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# Quantum corrections to conductivity observed at intermediate magnetic fields in a high-mobility GaAs/Al$_x$Ga$_{1-x}$As two-dimensional electron gas

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Magnetoresistance with the field perpendicular to the two-dimensional electron gas in a high-mobility GaAs/Al$_x$Ga$_{1-x}$As heterostructure at low temperatures is studied. At the lowest magnetic field we observe the weak localization. In particular we study the resistivity at magnetic fields, where the product of the mobility and the magnetic field is of the order of unity. The quantum correction to conductivity due to the electron-electron interaction [and other possible deviations from the free-electron (Drude) model], though having only a slight variation with magnetic field, nevertheless gives rise to a sizable quadratic magnetoresistance in this regime. Our experiments yield in this way not only the well-known electron-electron term with the expected temperature dependence, but also a new type of temperature-independent quantum correction, which varies logarithmically with mobility.

Localization and electron-electron interaction play a key role in the understanding of the transport properties of the two-dimensional electron gas (2D EG) at low temperatures. Three physical systems with a two-dimensional electron gas have repeatedly been studied in the literature, namely the quasi-two-dimensional metallic film and the more strictly two-dimensional electronic systems, the silicon metal-oxide-semiconductor field-effect transistor and the modulation-doped GaAs/Al$_x$Ga$_{1-x}$As heterostructure.[1] The quantum corrections to the electrical conductivity caused by localization and electron-electron interaction are small in the limit, where the electron mean free path is much longer than the Fermi wavelength, and the localization in this particular limit is normally called weak. Both quantum corrections can be studied by their dependence on temperature. However, the temperature dependence due to both effects is very similar and difficult to measure. Fortunately the weak localization correction turns out to be very sensitive to magnetic fields perpendicular to the 2D EG and is extinguished at very low magnetic fields. The electron-electron interaction quantum correction to the Drude conductivity is relatively unaffected by magnetic fields and gives rise to a magnetoresistance, which at magnetic fields corresponding to the inverse mobility is of the order of the reciprocal conductivity correction. This stems from the transformation from conductivity tensor to resistivity tensor. This magnetoresistance can therefore be studied best at intermediate fields, where the weak localization is completely quenched, and where the Shubnikov–de Haas oscillations and the quantum Hall effect are not yet dominating.

Figure 1 gives an overview of the type of experimental recordings which we have analyzed. Three representations of the same recording are shown with increased resolution, going from (a) to (b) to (c). Figure 1(a) shows the magnetoresistance up to $\mu B \sim 10$, where $\mu = e\tau_0/m^*$ is the mobility ($m^*$ is the effective mass, and $\tau_0$ is the transport scattering time). At the highest field the Shubnikov–de Haas oscillations have minima at the magnetic field $B = nh/(2ei)$, where $i = 1,2,3,\dots$, and where $n$ is the two-dimensional electron density. In order to measure the weak localization magnetoresistance one must have a high resolution in the resistance measurements, as in (c). In particular, this is the case if the conductance is large compared to the conductance quantum $e^2/\pi h$. The samples investigated were of the GaAs/Al$_x$Ga$_{1-x}$As modulation-doped type (30% Al content, 15-nm Al$_x$Ga$_{1-x}$As spacer layer) with a two-dimensional electron density of $3.5 \times 10^{15}$ m$^{-2}$ and mobilities of 22 m$^2$/V s, corresponding to a resistance per square of 80 Ω. A spread in mobility for a number of chips from the same wafer was obtained by ion implanting 80-keV $^4$He ions into the heterostructure. An implantation fluence of $10^{14}$ m$^{-2}$ reduced the mobility to 5.6 m$^2$/Vs, but only changed the carrier density to $3.2\times 10^{15}$ m$^{-2}$. The electron density measured by the Hall effect and by Shubnikov–de Haas oscillation was found to agree, showing that there was no parallel conduction channel in the doped Al$_x$Ga$_{1-x}$As layer. The localization correction (as well as the correction due to electron-electron interaction) is of the order $\sigma_{00} = e^2/\pi h$, and this corresponds to changes in the square resistance of the order of 1.2–0.08 Ω. In order to achieve the high sensitivity needed, we used a sensitive electrical dc bridge and matched the resistance of our samples to this bridge by using a mesa-etched 2D EG channel with a large length-to-width ratio.

