in configurational details have been proposed in the literature, they all share the common property that a large fraction of the electrons injected into the GaAs region must move through it ballistically for the device to work. In particular, one defines an amplification ratio $\beta=\alpha/(1-\alpha)$, where α is the fraction of electrons that moves through the GaAs region ballistically, and β must be larger than unity (i.e., α must be larger than 0.5) for the system to work as a transistor. The goal is to make α as large as possible to produce the highest amplification. If the thickness of the GaAs layer is d, then $\alpha \simeq e^{-d/l}$, where l is the electronic mean free path. Thus one needs $l \gg d$ for the hot-electron transistor to be a viable device. This can be achieved either by making l large or by making d small (or, of course, by a combination of both).

From the above discussion it is clear that one wants the GaAs layer to be as thin as possible (so as to make d very small), the temperature to be low (so as to minimize phonon absorption), and the system to be undoped (so as to minimize electron-electron scattering) for the ideal hot-electron transistor operation. Unfortunately, due to materials and technological problems, there are lower limits on how thin ($d \gtrsim 500-600$ Å) the GaAs layer can be and on how low the doping level ($n \sim 10^{17}$ cm^{-3}) can be in a three-terminal device with the GaAs layer serving as the base. With these limitations it becomes important to have quantitative information on the electronic mean free path in this system in order to have an idea about the technological feasibility of such hot-electron transistor operation. In this paper we provide a quantitative calculation of the hot-electron scattering time (τ) in doped GaAs.

In an interesting recent paper[1] Heiblum *et al.* reported the observation of a single-optical-phonon emission by monoenergetic hot electrons traversing thin doped GaAs layers. This was done by injecting ballistic electrons into the thin layers with energy around the threshold of the bare-optical-phonon emission and monitoring their exit energy. The scattering time estimated in Ref. 1 on the basis of the experimental data is about 185–210 fsec for electrons with energy around 85–90 meV in a doped GaAs region about 300–500 Å thick with the doping level around 8×10^{17} cm^{-3}. It was noted in Ref. 1 that these results are in approximate agreement with earlier experiments[6] done in bulk *undoped* GaAs using time-resolved Raman scattering. In this paper we provide a quantitative calculation of inelastic scattering in GaAs which is in very good agreement with the experimental results of Ref. 1.

An interesting and, in fact, quite puzzling aspect of the results reported in Ref. 1 is that the observed scattering time τ is essentially the same as that in an undoped GaAs system. Comparison between the experiment and the calculated inelastic-scattering rate[7] due to single-longitudinal-optical (LO)-phonon emission by a single electron in undoped GaAs shows very good agreement. This is very surprising, as indeed was noted by the authors of Ref. 1. The physics of inelastic scattering is very different in doped and undoped polar materials. In an undoped system, the only possible inelastic process at low temperatures is the emission of bare phonons—the scattering rate for this process is easily calculable[7] from the Fröhlich electron-phonon interaction matrix element by using Fermi's "golden rule." In a doped Fermi system this simple picture of bare-phonon emission gets modified in the following three important ways: (1) The electron-phonon interaction is now screened by the Fermi sea. (2) The emitted phonons are renormalized by the electrons, and, therefore, the simple picture of bare-phonon emission breaks down. (3) Electron-electron scattering gives rise to a new inelastic channel tending to increase the total scattering rate. In a degenerate system one has the additional complication of accounting for the Pauli principle. Clearly, all these effects of finite doping become more important at higher electron densities. Therefore, it is very difficult to understand why the observed scattering time in Ref. 1 at a fairly high electron density of

8×10^{17} cm^{-3} (corresponding to a Fermi energy $E_F\approx45$ meV and a plasma frequency $\omega_p\approx38$ meV, both of which are, in fact, larger than the GaAs LO-phonon energy $\omega_{LO}\approx36$ meV, with units in which $\hbar=1$ throughout this paper) agrees with the *unscreened* optical-phonon emission time in *undoped* GaAs.

