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Ballistic electronic conductance of a wide orifice

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For the ballistic motion of a two-dimensional noninteracting electron gas we compute the quantum-mechanical conductance of a rectangle of length L whose lateral dimension a is greater than that of the incoming and outgoing wires b. The global structure of the conductance when the lateral dimension increases reveals slow decay, contrary to what one expects from the simple law of addition of resistances in parallel. Closer inspection shows an oscillatory behavior with well-defined peaks at $k_F a/\pi \gtrsim n$. No quantized plateaus (which occur for $a < b$) exist. A simple interpretation is given in terms of localization and resonance tunneling.

Recent experiments by Wharam *et al.*[1] and by van Wees *et al.*[2] on ballistic motion of electrons through a narrow constriction revealed the phenomenon of conductance quantization, i.e., the conductance increases as a function of the constriction width by integer values in units of the fundamental conductance unit. This fascinating phenomena has been the subject of several theoretical works.[3] A natural question which arises (and can apparently be tested experimentally), is what happens when the lateral dimension of the sample, a, is greater than that of the wire leads connected to it, b. For $a < b$ the (quantized) conductance is roughly proportional to the magnitude of the lateral dimension, which demonstrates a quantum-mechanical law of addition of resistances in parallel similar to the classical law. The purpose of this Rapid Communication is to present the results of our exact quantum-mechanical calculations of the conductance of a rectangle of dimension $a \times L$, connected to long wire leads of width $b < a$ and to suggest a relatively simple explanation for the structure of the conductance and its dependence on geometry. The basic quantities we calculate are the transmission-amplitude matrix $\underline{t}$, from which the conductance is evaluated using the linear conductance formula, $G(2e^2/h)\mathrm{Tr}(\underline{t}\underline{t}^\dagger)$.[4] We show that the conductance is maximum when $a = b$ and decreases as $a > b$, which demonstrates a violation of the law of addition. Superposed on this global decrease with increasing width there is a rich oscillatory structure which is partly related to the spectrum of states of a single particle in a rectangular box. We correlate our results to concepts of resonance tunneling and localization in configurations without symmetry. Our calculational techniques can be easily extended to study samples containing impurities and the effect of a perpendicular magnetic field.[5]

The geometry in the present problem is shown in the inset of Fig. 1. Consider the quantum-mechanical motion of a particle with mass m and (Fermi) energy E in a planer region composed of two semi-infinite strips defined by $(-\infty < x \le 0,\ 0 \le y \le b)$, $(L \le x < \infty,\ 0 \le y \le b)$, and a wider strip separating them, with dimension defined by $(0 < x \le L,\ 0 \le y \le a)$ with $a > b$. We look for a solution of the Schrödinger equation $-\Delta\psi_n = (2mE/\hbar^2)\psi_n$ corresponding to an incoming initial wave moving from left to right in a definite channel n, with hard-wall boundary conditions. We use the formalism and the notation described in Ref. 3 by Avishai and Band, to solve for the reflection and transmission coefficients, R_{mn} and T_{mn} and form the flux-normalized amplitudes $r_{mn} = (k_m/k_n)^{1/2} \times R_{mn}$, $t_{mn} = (k_m/k_n)^{1/2} T_{mn}$.

We evaluate $\mathrm{Tr}(\underline{t}\underline{t}^\dagger)$ for a rectangle of (dimensionless) length $kL = 500$ and varying (dimensionless) width $50 < ka < 100$ (aspect ratio between 10 and 5) using dimensionless wire width $kb = 50$. Actual values of $k = k_F$ for the experiment reported in Ref. 1 using the electron density of 3.56×10^{15} electrons/m^2 is $k_F = 0.015$ Å^{-1}. The number of evanescent waves in the wide regions and in the orifice has been chosen such that the effect of adding a few more waves was miniscule. In all cases unitarity was maintained to 13 digits.

In Fig. 1 we plot the conductance as a function of the dimensionless parameter ka/π (which counts the number of channels with real momenta in the rectangle) in the range $16 < ka/\pi < 32$. The structure of the conductance reveals three interesting features. (i) There are no traces of quantization [the dominant feature for the narrow constriction $(a < b)$ geometry]. Indeed, for $a < b$ (see Glazman and Lesovick[3]), the conductance was proportional to the number of propagating waves in the narrow constriction, and the results were insensitive to the width of the wires. For each mode j in the orifice $(j = 1,2,\ldots,J)$ with wave number q_j there was a mode $m(j)$ in the wire leads whose wave number $k_{m(j)}$ was the closest one to q_j. [Actually, $m(j) = (jb/a)$.] The mode $m(j)$, which came from the left-hand side, excited the mode j and the degree of excitation was determined by the impedance matching between the two wave numbers. On the right-hand side of the orifice, this mode matching picture was repeated, this time the mode j excited the mode $m(j)$. What we had then is a transmission through a barrier of length L such that the wave number outside the barrier is $k_{m(j)}$ and in-