The magnetic field dependence of the weak localization quantum correction can be written in the form

$$\Delta\sigma_{\mathrm{WL}}(B) - \Delta\sigma_{\mathrm{WL}}(0) = \sigma_{00} f\left(\frac{B}{B_0}, \frac{B}{B_\phi}\right), \tag{1}$$

where $B_\phi = \hbar(4eD\tau_\phi)^{-1}$ is a characteristic field corresponding to the phase relaxation time $\tau_\phi$, and $B_0 = \hbar(4eD\tau_0)^{-1}$ is a characteristic field, which corresponds to the transport relaxation time $\tau_0$. $D$ is the two-dimensional diffusion constant. $f(x,y)$ is a function which is defined to be zero for $B = 0$. In the paper by Hikami, Larkin, and Nagaoka,[2] $f(x,y)$ was expressed by digamma functions and found to saturate at the value

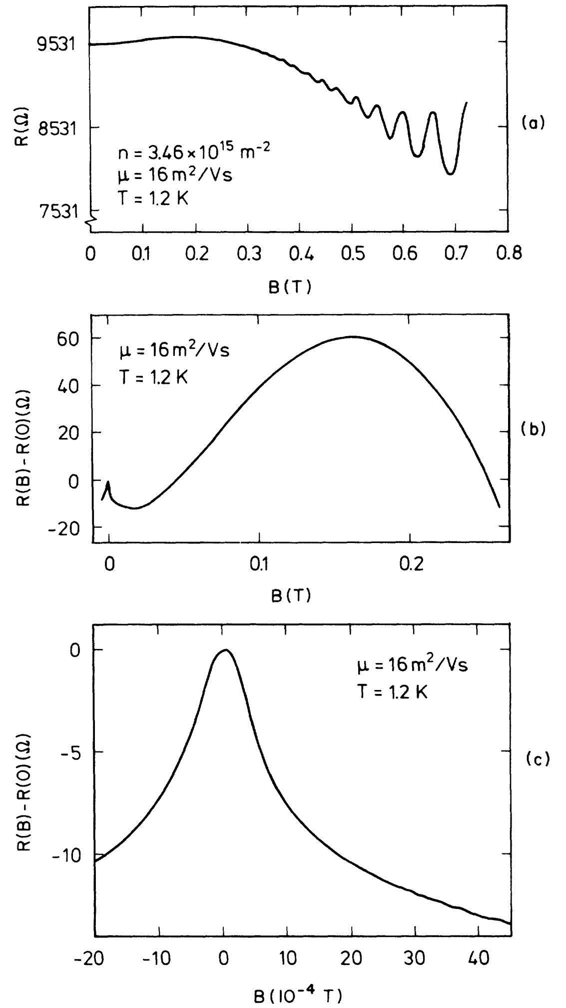

FIG. 1. Magnetoresistance for a GaAs/Al$_x$Ga$_{1-x}$As heterostructure with the magnetic field perpendicular to the two-dimensional electron gas. The sample was irradiated by a dose of $5\times10^8$ cm$^{-2}$ 80-keV $^4$He ions in order to reduce the mobility. The starting probability before irradiation was 22 m$^2$/V s. The samples were intentionally made long compared to their width in order to improve the experimental resolution of the quantum corrections. The resistance was concomitantly measured over 84 squares. (a) depicts the overall behavior at low and intermediate magnetic fields including the Shubnikov–de Haas oscillations at the highest magnetic field. (b) and (c) show the magnetoresistance measured with increased resolution revealing the magnetoresistance associated with the interaction effect (b) and the weak localization effect (c).

$f(\infty,\infty)=\ln(\tau_\phi/\tau_0)$, when $B\gg B_0$; however, this theory is valid only in the region $B\ll B_0$. In the paper by Kawabata,[3] $f$ has a different form and saturates at a somewhat smaller value depending on the ratio $B_\phi/B_0$. This theory is valid, as long as $B\ll\mu^{-1}$, i.e., to a much higher magnetic field than the theory of Hikami *et al.*[2] The major reason to start this experimental investigation was to study weak localization as a function of temperature and mobility. It turned out, however, that the most intriguing results were found in an intermediate magnetic field region, and it is these results that we shall concentrate on here. The weak localization results will be published in detail in a forthcoming publication. The lower curve [Fig. 1(c)] shows an example of the weak localization magnetoresistance. Such curves can be well fitted to the theory of Kawabata,[3] thereby determining the phase breaking time $\tau_\phi$. Electron-electron scattering is the dominating contribution to $\tau_\phi$. From such measurements and using the theoretical expression for the electron-electron scattering times,[4] we determine the so-called interaction parameter $F$.[5] The measurements of $\tau_\phi$ will, as mentioned, be reported in detail in another publication.