We resolve this puzzle in this paper by showing that this agreement is *purely fortuitous* with no fundamental significance at all. For a different electron energy at the same doping level, or, for a different doping level with the same electron energy, the scattering time will be significantly different from that in the simple unscreened theory. It is an unfortunate coincidence that the electron energy and the doping level happen to be such that the unscreened and the screened results come accidentally close together for the experimental values of the parameters used in Ref. 1. Our calculated results, which are essentially in exact agreement with the few available experimental data points, make specific quantitative and experimentally verifiable predictions about the scattering rate as a function of electron energy for various doping levels. As one expects, our results are significantly different from the simple result for the bare-optical-phonon emission rate in an undoped and unscreened system.

Our model is that of parabolic GaAs conduction-band electrons in the Γ valley interacting with each other via the direct Coulomb interaction and with bulk LO phonons of GaAs via the long-range Fröhlich interaction. We consider the zero-temperature situation (it should, however, be valid as long as $k_BT\ll E_F,\omega_{LO}$) and neglect acoustic-phonon and impurity-scattering effects (except as noted below). We are interested in calculating the inelastic-scattering rate for electrons in the GaAs well region only and, as such, we neglect all complications (e.g., quantum reflection from the barrier, etc.) arising from the existence of the barrier regions on two sides of the GaAs base. We consider the electron system to be *three dimensional,* neglecting any size quantization imposed by the quantum-well confinement. This approximation restricts the applicability of our calculation to fairly thick (>300 Å) GaAs layers and/or to fairly large doping densities ($>10^{17}$ cm^{-3}). In actual hot-electron transistors the base region is thicker than 500 Å and the doping level is around 10^{18} cm^{-3}. Our three-dimensional approximation is extremely well valid for these structures, because roughly *ten* two-dimensional subbands are occupied by electrons, and the electronic motion in the vertical direction is better represented as three-dimensional than as two-dimensional intersubband quantum transitions in this multisubband occupancy situation. We believe that our three-dimensional approximation is well valid for the experimental situation in Ref. 1.

We obtain the inelastic-scattering rate by calculating the electronic self-energy in a leading-order expansion in the total effective dynamical interaction $V_{eff}(q,\omega)$:

$$\Sigma(k,E)=\frac{i}{(2\pi)^4}\int d^3q\int d\omega\, V_{eff}(q,\omega)G_0(\mathbf{k}-\mathbf{q},E-\omega)\ ,$$

where G_0 is the time-ordered noninteracting electronic Green's function and $V_{eff}(q,\omega)=V_q/\epsilon(q,\omega)+|M_q|^2D(q,\omega)/\epsilon^2(q,\omega)$. The dielectric function is obtained in the random-phase approximation (RPA): $\epsilon(q,\omega)=1-V_q\Pi_0(q,\omega)$, where Π_0 is the irreducible bare polarizability function (the Lindhard function) of the three-dimensional electron gas. In the above expression for V_{eff}, the first term is the dynamically screened Coulomb interaction (V_q is the bare Coulomb interaction), whereas the second term is the renormalized phonon-mediated dynamically screened Fröhlich interaction (M_q is the polar Fröhlich interaction). The renormalized phonon propagator $D(q,\omega)$ is given by $D(q,\omega)=2\omega_{LO}(\omega^2-\omega_{LO}^2)^{-1}[1-2\omega_{LO}(\omega^2-\omega_{LO}^2)^{-1}M_q^2\Pi_0(q,\omega)/\epsilon(q,\omega)]^{-1}$, with $2\omega_{LO}(\omega^2-\omega_{LO}^2)^{-1}$ defining the bare-phonon propagator. Finally, the inelastic-scattering rate is defined by $\Gamma(E)\equiv|\mathrm{Im}\Sigma(k,E=k^2/2m)|$ and the scattering time is defined by $\tau(E)=(2\Gamma)^{-1}$. The inelastic mean free path is defined as $l=v\tau$, where $v=(2E/m)^{1/2}$ is the electron velocity. Before presenting our results we want to emphasize that the calculated Γ for the dynamically *coupled* electron-phonon system in the doped situation is the only experimentally relevant scattering rate in the problem, and cannot be separated out into a phonon-emission rate and an electron-electron scattering rate because of the nonmultiplicative and nonadditive nature of the mutual renormalization effect between electrons and phonons.

