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Improved form of static scaling for the nonlinear magnetization in spin glasses

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An improved form of static scaling for the nonlinear susceptibility, $\chi_{\rm NL} \equiv \chi_0 - (M/H)$, is given in which the argument of the scaling function is linear in t, i.e., $\chi_{\rm NL} \sim H^{2\beta/\phi}\tilde{G}(t/H^{2/\phi})$ with $\phi = \gamma + \beta$. This allows for a linear instead of the usual logarithmic scaling plot and a closer examination of the scaling fit relative to experimental error. Application is made to several spin glasses including $Cd_{0.6}Mn_{0.4}Te$. Significantly lower T_c and larger γ are found compared to previous analyses.

Both dynamic and static critical scaling have been extensively used as supporting evidence for phase transitions in spin glasses and other random magnets. We have recently presented a new approach to dynamic critical scaling[1] in which the reduced temperature $t = (T - T_c)/T_c$ appears linearly in the argument of the scaling function, and for conventional dynamics the scaling is now expressed as[1]

$$\chi'' T \approx \omega^{\beta/z\nu} f(t/\omega^{1/z\nu}) . \tag{1}$$

This formulation allowed for a linear scaling plot wherein a more critical evaluation of the scaling fit relative to experimental error could be made, instead of the usual log-log plot which tends to conceal departures from good scaling. In addition, the presence of a clearly identifiable and accurately measured feature such as the peak of $\chi''(\omega, T)$ led to a procedure for separately determining $\beta/z\nu$, $z\nu$, and T_c independent of each other. We wish to consider here an analogous formulation of *static* scaling which again leads to a much more revealing *linear* scaling plot. As we found in the dynamic scaling with T_c and $z\nu$,[1] the data are generally shown to be consistent with a significantly lower T_c and larger γ than in previous analyses.

Static scaling of the nonlinear susceptibility, $\chi_{\rm NL} \equiv (\chi_0 - M/H)$, where χ_0 is the zero-field susceptibility, has been traditionally expressed as[2]

$$\chi_{\rm NL} \approx t^{\beta} F(H^2/t^{\phi}) \tag{2a}$$

or

$$\chi_{\rm NL} \approx H^{2\beta/(\gamma+\beta)} G(H^2/t^{\phi}) , \tag{2b}$$

where β is the order-parameter exponent and $\phi = \gamma + \beta$. T_c is the critical temperature and γ is the critical exponent of the leading term $\chi_2 H^2$ in the expansion of $\chi_{\rm NL}$ in powers of H^2, i.e., $\chi_2 \sim t^{-\gamma}$. If for small H an experimental region can be accessed where $\chi_{\rm NL} \sim H^2$, the critical exponent γ may be determined by the selection of T_c to give the best straight-line fit on a plot of $\ln\chi_{\rm NL}$ vs $\ln t$, i.e., a two-parameter fit. However, this usually involves the greater inaccuracy of working at small $\chi_{\rm NL}$ and further from T_c where corrections to scaling may be important. Of course to determine β as well, the next higher-order term is needed and ultimately one has recourse to the full scaling plots with Eqs. (2a) and (2b).[3] γ, β, and T_c are then selected to give the best collapse of all the data onto a single curve in a plot of $\chi_{\rm NL}/t^{\beta}$ or $\chi_{\rm NL}/H^{2\beta/(\gamma+\beta)}$ vs $H^2/t^{\gamma+\beta}$. As $t \to 0$, the abscissa extends over very many decades so that it is plotted on a log scale. In addition, the log scale for $\chi_{\rm NL}/H^{2\beta/(\gamma+\beta)}$ compresses the larger values of $\chi_{\rm NL}$ where the data are most accurate. Both effects conspire to hide departures from good scaling which may easily exceed experimental error by significant amounts.

To avoid these pitfalls of the log plot, we follow our recent suggestion for dynamic scaling and simply recast the argument of the scaling function for $\chi_{\rm NL}$ to be linear in t, i.e.,

$$\chi_{\rm NL} \sim H^{2\beta/(\gamma+\beta)} \tilde{G}(t/H^{2/(\gamma+\beta)}) , \tag{3}$$

where $\tilde{G}$ is a scaling function.[4] Since typically $(\gamma+\beta) \gtrsim 4$ and $\beta \simeq 0.6$, even for H varying over a factor of 10^2, a linear scaling plot $\chi_{\rm NL}/H^{2\beta/(\gamma+\beta)}$ vs $t/H^{2/(\gamma+\beta)}$ is feasible and qualitatively resembles $\chi_{\rm NL}$ vs T itself, allowing a more critical examination of any departures from good scaling relative to assumed experimental error. If $\chi_{\rm NL}$ had some unique and accurately measurable identifying feature[5] such as the peak in $\chi''(\omega, T)$, one could follow our procedure for dynamic scaling[1] and separately and independently determine $2\beta/(\gamma+\beta)$, $2/(\gamma+\beta)$, and T_c [note the correspondence in Eqs. (1) and (3) of $(\gamma+\beta)/2 \to z\nu$ and $H \to \omega$]. However, even though the absence of such a feature prevents this, so that the linear scaling plot still involves a three-parameter fit, its great advantage over the log plot remains in that it provides a much more revealing test of scaling with the most accurate portion of the data. This will now be illustrated with a few examples.