The correction to the conductivity from electron-electron interaction has the form[6–9]

$$\Delta\sigma_{ee}=\sigma_{00}f(F)[\ln(k_BT\tau_0/\hbar)]\,, \tag{2a}$$

or

$$\Delta\sigma_{ee}=\sigma_{00}f(F)[\ln(k_BT\tau_0/\hbar)-\tfrac{1}{2}\ln(2\mu B/\pi)] \tag{2b}$$

at $B\gg\mu^{-1}$. $F$ depends on the ratio of the Fermi wave vector and the inverse screening length in GaAs. According to Finkelshtein,[5] $f(F)=4-3(F+2)\ln(1+F/2)/F$. The electron-electron interaction is independent of magnetic field up to $B\sim\mu^{-1}$ and given by Eq. (2a). At magnetic fields $B\gg\mu^{-1}$, the second term in Eq. (2b) enters [by the substitution[8] $\tau_0\rightarrow\tau_0/(2\mu B/\pi)^{1/2}$], and this term eventually changes the sign of this quantum correction. At even higher magnetic fields the Zeeman splitting becomes comparable to $\hbar/\tau_0$, and Eq. (2) is no longer valid. At such high magnetic fields the Shubnikov–de Haas effect is observed, and we are here in a completely different regime, where the semiclassical electron approximation breaks down, and everything is dominated by the Landau quantization. The upper curve [Fig. 1(a)] shows the Shubnikov–de Haas oscillations.

The major theme of this Rapid Communication is to investigate the magnetoresistance at intermediate magnetic fields as most clearly seen in Fig. 1(b). The classical magnetoresistance turns out to be very sensitive to deviation from the Sommerfeld free-electron model. It is well known that the free-electron model predicts a magnetoconductivity tensor of the form

$$\underline{\sigma}=\frac{\sigma_0}{1+(\mu B)^2}\begin{Bmatrix}1 & -\mu B\\ \mu B & 1\end{Bmatrix}, \tag{3}$$

where $\sigma_0=ne^2\tau_0/m^*$. Inverting the conductivity tensor into a resistivity tensor yields a Hall resistivity $\mu B/\sigma_0$, but no magnetoresistance. A correction $\Delta\sigma$, such as that of Eq. (2), added to the diagonal terms in the conductivity tensor Eq.(3) yields, after inversion and expansion to first order in $\Delta\sigma/\sigma_0$, a magnetoresistance of the form

$$\rho_{xx}=R_\Box\{1-R_\Box\Delta\sigma[1-(\mu B)^2]\}\,, \tag{4}$$

where $R_\Box=\sigma_0^{-1}$ is the two-dimensional resistivity or square resistance. A similar correction also enters the low-field linear Hall resistivity, though this can be experimentally observed only in samples with large quantum corrections.[10] Armed with this knowledge it is of interest to investigate the magnetoresistance at various mobilities and temperatures of the 2D EG of GaAs/Al$_x$Ga$_{1-x}$As selectively doped heterostructures in intermediate magnetic fields, where the magnetoresistance Eq. (4) can be observed. Our investigation leads to a consistent picture of the magnetoresistance of a high-mobility 2D EG at low temperatures on the basis of Eq. (4), and reveals a new type of quantum correction.