In Fig. 1(a) we show the calculated scattering (or, the damping) rate Γ for GaAs in inverse picoseconds as a function of the energy (E) of the hot electrons measured from the bottom of the band for three different electron densities: $n=4\times10^{16}$, 10^{17}, and 8×10^{17} cm^{-3}. For each density the thick line indicates the total damping, whereas the thin line gives the results for just the dynamically screened Coulomb interaction (i.e., the thin lines include just the electron-electron scattering and no phonon-emission process). We also show as dashed-dotted lines the corresponding unscreened scattering rate for the one-electron undoped situation. The unscreened result is obviously for the phonon emission only. The lower curve for the unscreened results includes[8] effects of conduction-band nonparabolicity and indicates approximately the magnitude of the error involved in making the parabolic approximation. The experimental points from Ref. 1 are indicated by the triangles and they fall right on our calculated result for the same electron density. As one can see from Fig. 1(a) the experimental points, because of the choice of electron energies involved, fortuitously agree with the unscreened one-electron result as well. We note, however, that the correct trend in Γ with increase in E is given only by our results. Also, if the electron energy is measured from the Fermi energy E_F rather than the band bottom (which is the usual norm in a doped situation because all states inside the Fermi sea are occupied), there is very strong disagreement between experimental results and the unscreened theory, whereas the agreement with our results remains unaffected. Clearly, more experimental results at other (particularly higher) energies are needed for a definitive confirmation of our theoretical predictions.

In briefly discussing our theoretical results we point out the following salient features: (1) Γ goes to zero for

$E = E_F$ as it should for the imaginary part of a proper self-energy; (2) $\Gamma(E)$ is small, but nonzero for $E < E_F + \omega_{LO}$ unlike the unscreened ($E_F = 0$) situation where $\Gamma(E) = 0$ for $E < \omega_{LO}$ since no phonon emission is possible below this threshold; (3) the shoulderlike structure in Γ for $E = E_F + \omega_{LO}$ is due to the phonon-emission threshold; (4) at the highest density ($n = 8 \times 10^{17}$ cm^{-3}) one can see an additional structure in Γ at higher energy which arises from the plasmon-phonon coupling and the plasmon-emission threshold ($\omega_p \approx \omega_{LO}$ for $n = 8 \times 10^{17}$ cm^{-3}); (5) at high energies ($E \gg E_F + \omega_{LO}$), Γ is higher at higher doping densities due to stronger electron-electron scattering effects; and, Γ saturates and starts decreasing slowly.