First, we consider $Cd_{0.6}Mn_{0.4}Te$. A recent suggestion[6] that the critical behavior of this system may be more akin to a random field, rather than spin-glass transition, was based on the very large $z\nu \simeq 13$ found in a scaling with conventional dynamics of $\Delta\chi' = [\chi_{\rm eq} - \chi'(\omega, T)]/\chi_{\rm eq}$ on a log-log plot, the equally good scaling on a log plot using activated dynamics, and the presence of significant short-range type-III antiferromagnetic order, especially for higher Mn concentration (see Ref. 6 for further references and details). However, more recently,[1] the much more accurate analysis of $\chi''(\omega, T)$ proceeding from Eq. (1) showed a clear preference for conventional over activated dynamics over most of the critical region. In Ref. 1, in addition to addressing the question of conventional versus activated dynamics it was also shown that there was a general tendency in previous work to overestimate T_c and underestimate $z\nu$ in dynamic scaling. We now demonstrate a similar situation in the static scaling of a frequent overestimate of T_c and underestimate of γ.

χ_{NL} has recently been measured by Mauger, Ferré, and Beauvillain[7] in $Cd_{0.6}Mn_{0.4}Te$ and using a log-log scaling plot the following parameters were deduced: $T_c = 12.37 \pm 0.05$, $\gamma = 3.3 \pm 0.3$, and $\beta = 0.9 \pm 0.2$. Figure 1(a) is a scaling plot according to Eq. (2) (on the more revealing linear scale) of the χ_{NL} data presented in Fig. 1 of Ref. 7 with the parameters just cited. In Fig. 1(b) we have plotted the very same data with a rather different selection of parameters, i.e., $T_c = 12.14$, $\gamma = 4.4$, and $\beta = 0.6$. Clearly, the latter selection gives as good a scaling fit as in Fig. 1(a), indicating insufficient exploration of the parameter space in estimating the error bars reported in Ref. 7. It is unfortunate that the χ_{NL} data in Fig. 1 of Ref. 7 did not cover a wider range of field and get closer to T_c so that from the linear plot alone one could distinguish between two choices. We were guided in our selection of β and T_c by the values $\beta = 0.59 \pm 0.05$, $T_c = 12.13 \pm 0.1$, and $z\nu = 11.4 \pm 1$, which we determined from our new more accurate approach to dynamic scaling.[1] While Ref. 7 also claimed agreement of their value of β with dynamic scaling, the quoted work[8] covered only slightly more than three decades of frequency compared to five in Ref. 1, and even on a log plot a less accurate fit to good dynamic scaling may be detected compared to Ref. 1. With regard to T_c, Mauger *et al.*[7] claim agreement with the value found from the temperature at which the slow relaxation of the field-cooled magnetization changes sign with step cooling of 0.05 K. However, this procedure overestimates T_c in the same way as a determination of T_c from the peak in the so-called dc susceptibility, "χ_{dc}." Due to the inordinately long relaxation time near T_c, true equilibrium is impossible to achieve on laboratory time scales and the peak in χ_{dc} keeps moving to lower temperature as the temperature is swept more slowly.[9] The change of sign referred to is clearly exactly what one would expect on passing through the peak of χ_{dc}. Moreover, even if there is no peak but only a break in χ_{dc} this same change in sign is likely to occur near the break as well because of lack of equilibrium. Thus leaving aside questions about behavior *very* close to T_c, we believe that at least within the framework of conventional dynamic scaling the appropriate pa-