On the rough scale, as shown in Fig. 1(a), the general trend of the magnetoresistance, before the Shubnikov–de Haas oscillations set in, is a negative almost parabolic magnetoresistance. This has been observed earlier,[11,12] and interpreted[11] as related to electron-electron interaction; however, as we shall demonstrate below, the interpretation in Ref. 11 is true only with some severe modifications. With higher resolution we observe more details in the magnetoresistance as shown in Figs. 1(b) and 1(c). At the lowest magnetic field we observe the weak localization magnetoresistance mentioned above. In the region around $\mu B \simeq 1$ displayed in Fig. 1(b), we see a positive magnetoresistance for the high-mobility samples and a negative magnetoresistance for the low mobility samples, whereas at higher fields ($\mu B > 5$) the negative magnetoresistance mentioned above always dominates. The reason for this behavior can be explained by a combination of Eqs. (2) and (4) and the fact that the weak localization correction is quenched already at much lower magnetic fields. In order to test the validity of Eqs. (2) and (4) we have performed a number of experiments, where curves like the one shown in Fig. 1(b) are measured at different temperatures. Figure 2 shows such a family of curves, where we have plotted the magnetoresistance [in conductivity units; note that $\sigma_{00} = e^2/(2\pi^2\hbar) = 0.0000126$ $\Omega^{-1}$] as a function of $(\mu B)^2$. If we determine the magnetoresistance coefficient corresponding to the lines indicated in Fig. 2 as a function of temperature, we can compare this result with Eq. (2a). This has been done for five different samples with almost the same carrier densities, but different mobilities. If we plot these coefficients as a function of $\ln(k_B T \tau_0/\hbar)$, we obtain a series of parallel lines. From the slopes of these lines we have quite reliably determined $F$ in $f(F)$ in Eq. (2) to be $f(F) = 0.9 \pm 0.2$ giving $F = 0.4$ in reasonable agreement with theory[5] as well as our determination from the phase-breaking rate. In both these situations we obtain $F = 0.6$. It turns out that although the slope of $\Delta\sigma$ vs $\ln(k_B T \tau_0/\hbar)$ was identical in all cases, the $\ln(k_B T \tau_0/\hbar) = 0$ intercept varies a great deal from sample to sample. In Fig. 3 we plot the magnetoresistance as a function of $\mu B$ at a fixed temperature for the five investigated samples cut from the same wafer, but implanted to have different mobilities. Here it is already clear that there is a significant variation of the parabolic magnetoresistance as the mobility changes. The coefficient of the parabolic magnetoresistance even changes sign from positive to negative, as the disorder increases. In Fig. 4 we have plotted the coefficient of $(\mu B)^2$ in units of $\sigma_{00}$ as a function of $\ln(k_B T \tau_0/\hbar)$ with the temperature kept constant and mobility as the variable parameter. The measured points almost follow a straight line. Therefore, in addition to the slope expected on the basis of Eq. (2), we find an unexpected quantum correction of the form

$$\Delta\sigma = \beta\sigma_{00} \ln(\tau_0/\tau_x) , \tag{5a}$$

or

$$\Delta\sigma = \beta\sigma_{00}[\ln(\tau_0/\tau_x) - \tfrac{1}{2} \ln(2\mu B/\pi)] \tag{5b}$$

at $B \gg \mu^{-1}$. The coefficient and the characteristic time was found to be $\beta = 4.0$ ($\pm 0.2$) and $\tau_x \simeq 4 \times 10^{-12}$ s, respectively. The sum of the conductivity corrections in

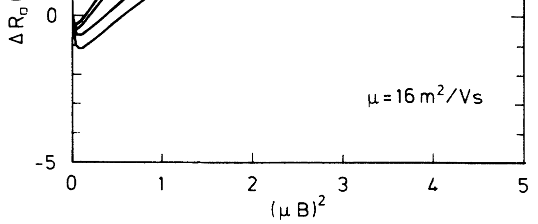

FIG. 2. Magnetoresistance $\Delta R_\square(B)$ (in conductivity units) recorded as a function of $(\mu B)^2$ for four different temperatures (same sample as in Fig. 1). The linear part, which corresponds to the quadratic magnetoresistance mentioned in the text, is indicated by the straight lines. The slopes of these lines correspond to the deviation of the conductivity from the prediction of the free-electron model. The slope increases with temperature in accordance with Eq. (2a).

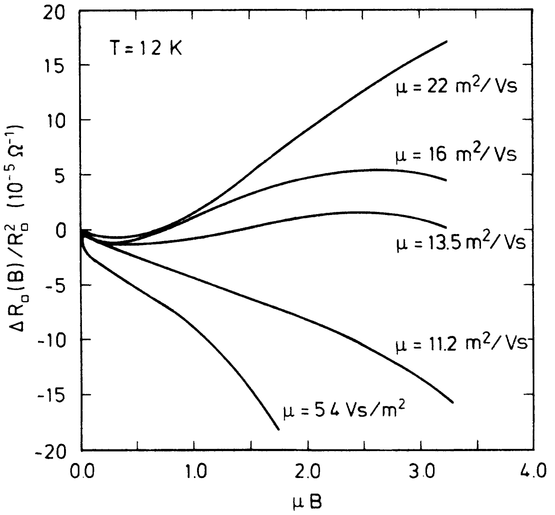

FIG. 3. Magnetoresistance $\Delta R_\square(B)$ (in conductivity units) for five GaAs/Al$_x$Ga$_{1-x}$As heterostructure samples measured at the same temperature. The samples were $^4$He ion implanted to obtain the indicated mobilities. The two-dimensional carrier concentration was only significantly changed by the implantation. For $\mu B < 0.5$ the magnetoresistance is dominated by weak localization. For higher magnetic fields the magnetoresistance is positive for high-mobility samples and negative for low-mobility samples. The negative curvature for all samples at the highest magnetic field shown here corresponds to a magnetic field dependence of the electron-electron interaction as given by Eq. (2b).