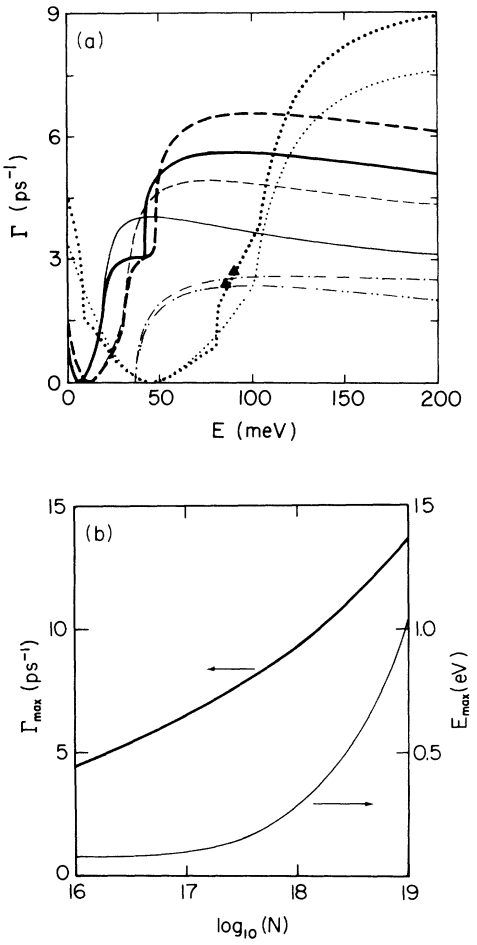

FIG. 1. (a) Shows the inelastic-scattering rate (calculated as the imaginary part of the electronic self-energy) Γ in inverse picoseconds as a function of the hot-electron energy E (in meV) measured from the band bottom for three different electron densities: $n = 4 \times 10^{16}$ cm^{-3} (solid lines); 10^{17} cm^{-3} (dashed lines); 8×10^{17} cm^{-3} (dotted lines) in GaAs. The thick and the thin lines represent, respectively, the total and the electron-electron scattering rates. The dashed-dotted lines are the unscreened results appropriate for undoped GaAs. The mean-free-path l is given by $l \approx 360\, E^{1/2}/\Gamma$ with E in meV and Γ in ps^{-1}. The solid triangles are experimental results from Ref. 1. (b) Shows the maximum value of Γ and the energy at which this maximum occurs as a function of the electron density.

To characterize the variation of the damping with the electron density n, we show in Fig. 1(b) the maximum value Γ_{max} of Γ for various densities plotted as a function of n (we also show the energy value E_{max} at which this maximum Γ occurs for a specific density). We find that Γ_{max} increases with n slightly superlinearly. At lower energies, however, Γ does not vary monotonically with n (as can be seen from Fig. 1) since it is a function of both E and E_F. We note that E_{max} also goes up with n.

Before concluding, we point out that we have included the effect of impurity scattering on these results by incorporating an elastic lifetime in our theory. Results change very little provided the elastic mean free path is much longer than the inelastic one calculated here (which, of course, is the basic assumption of the whole calculation). Also, scattering by acoustic phonons in negligible compared with the electron-electron and electron–optical-phonon scattering results in III-V-compound polar materials of interest here. We believe that for the electron densities above 10^{17} cm^{-3} the RPA employed here is a good approximation (at low n, contributions from screening and electron-electron scattering are negligible anyway) and the leading-order self-energy calculation in the effective dynamical interaction should be quite accurate. For comparison with the single-optical-phonon emission experiment,[1] obviously, the leading-order self-energy diagram is the correct quantity to calculate. The higher-order phonon vertex corrections (multiphonon emission processes) should be important only at very high energies ($E > 300$ meV) where one has to take into account intervalley scattering (Γ to L) as well. Thus, our results should be quite accurate for GaAs up to $T = 50$–100 K and for $E < 300$ meV. We should mention that, because the LO phonons interact with the electrons via the long-range Fröhlich interaction, the RPA theory of dynamical screening employed here is well valid within the effective-mass approximation, and the recent objection[9] of Combescot *et al.*, which is applicable to acoustic phonons with their short-range deformation-potential coupling, is not relevant here.

In conclusion, we have calculated the inelastic-scattering rate of hot electrons in doped GaAs by including both electron-electron and electron–optical-phonon interactions on an equal footing. Our results, which include quantum degeneracy, dynamical screening, plasmon-phonon coupling, Coulomb and Fröhlich interactions, are in excellent agreement with the available experimental results on the single-optical-phonon emission time in doped GaAs. We resolve a puzzle about why the observed results in highly doped material agree well with the unscreened theory (appropriate only for *undoped* GaAs) and make specific quantitative predictions about how the scattering time varies with the electron energy and the doping level. Our results should be useful in choosing judicious parameters for the design and fabrication of hot-electron transistors.

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