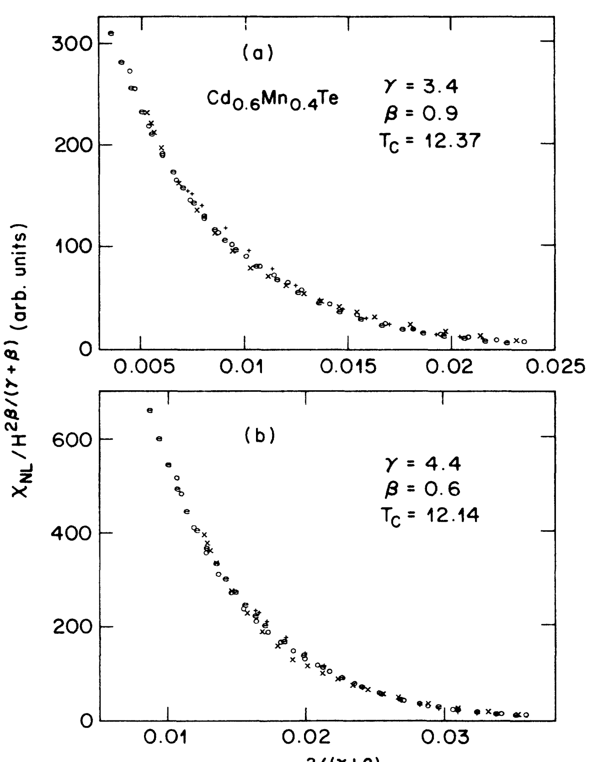

FIG. 1. (a) Linear scaling plot of the dc nonlinear susceptibility, $\chi_{NL} = \chi_0 - M/H$ data for $Cd_{0.6}Mn_{0.4}Te$ from Fig. 1 of Ref. 7. H is in units of G. (b) Equally good scaling fit of same data with more reliable parameters T_c and β determined from dynamic scaling of $\chi''(\omega,T)$ in Ref. 1. Note larger γ and lower T_c than in (a). Symbols +, x, o, and e correspond, respectively, to $H = 10$, 18, 30, and 55 G.

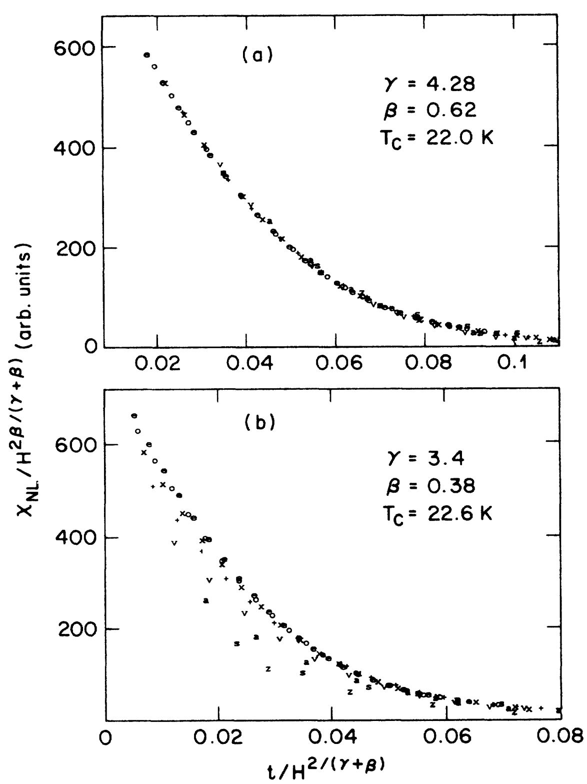

FIG. 2. Linear scaling plots of the nonlinear susceptibility $\chi_{NL} = \chi_0 - M/H$ in $(Fe_{0.15}Ni_{0.85})_{75}P_{16}B_6Al_3$. H is in units of G. Data scaled is that above 23 K from Fig. 4 of Svedlindh *et al.* (Ref. 11). Symbols z, s, a, v, +, x, o, and e correspond, respectively, to $H = 0.4$, 0.6, 1, 2, 4, 6, 8, and 10 G. (a) Scaling plot using β and T_c as determined from dynamic scaling in Ref. 1 of $\chi''(\omega,T)$ data from Ref. 12. (b) Linear scaling plot using parameters γ, β, and T_c given in Ref. 11 as determined from $\chi_{NL,ac} = \chi_0 - dM/dH$. Clear departures from good scaling are seen.

rameters for $Cd_{0.6}Mn_{0.4}Te$ are $z\nu = 11.3 \pm 1$, $\beta = 0.59 \pm 0.05$, $\gamma = 4.4 \pm 0.8$, and $T_c = 12.14 \pm 0.15$. Levy[10] finds $\gamma = 4.8$ in $Cd_{0.75}Mn_{0.25}Te$. In any quoting of T_c it is important that it be specified relative to, say, the peak in $\chi'(\omega)$ for some ω in order to allow comparison of our particular sample and temperature scale with other work on this compound. We found[1] the peak in χ' (97.5 Hz) to be at $T = 13.19$ K, for example, compared with 13.37 K in the sample and temperature scale used in Ref. 6. However, this has no bearing on the critical exponents.