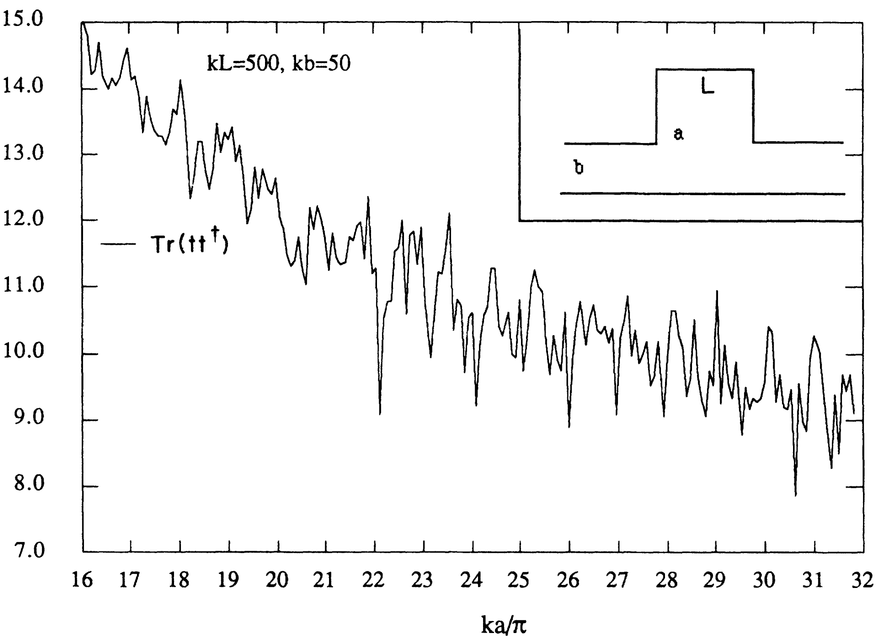

FIG. 1. Conductance (in units e^2/h) of a rectangle of length L and width a situated between two wire leads of width $b < a$ (inset) for a two-dimensional electron gas with Fermi momentum k. The conductance is plotted as a function of ka/π between 16 and 32, for $kL = 500$.

side the barrier is q_j. (Equivalently, a plane wave whose wave number is $k_{m(j)}$ penetrates through a barrier of "potential" whose height V_j is given by $2mV_j/\hbar^2 = k^2_{m(j)} - q_j^2$.) The transmission coefficient for this penetration was close to unity and the expression for the conductance just summed incoherently the contributions from all modes in the orifice. This is no longer the case for $a > b$. The number of contributing waves is now determined by the width of the wire. Increasing the width of the rectangle does not add any definite conductance, and hence quantization is absent. The dependence of the conductance on the width of the rectangle is addressed in the next two points. (ii) There is a global trend for the conductance to decay as the width of the rectangle increases, contrary to what we expect from the law of parallel addition of conductances. This can be understood intuitively. It is clear that the larger the width of the rectangle, the greater the chance of the electrons being trapped in the box. The electrons find their way out of the box after some time, and this trapping time gets longer as the ratio of the dimensions of the box and the diameter of the pipe increase. It has been argued recently[6] that electrons stand a chance of becoming trapped in a resonance state, since modes with wavelength larger than the wire diameter will prefer to stay within the box rather than be squeezed into the wire. Yet another view on the decay of the conductance is provided by the connection between decay of conductance and localization. This possibility of obtaining localized states without disorder has been substantiated recently by Entin-Wholman and Azbel.[7] They considered electronic spectra of infinite quasi-one-dimensional disorder-free systems of varying width and of low geometrical symmetry, and showed that the low-lying states are localized. The fact that the decay is not monotonic is discussed next. (iii) Figure 1 shows a rich oscillatory structure which is dominated by peaks close to integer values of ka/π. Unlike the steep increase of the conductance for $a < b$ at integer values of ka/π (the onset of the next plateau), the conductance here is roughly symmetric around the peaks, indicating a resonance phenomenon. To explain this result we recall that in one-dimensional double-barrier tunneling, the transmission coefficient peaks at the resonance energies of the double barrier, which in the limit of infinite barrier height become bound states. In the present case, the states in the rectangle are almost bound states with energy k^2 and standing-wave numbers $q_x = n\pi/L$ and $q_y = r(k^2 - q_x^2) = m\pi/a$. Thus, there exists a bound state if $ka/\pi = (m^2 + n^2a^2/L^2)^{1/2}$. The dominant peaks will be those with small n, which for a small value of a/L are close to integer m. When a/L becomes larger, the peaks are slightly shifted to the right as is verified by closer inspection of the numerical results. The smaller peaks in Fig. 1 are apparently those belonging to $n = 2,3,\ldots$. Thus, the dependence of the conductance versus ka is in this case an excellent probe for scanning the spectrum of states in the sample. The idea of relating the conductance directly to the geometrical size of the sample through its spectrum reminds us of the scaling theory of conductance.[8]

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