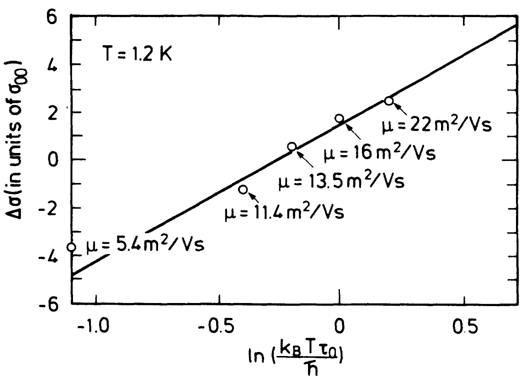

FIG. 4. The figure depicts the slope $\Delta\sigma$ of $\Delta R_\square/R_\square^2$ vs $(\mu B)^2$ (from lines as those indicated in Fig. 2) plotted as a function of $\ln(k_B T\tau_0/\hbar)$, where the argument of the logarithm is changed by implanting $^4$He ions into the GaAs/Al$_x$Ga$_{1-x}$As heterostructures. The temperature is kept constant: $T = 1.2$ K. The ion implantation only changes $\tau_0$ (or the mobility) and not the carrier density. The measured points fall on a straight line. The slope of this line is much larger than expected from Eq. (2a), revealing the presence of an additional quantum correction to conductivity of the form given by Eq. (5).

Eqs. (2) and (5) describes the measured magnetoresistance at intermediate magnetic fields (0.02–0.5 T) quantitatively very well for varying $B$, $T$, and $\tau_0$. It should be emphasized that Eq. (2), apart from an adjustable constant, is fully adequate, if only $T$ and $B$ are varied.

At magnetic fields where $\mu B \gg 1$, the second term in Eqs. (2b) and (5b) enters strongly and gives rise to a magnetoresistance via Eq. (4). This contribution gets logarithmically more negative, as the magnetic field is increased, and gives rise to the characteristic negative curvature observed. This is, in fact, the dominating contribution one observes with the resolution in Fig. 1(a). This negative magnetoresistance versus temperature was the basis of the interpretation made by Paalanen, Tsui, and Hwang,[11] from which they determined $f(F)$. It turns out that such a plot, in fact, gives roughly the right value of $f(F)$, since the temperature variation exclusively samples the first term in Eq. (2). However, their physical interpretation is not quite correct and misses the point that at fields around $\mu B \simeq 1$, the true quadratic magnetoresistance is positive for clean samples and negative for dirty samples.

It is interesting to find a temperature- and magnetic-field-independent quantum correction in a magnetic field region, where the weak localization magnetoresistance has long since been extinguished. The new quantum correction, which we have evidence for, occurs in a regime where the electronic wave function changes character, and we have no clear picture of the nature of the quantum correction Eq. (5). In a very recent paper by Chalker, Carra, and Benedict[13] a percolation model of the integer quantum Hall effect has been investigated in a way which is very similar to the weak localization picture at zero magnetic field. The "weak localization" they[13] predict [their Eq. (4)] has the same logarithmic dependence on disorder ($\tau_0$) as we find. They also obtain the prefactor $e^2/(\pi h)$. However, the numerical constant $\beta$, which in our case is of the order of unity, is in their case roughly inverse proportional to the Landau-level number at the Fermi energy, i.e., much smaller.

To summarize, we have studied the quantum correction to the conductivity at intermediate magnetic fields $\mu B \simeq 1$ and have found the expected electron-electron interaction term from the plot of the magnetoresistance versus temperature. The determined value of the interaction parameter $F$ is in agreement with theory. At high magnetic fields $\mu B \gg 1$, but before the Shubnikov–de Haas oscillations and the quantum Hall effect set in, we also show evidence for the first correction term to the electron-electron interaction due to the cyclotron period entering on expense of the transport relaxation time. When the quantum corrections are studied for different sample mobilities, we discover at intermediate magnetic fields a new type of quantum correction, which has not been reported earlier in the literature. This correction depends logarithmically on the transport relaxation time, but is independent of temperature and depends on the magnetic field in the same way as the electron-electron interaction term. We expect this quantum correction to be related to a new type of localization, which is not quenched by the high magnetic field.

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