As a second illustration of the utility of the linear scaling plot using Eq. (3) we reanalyze the very detailed measurements of χ_{NL} in the amorphous metallic spin glass $(Fe_{0.15}Ni_{0.85})_{75}B_{16}P_6Al_3$ by Svedlindh *et al.*[11] They measured $\chi_{NL,ac}(T,H) = \chi_0 - dM/dH$ with great accuracy by using a small ac modulation (0.01 G) superimposed on the much larger dc field and using static scaling found $\gamma = 3.4$, $\beta = 0.38$, and $T_c = 22.6$ K. These values of β and T_c contrast with $\beta = 0.62^{+0.03}_{-0.05}$ and $T_c = 22.0^{+0.2}_{-0.1}$ found in our reanalysis[1] of $\chi''(\omega,T)$ data of Svedlindh *et al.*[12] on the same material using our new approach to dynamic scaling of $\chi''(\omega,T)$ mentioned above. However, Svedlindh *et al.* in Ref. 11 also measured the dc susceptibility $\chi_{NL} = \chi_0 - (M_{FC}/H)$, where M_{FC} is the field-cooled susceptibility, but did not scale this data. While, of course, one needs to be concerned with irreversible (nonequilibrium) effects close to T_c as mentioned in Ref. 11, examination of their Figs. 1 and 4 would indicate good equilibrium for the χ_{NL} data in Fig. 4 above 23 K. We have, therefore, scaled this data for $T > 23$ K guided by the β and T_c cited above as determined from our dynamic scaling of $\chi''(\omega,T)$ and the result is shown in Fig. 2(a). The scaling fit with $\gamma = 4.3^{+0.3}_{-0.5}$, $\beta = 0.62 \pm 0.05$, and $T_c = 22.0 \pm 0.2$ is excellent and contrasts with the scaling of the same χ_{NL} data shown in Fig. 2(b) using the parameters from the analysis of $\chi_{NL,ac}$ quoted in Ref. 11 above. Thus again there seems to be an overestimate of T_c and an underestimate of γ. The departure from good scaling in Fig. 2(b) would appear far less severe on the usual log plot.

We finally address the discrepancy cited above of the parameters from dynamic scaling[1] of $\chi''(\omega,T)$ and static scaling of χ_{NL} on the one hand, as compared with those from $\chi_{NL,ac}$ on the other. In the scaling of the $\chi_{NL,ac}$ data, temperatures far above T_c were used ($T \sim 34$ K), i.e., $t \gtrsim 0.5$, where corrections to scaling are likely to be significant. If the data are restricted to $23.7 \leq T \leq 27.7$ K one gets a comparable scaling fit of $\chi_{NL,ac}$ with $\gamma = 4.18$, $\beta = 0.45$, and $T_c = 22.2$ as with the $\gamma = 3.4$, $\beta = 0.38$, and $T_c = 22.6$ found by Svedlindh *et al.*[11] Finally, it is worth remarking that one can get an equally good fit with $\gamma = 4.28$, $\beta = 0.62$, and $T_c = 22.0$ [our choice from χ_{NL} and $\chi''(\omega,T)$] by omitting the 23.7 K $\chi_{NL,ac}$ data, although we have no other reason to advocate this.

In summary, we have presented a modification of the usual static scaling which allows for a linear scaling plot and more direct examination of any departure from scaling relative to experimental error and have shown its utility with several examples. In addition, we suggest a past general tendency to overestimate T_c and underestimate γ.

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[3] One may achieve greater accuracy in measuring χ_{NL} by observing the 3ω signal in the ac response to an ω drive signal [see L. Lévy, Phys. Rev. B **38**, 4963 (1988)], but for large γ the higher-order terms enter very quickly.

[4] The same argument of the scaling function $t/H^{2/(\gamma+\beta)}$ was used by Montenegro *et al.*, in *Proceedings of the International Conference on Magnetism, Paris, France, 1988*, edited by D. Givord (Editions de Physique, Paris, 1988), p. 1007, but unfortunately they failed to exploit its potential by again using a log-log plot with its attendant inadequacies. For example, their scaling gives the appearance that $\chi_{NL}/H^{2\beta/(\gamma+B)(\tau+\beta)}$ flattens as $t \rightarrow 0$ whereas this is primarily an effect of the log-log plot.

[5] While some may have reported peaks or breaks in χ_{NL} vs T, these occur so close to T_c that they are likely to be due to nonequilibrium effects at T_c and the true equilibrium behavior at T_c and near T_c at nonzero H, and thus the behavior of the scaling function $\tilde{G}(x)$ for $x \lesssim 0$, is uncertain.

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[7] A. Mauger, J. Ferré, and P. Beauvillain, Phys. Rev. B **40**, 862 (1989).

[8] Y. Zhou *et al.*, Phys. Rev. B **40**, 8111 (1989).

[9] A. Mauger, J. Ferré, M. Ayadi, and P. Nordblad, Phys. Rev. B **37**, 9022 (1988); L. Sandlund *et al.*, *ibid.* **40**, 869 (1989); see Fig. 1(b).

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